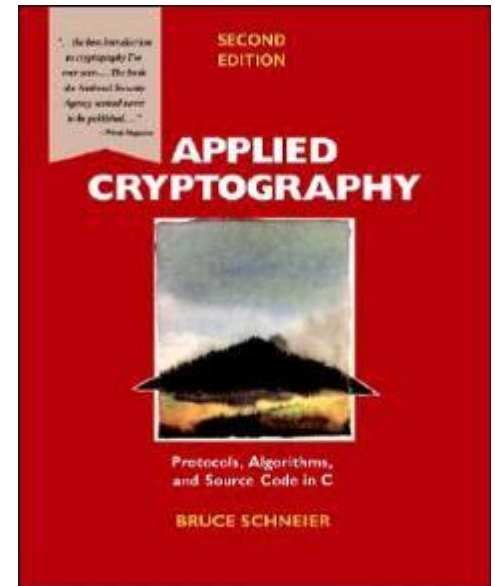
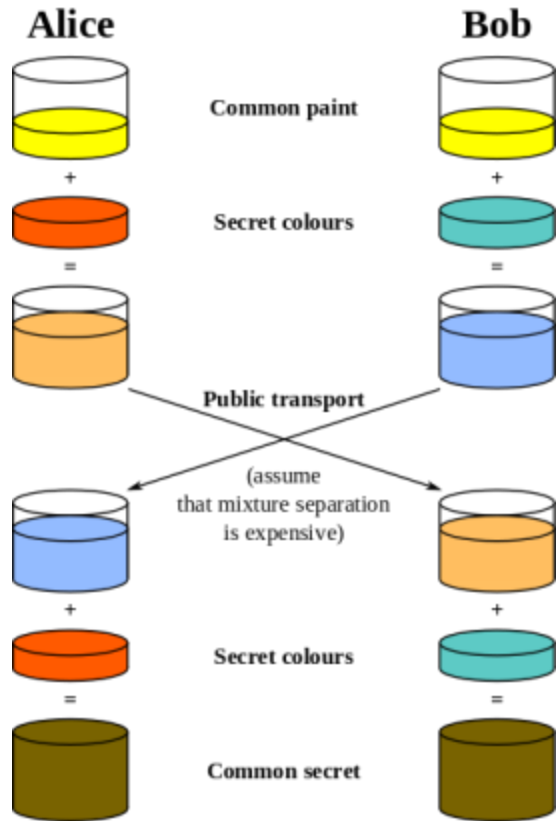


Modern cryptography



Overview

- Modern cryptography
 - Symmetric cryptography
 - DES
 - 3DES
 - AES
 - Asymmetric cryptography
 - Diffie-Hellman key exchange

Modern cryptography

- Moving into computer age

- Not limited to physical engineering constraints
 - 100's of rotors instead of 3, changing in complex ways
 - Much faster
 - Scrambling at the bit level

- Symmetric encryption (what we've seen thus far)

- Encrypting message M with key K : $E_K(M) = C$
- Decrypting ciphertext C with key K : $D_K(C) = M$
- $D_K(E_K(M)) = M$
- Stream cipher: operates one bit/byte at-a-time
- Block cipher: operates on a group of bits/bytes

Bit encryption / decryption example

Message : HELLO

Sender

Binary : 1001000 1000101 1001100 1001100 1001111
KEY = DAVID : 1000100 1000001 1010110 1001001 1000100
Encrypted (XOR) : 0001100 0000100 0011010 0000101 0001011

Receiver

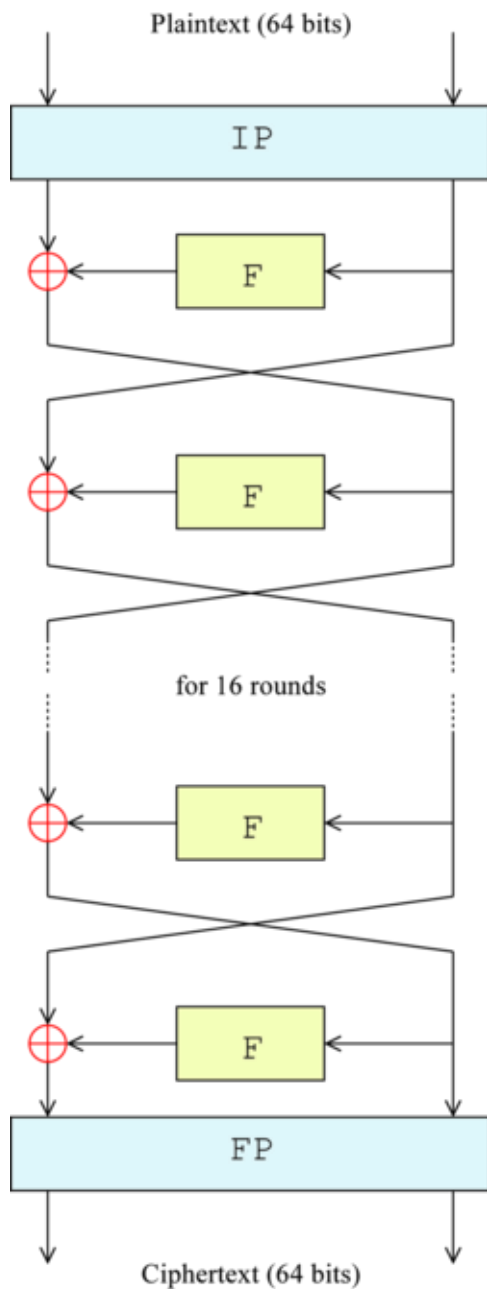
Encrypted : 0001100 0000100 0011010 0000101 0001011
KEY = DAVID : 1000100 1000001 1010110 1001001 1000100
Decrypted (XOR) : 1001000 1000101 1001100 1001100 1001111

DES

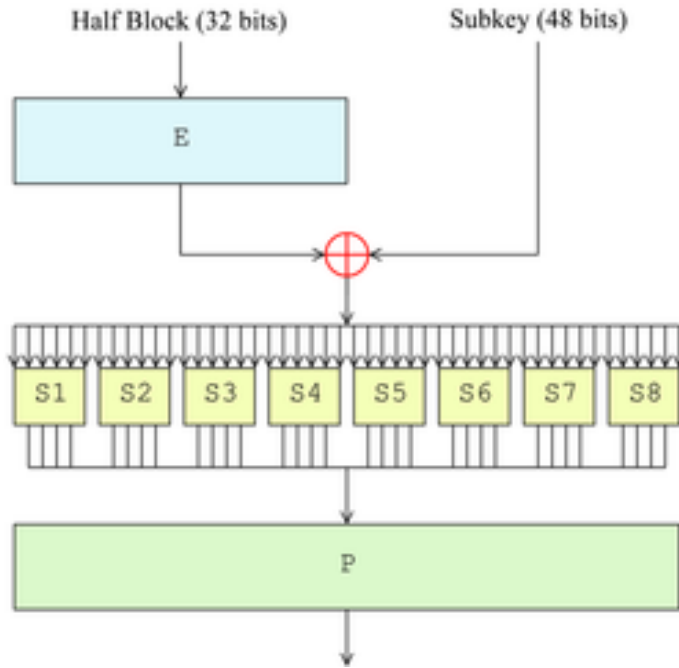
- Data Encryption Standard (DES)
 - NIST wanted a government standard
 - Based on IBM's Lucifer cipher
 - 16 round Feistel network
 - Security provided by a key
 - With "cooperation" from NSA:
 - Improved S-boxes
 - Reduced key length to 56 bits
 - 1976 approved as a standard
 - Same hardware/software can encrypt/decrypt

"DES did more to galvanize the field of cryptanalysis than anything else. Now there was an algorithm to study: one that the NSA said was secure"

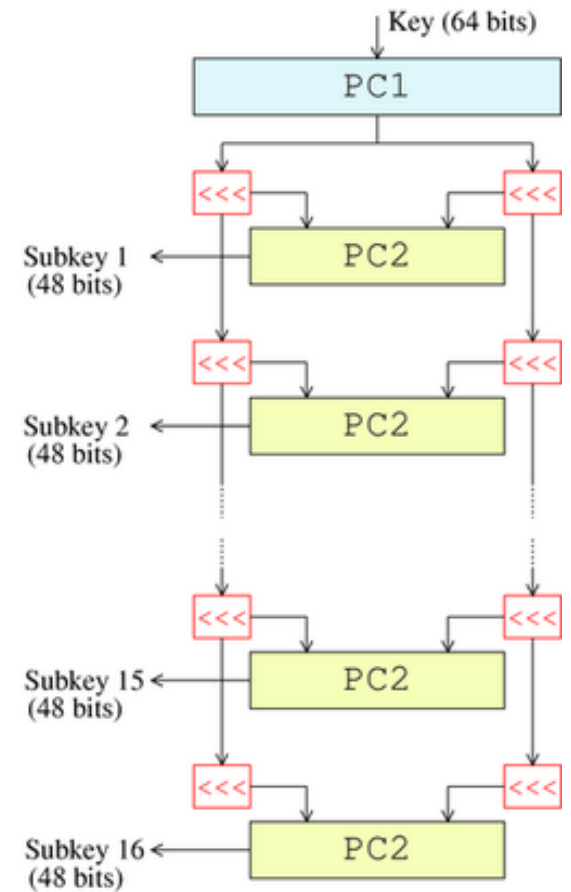
-Bruce Schneier



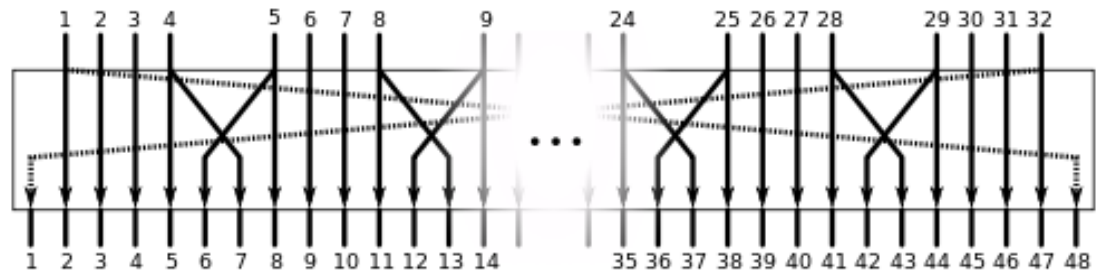
Overall structure



The Feistel function (F-function)



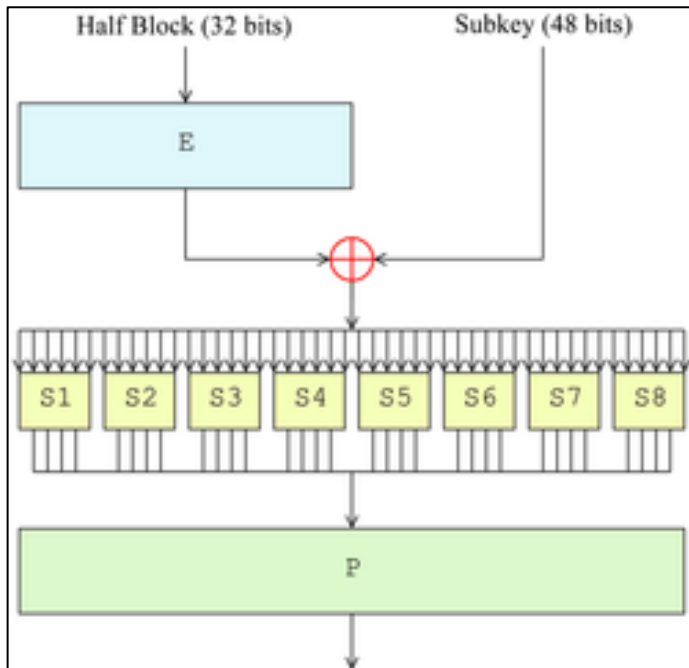
Key schedule



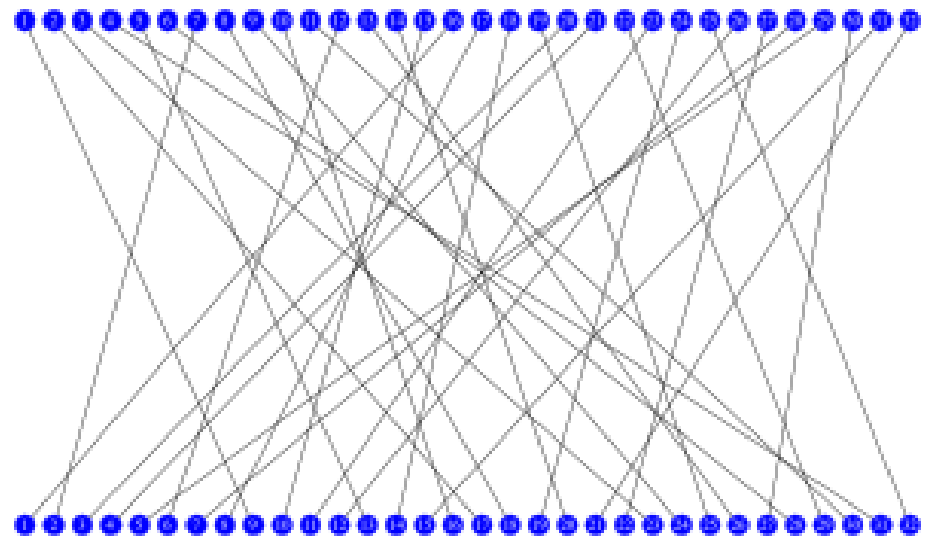
E-box, expansion permutation, 32 -> 48 bits

S ₅																
	x0000x	x0001x	x0010x	x0011x	x0100x	x0101x	x0110x	x0111x	x1000x	x1001x	x1010x	x1011x	x1100x	x1101x	x1110x	x1111x
0yyyy0	2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
0yyyy1	14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
1yyyy0	4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
1yyyy1	11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3

S-box #5, 6 bits -> 4 bits, e.g. 011011 -> 1001 (9)



The Feistel function (F-function)



P-box, straight permutation, 32 bits -> 32 bits

Breaking DES

- Key size, 72 quadrillion

- $2^{56} = 72,057,594,037,927,936$

- DES Challenges (brute force)

- Sponsored by RSA Security

- Challenge I: 96 days, Internet users

- Challenge II: 41 days, distributed.net

- Challenge II-2: 56 hours, EFF deep crack

- \$250,000 to develop, \$10,000 prize

- 90 billion keys/second

- Challenge III: 22 hours, EFF+distributed.net

- 2008, FPGA, 1 day



Stronger symmetric schemes

- Triple DES (3DES)

- Ciphertext: $E_{K3}(D_{K2}(E_{K1}(\text{plaintext})))$

- Plaintext: $D_{K1}(E_{K2}(D_{K3}(\text{ciphertext})))$

- Keying option 1: $K1 \neq K2 \neq K3$

- 168 bits = 56 bits x 3

- Advantages:

- Uses DES, most analyzed encryption algorithm

- No known effective attack (besides brute force)

- Disadvantages:


- Slow in software, DES designed for 1970's hardware

- Small block size of 64-bits

AES

- Advanced Encryption Standard (AES)
 - 2001 new NIST standard, Rijndael
 - Symmetric block cipher
 - Key lengths of 128, 192, and 256 bits
 - Approved by NSA for top secret information

Mix Columns is the hardest. I treat each column as a polynomial. I then use our new multiply method to multiply it by a specially crafted polynomial and then take the remainder after dividing by x^4+1 . This all simplifies to a matrix multiply:



$$b(x) = c(x) \cdot a(x) \pmod{x^4+1}$$

$$= (03x^3 + 01x^2 + 01x + 02) \cdot (a_3x^3 + a_2x^2 + a_1x + a_0) \pmod{x^4+1}$$

special polynomial the column

$$= \frac{03a_3x^6 + 03a_2x^5 + 03a_1x^4 + 03a_0x^3 + 01a_3x^5 + 01a_2x^4 + 01a_1x^3 + 01a_0x^2 + 01a_3x^4 + 01a_2x^3 + 01a_1x^2 + 01a_0x + 02a_3x^4 + 02a_2x^3 + 02a_1x^2 + 02a_0x + 02a_0}{x^4+1}$$

$$\oplus \frac{03a_3x^2 + 03a_2x^2}{3a_2x^3 + 3a_1x^2 + 3a_0x^2 + a_3x^2 + a_2x^2 + a_1x^2 + a_0x^2 + a_3x^2 + a_2x^2 + a_1x^2 + a_0x^2 + 2a_3x^3 + 2a_2x^2 + 2a_1x^2 + 2a_0x^2}$$

$$\oplus \frac{3a_3x^2 + a_3x^2 + 3a_2x^2 + a_2x^2}{3a_3x^2 + 3a_2x^2 + a_3x^2 + a_2x^2 + a_1x^2 + a_0x^2 + a_3x^2 + a_2x^2 + a_1x^2 + a_0x^2 + 2a_3x^2 + 2a_2x^2 + 2a_1x^2 + 2a_0x^2}$$

$$\oplus \frac{(3a_3 + a_2 + a_1)x^2 + (3a_2 + a_1 + a_0)x^2}{(2a_3 + a_2 + a_1 + 3a_0)x^3 + (3a_2 + 2a_1 + a_0)x^2 + (a_3 + 3a_2 + 2a_1 + a_0)x + (a_3 + a_2 + 3a_1 + 2a_0)}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix}$$

Plaintext in 4x4 grid

AES Crib Sheet (Handy for memorizing)

Initial Round

General Math

1.9B = AES Polynomial: $m(x)$

Fast Multiply

$x^2 + x^3 + x^3 + x^3 + 1$

$x \cdot ax^3 = (a \ll 1) \oplus (a_2 = 1) ? 1B : 00$

$\log(x \cdot y) = \log(x) + \log(y)$

Use $(x+1) = 03$ for log base

S-Box (SRD)

$SRD[a] = f(g(a))$

$g(a) = a^{-1} \pmod{m(x)}$

5 is and 3 is $[0110\ 0011]^T$

Key Expansion: Round Constants

Other Columns:

Prev Col @ Col from Previous round Key

Intermediate Rounds

#	Key
9	128
11	192
13	256

Final Round

Ciphertext

Mix Columns:

2113	a_3
2113	a_2
3211	a_1
1321	a_0
1132	a_0

Inverse Mix

EBD9

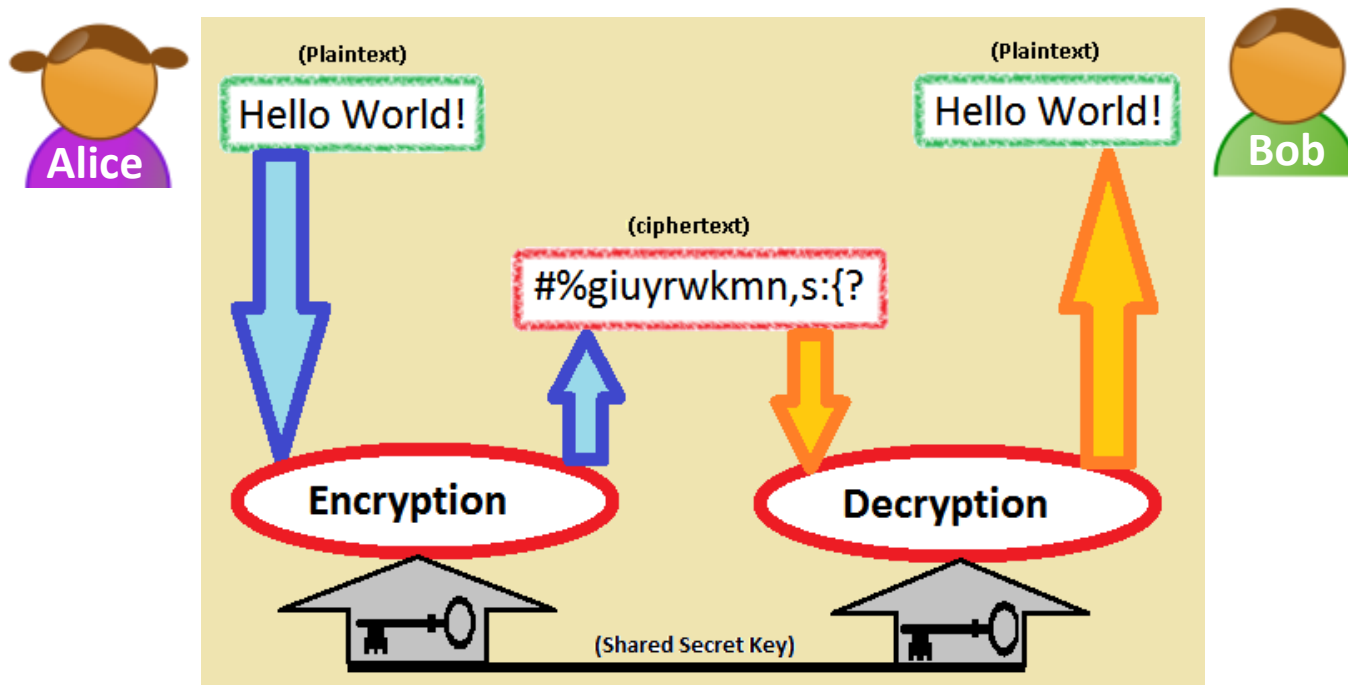
E B D 9	a_3
9 E B 0	a_2
0 9 E 0	a_1
B 0 9 B	a_0

Attack types

- Ciphertext only
 - Ciphertext of message(s), plaintext unknown
- Known plaintext
 - Ciphertext plus corresponding plaintext
- Chosen plaintext
 - Ciphertext plus plaintext of your own choosing
 - Adaptive chosen plaintext
 - Modify plaintext based on previous decryptions
- Rubber hose
- ...

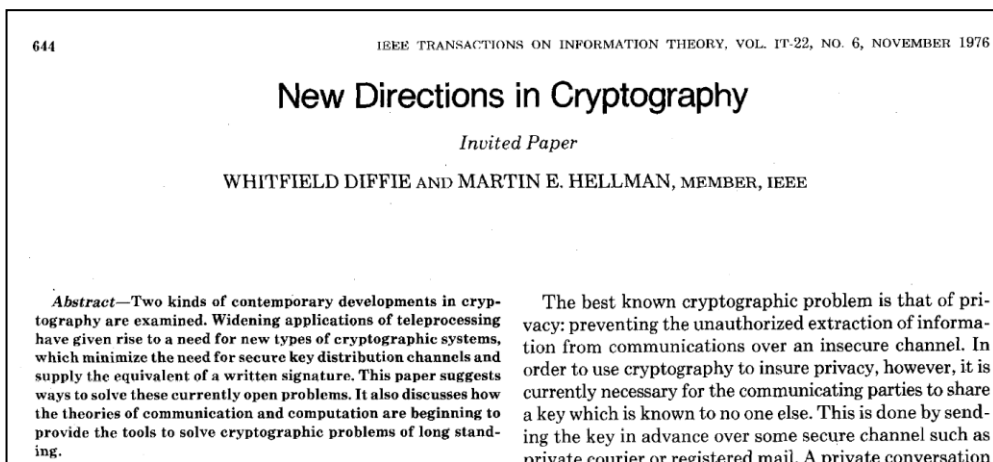
Key exchange

- Thus far: **symmetric encryption**
 - Alice and Bob need to have shared secret
 - But how do you distribute?
 - Doesn't scale



Diffie-Hellman

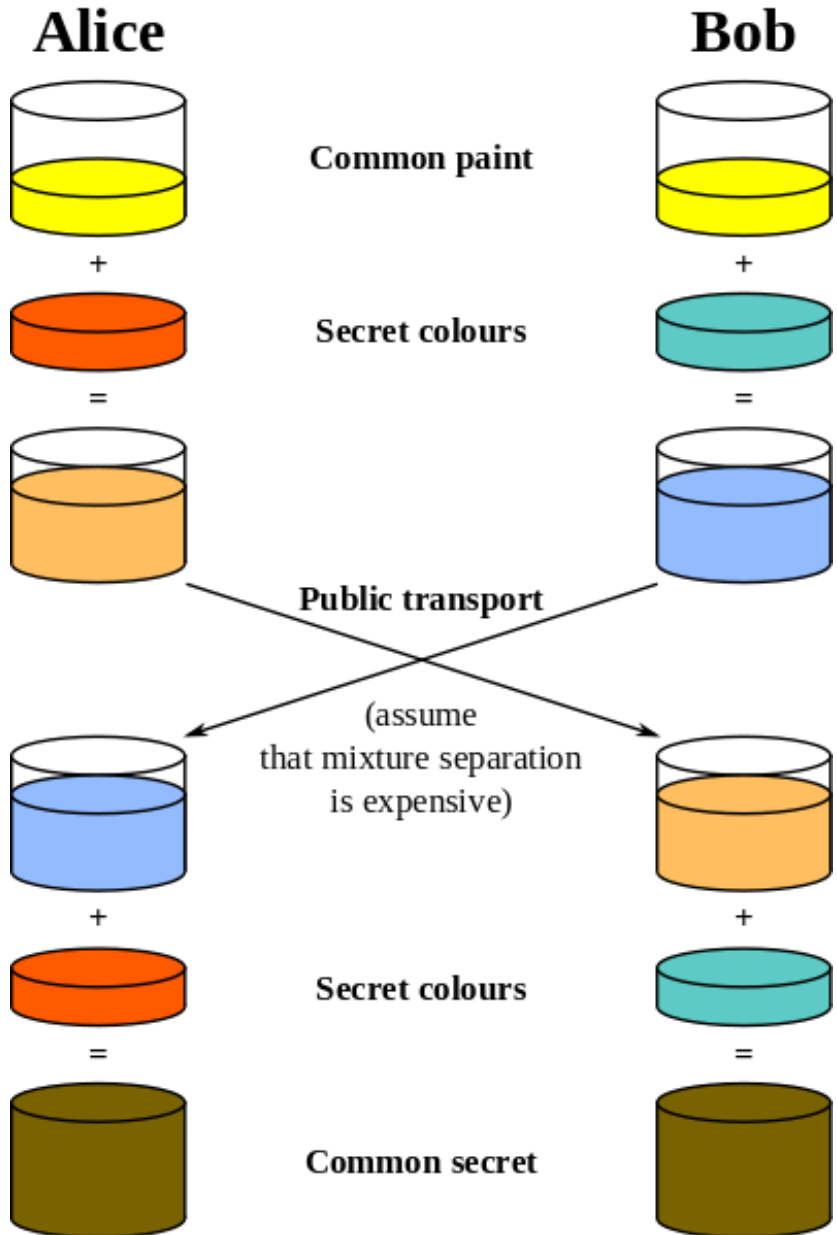
- Diffie-Hellman (DH) key exchange
 - 1976, Whitfield Diffie & Martin Hellman
 - Alice and Bob **agree on a private secret:**
 - On a public channel <http://www.youtube.com/watch?v=3QnD2c4Xovk>
 - Where Eve hears all the traffic
 - Only Alice and Bob end up knowing the secret
 - Relies on **one-way function**
 - Function must be easy to do, but difficult to undo



Whitfield Diffie



Martin Hellman



Alice	Bob
Alice and Bob agree publicly on values for Y and P for the one-way function: $Y^x \pmod{P}$, e.g. $Y=7, P=11$	
Alice chooses secret number: $A = 3$	Bob chooses secret number: $B = 6$
$\alpha = 7^A \pmod{11}$ $= 7^3 \pmod{11}$ $= 343 \pmod{11}$ $= 2$	$\beta = 7^B \pmod{11}$ $= 7^6 \pmod{11}$ $= 117649 \pmod{11}$ $= 4$
Sends $\alpha = 2$ to Bob	Sends $\beta = 4$ to Alice
Using Bob's result: $\beta^A \pmod{11}$ $4^3 \pmod{11} = 9$	Using Alice's result $\alpha^B \pmod{11}$ $2^6 \pmod{11} = 9$
$7^{B \cdot A} \pmod{11}$	$7^{A \cdot B} \pmod{11}$

Public key cryptography

- Diffie-Helman key exchange
 - Both parties had to be around to negotiate secret
- Symmetric encryption
 - Encrypting message M with key K : $E_K(M) = C$
 - Decrypting ciphertext C with key K : $D_K(C) = M$
- Asymmetric encryption
 - 1975, Diffie conceives of idea
 - Users have a **private key** and a **public key**
 - Alice encrypts plaintext with Bob's public key
 - Only Bob can (tractably) decrypt using his private key
 - Special one-way function
 - Hard to reverse unless you know something special

Summary

- Modern cryptography
 - Computer-based symmetric ciphers
 - DES, 3DES, AES
 - Rise of asymmetric cryptography
 - Diffie-Hellman