



Linear Algebra Review

# CSCI 447/547 MACHINE LEARNING

# Outline

- Matrices
- Determinants
- Eigen Something Something
- <https://betterexplained.com/articles/linear-algebra-guide/>
- What is “Linear”?
- Inputs and Operations
- Operations as Inputs

# What is Linear?

- An operation is linear if scaling the inputs scales the output, and adding the inputs adds the outputs

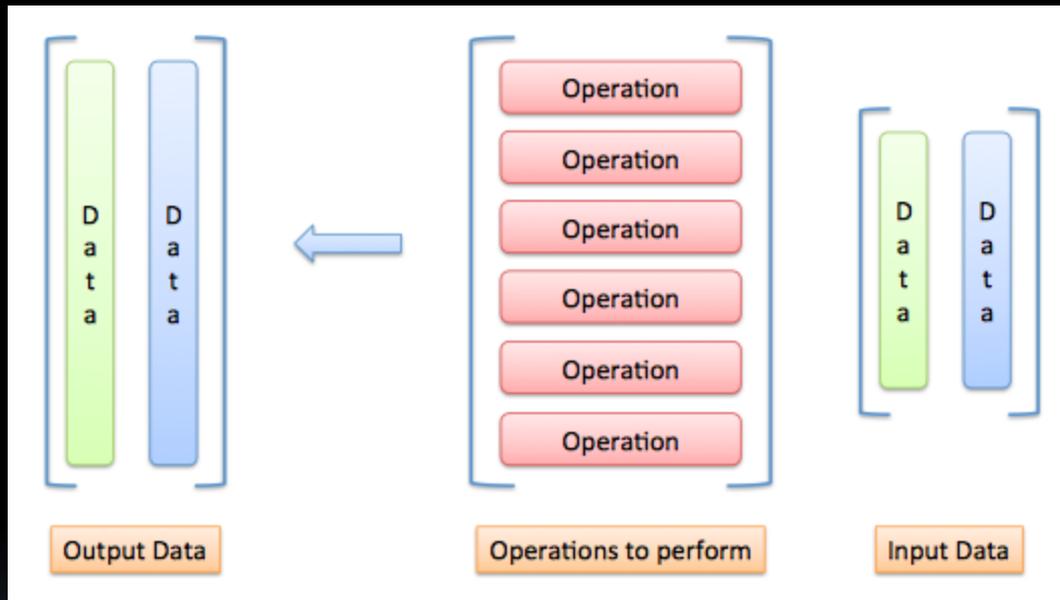
$$F(ax) = a \cdot F(x)$$
$$F(x + y) = F(x) + F(y)$$

- Are the following linear?
  - $F(x) = x + 3$
  - $F(x) = 1/3x$
  - $F(x, y, z) = 1/3x + 1/3 y + 1/3 x$

# Inputs and Operations

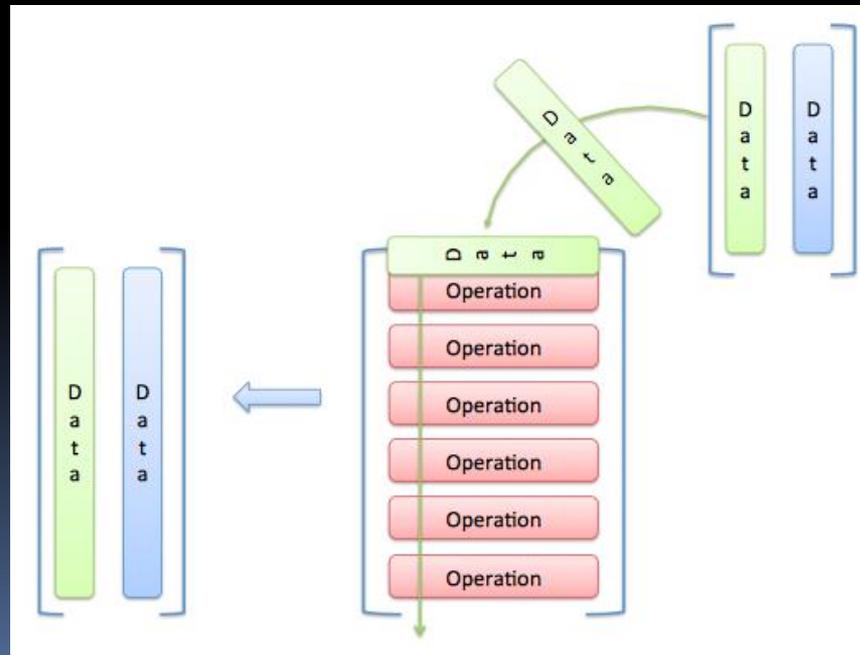
- We have inputs
  - Presumably our input data instances
- We want to relate these inputs to some outputs
  - Determine operations from input to output
- Organize inputs as a series of (column) vectors
- Organize operations as a matrix – each row is an operation
- Multiply operations and inputs to get outputs
- Output is automatically a matrix

# Inputs and Operations



# Inputs and Operations

- Imagine “pouring” the input data through the operation matrix
  - As it passes through each operation it produces an output



# Input and Operations

- If

- $F(x, y, z) = 3x + 4y + 5z$

- $G(a, b, c) = 3a$

- Input is: 
$$\begin{bmatrix} a & x \\ b & y \\ c & z \end{bmatrix}$$

- Operation is: 
$$\begin{bmatrix} 3 & 4 & 5 \\ 3 & 0 & 0 \end{bmatrix}$$

- Output is: 
$$\begin{bmatrix} 3a + 4b + 5c & 3x + 4y + 5z \\ 3a & 3x \end{bmatrix}$$

# Picky Stuff

- Inner dimensions of matrices in multiplication must match
- Result will be the size of the outer dimensions

$$[m \times n] [p \times q] = [m \times q]$$

$$[2 \times 3] [3 \times 2] = [2 \times 2]$$

# Examples

- Assuming three inputs...

- Adder operation:

$$[1 \ 1 \ 1]$$

- Averager:

$$[1/3 \ 1/3 \ 1/3]$$

- Extract first input only:

$$[1 \ 0 \ 0]$$

- Extract second input only:

$$[0 \ 1 \ 0]$$

- Extract third input only:

$$[0 \ 0 \ 1]$$

- Looking a lot like an identity matrix

# Operations as Input to Operations

- OK... maybe applies more to my Graphics class, but...
  - We have input data that represents a set of points that represent an objects vertices
  - Want to scale the object, then rotate it, then translate it
  - Instead of applying three matrices to each point, multiply the three together then multiply each point
  - Caveat: Order matters!

# Eigen Something Something

- Eigenvectors don't change direction when run through a transformation (operation) matrix
  - They may change size, though, and this size change is the eigenvalue
- <https://en.wikipedia.org/wiki/File:Eigenvectors.gif>

# Summary

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