

Theory of Computation, CSCI 438 spring 2022

More examples proving that languages are not regular, Feb. 4

1. Prove that $L = \{ww^R \mid w \in \{a,b\}^*\}$ is not regular.
(Note that the w^R is string w reversed.)

2. Exercise 1.29 b (page 88)

Use the pumping lemma to show that the following language is not regular.

$$A_2 = \{www \mid w \in \{a, b\}^*\}$$

3. Exercise 1.30 Describe the error in the following “proof” that 0^*1^* is not a regular language. (An error must exist because 0^*1^* is regular.) The proof is by contradiction. Assume that 0^*1^* is regular. Let p be the pumping length for 0^*1^* given by the pumping lemma. Choose s to be the string 0^p1^p . You know that s is a member of 0^*1^* , but Example 1.73, page 80, shows that s cannot be pumped. Thus you have a contradiction. So 0^*1^* is not regular.

EXAMPLE 1.73

Let B be the language $\{0^n1^n \mid n \geq 0\}$. We use the pumping lemma to prove that B is not regular. The proof is by contradiction.

Assume to the contrary that B is regular. Let p be the pumping length given by the pumping lemma. Choose s to be the string 0^p1^p . Because s is a member of B and s has length more than p , the pumping lemma guarantees that s can be split into three pieces, $s = xyz$, where for any $i \geq 0$ the string xy^iz is in B . We consider three cases to show that this result is impossible.

1. The string y consists only of 0s. In this case, the string $xyyz$ has more 0s than 1s and so is not a member of B , violating condition 1 of the pumping lemma. This case is a contradiction.
2. The string y consists only of 1s. This case also gives a contradiction.
3. The string y consists of both 0s and 1s. In this case, the string $xyyz$ may have the same number of 0s and 1s, but they will be out of order with some 1s before 0s. Hence it is not a member of B , which is a contradiction.

Thus a contradiction is unavoidable if we make the assumption that B is regular, so B is not regular. Note that we can simplify this argument by applying condition 3 of the pumping lemma to eliminate cases 2 and 3.

In this example, finding the string s was easy because any string in B of length p or more would work. In the next two examples, some choices for s do not work so additional care is required. □