Theory of Computation, CSCI 438 spring 2022
Regular expressions, pg. 63-66, Jan. 26

Exercise 1.18 (this refers to 1.6) b, f, g, k, m, & n and 1.20 a, c & f, 1.31 (page 88)

1.18 Create regular expressions for the following. The alphabet is {0, 1}.

b. {w | w contains at least three 1s}

$0^*10^*10^*1\ (0 \cup 1)^*$

f. {w | w doesn't contain the substring 110}

$0^* \cup 1^* \cup 0^*1^* \cup (0^*(10)^*)^*1^*$
Alternatively, $(0^*(10)^*)^*1^*$
Alternatively, $(0 \cup (10))^*1^*$

g. {w | the length of w is at most 5}

$\varepsilon \cup (0 \cup 1) \cup (0 \cup 1)(0 \cup 1) \cup (0 \cup 1)(0 \cup 1)(0 \cup 1) \cup$
$(0 \cup 1)(0 \cup 1)(0 \cup 1)(0 \cup 1) \cup (0 \cup 1)(0 \cup 1)(0 \cup 1)(0 \cup 1)(0 \cup 1)$

Alternatively,
$(\varepsilon \cup 0 \cup 1) \cup (\varepsilon \cup 0 \cup 1) \cup (\varepsilon \cup 0 \cup 1) \cup (\varepsilon \cup 0 \cup 1) \cup (\varepsilon \cup 0 \cup 1)$

k. {ε, 0}

$\varepsilon \cup 0$

m. The empty set

Φ

n. All strings except the empty string

$(0 \cup 1)(0 \cup 1)^*$
Alternatively, $\Sigma\Sigma^*$

1.20 Give two strings which are members and two string that are not members. The alphabet is {a, b}.

a. a*b*

   Are members: ε, a
   Not members: aba, ba

b. a* ∪ b*

   Are members: ε, a
   Not members: ab, ba

f. aba ∪ bab

   Are members: aba, bab
   Not members: ε, a

1.31 For any string $w=w_1w_2\ldots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n\ldots w_2w_1$. For any language A, let $A^R=\{w^R \mid w\in A\}$. Show that if A is regular, so is A^R.

This question is asking if regular languages are closed under the reverse operation. Regular languages are closed under the reverse operation.

Proof: Suppose that A is regular. By the definition of regular languages, there is a DFA which recognizes A. To show A^R is regular, we need to define a DFA that recognizes A^R.

Since A is recognized by a DFA, there is also an NFA that recognizes A. Using the results of Exercise 1.11 from last time, there is also an NFA with only one accepting state that recognizes A. Call this $M = (Q, \Sigma, \delta, q_0, \{q_{accept}\})$. Thus $\mathcal{L}(M)=A$.

Consider the NFA $M^R = (Q, \Sigma, \delta^R, q_{accept}, \{q_0\})$ where

$$p\in\delta^R(q,a) \text{ whenever } q\in\delta(p,a).$$

That is, consider the new NFA, M^R, whose start state is the accept state of M, where all transitions are reversed, and that has the single accept state, that was the start state of M. With some thought it can be seen that $\mathcal{L}(M^R)=A^R$.

We have defined an NFA that recognizes A^R. Using the theorem that says that for every NFA, there is an equivalent DFA, there is a DFA which recognizes A^R, and A^R is regular.