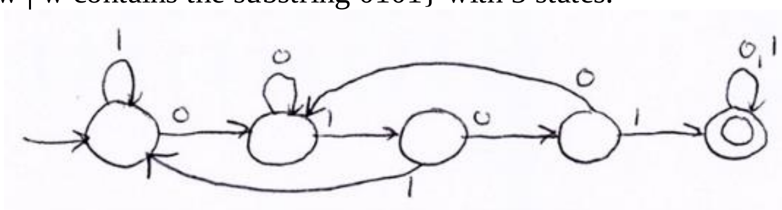
Theory of Computation, CSCI 438 spring 2022
Nondeterminism, pg. 47-54, Jan. 21

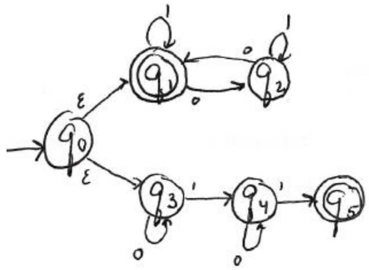
Exercise 1.7 b, c, e & 1.11

1.7 Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is {0, 1}.

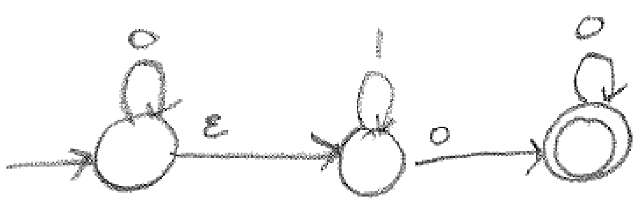
b. {w | w contains the substring 0101} with 5 states.



c. {w | w contains an even number of 0s, or contains exactly two 1s} with 6 states.

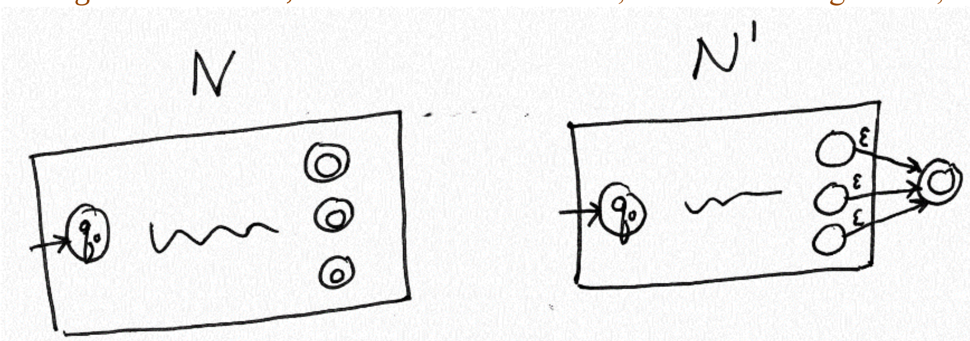


e. The language $0^*1^*0^+$ with three states.



1.11 Prove that every NFA can be converted to an equivalent one that has a single accept state.

Plan: given an NFA N, convert it to the NFA N', which has a single state, as shown.



Proof: Let N= (Q, Σ, δ, q_0, F). Consider the NFA N'= (Q∪{q_{accept}}, Σ, δ', q_0, {q_{accept}}) where δ' is defined by:

$$\delta'(q, x) = \begin{cases} \delta(q, x) & \text{for } q \in Q \setminus F \text{ and } x \in \Sigma_\varepsilon \\ \delta(q, x) & \text{for } q \in F \text{ and } x \in \Sigma \\ \delta(q, x) \cup \{q_{accept}\} & \text{for } q \in F \text{ and } x = \varepsilon \\ \{\} & \text{for } q = q_{accept} \text{ and } x \in \Sigma_\varepsilon \end{cases}$$

Clearly N' recognizes the same language as N, yet N' only has a single accept state. Thus, given an arbitrary NFA, it can be converted to an equivalent one that has a single accept state.