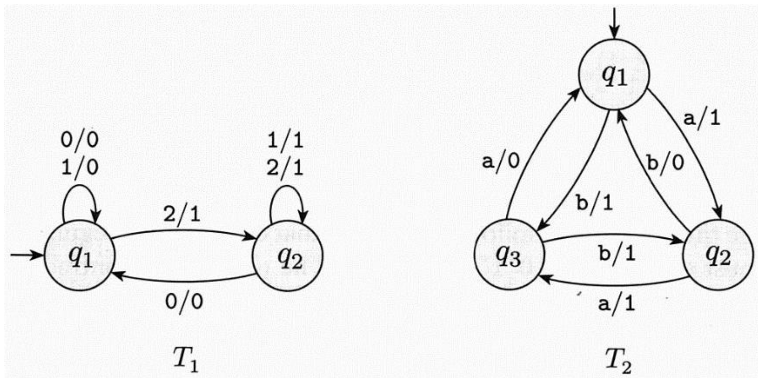
Theory of Computation, CSCI 438 spring 2022
Definition of Regular languages, pg. 40-44, Jan. 14

Exercise 1.24 a, b, f & g, 1.25 & 1.26 (page 87)

1.24 Given the two transducers T_1 and T_2



Give the sequence of states entered and the output produced in each of the following parts.

a. T_1 on input 001 Enters the sequence of states q_1, q_1, q_1, q_1 and outputs 000

b. T_1 on input 211 Enters the sequence of states q_1, q_2, q_2, q_2 and outputs 111

f. T_2 on input bbab Enters the sequence of states q_1, q_3, q_2, q_3, q_2 and outputs 1111

g. T_2 on input ε Enters the sequence of states q_1 and outputs nothing

1.25 Read the informal definition of the finite state transducer given in Exercise 1.24. Give a formal definition of this model, following the pattern in Definition 1.5 (page 35). Assume that an FST has an input alphabet Σ and an output alphabet Γ but not a set of accept states.

A finite state transducer, FST, is a 5-tuple $(Q, \Sigma, \delta, \Gamma, q_0)$ where

1. Q is a finite set of states
2. Σ is a finite set called the input alphabet
3. Γ is a finite set called the output alphabet
4. $\delta: Q \times \Sigma \rightarrow Q \times \Gamma$ is the transition function
5. $q_0 \in Q$ is the start state

Include a formal definition of the computation of an FST.

Computation of a FST begins in state q_0, reading symbols from Σ^* and outputting symbols from Γ^*. For each symbol read from the input, the FST applies δ to the current state and symbol, moving to the new state, and outputting the indicated symbol from Γ. The FST quits when all of the input has been consumed.

Alternatively, describing the computation more formally:

Given a finite state transducer, FST, $T = (Q, \Sigma, \delta, \Gamma, q_0)$, where

δ^* is the application of δ multiple times and has the signature

$$\delta^*: Q \times \Sigma^* \rightarrow Q \times \Gamma^*$$

On input w, $w \in \Sigma^*$, the FST outputs $o \in \Gamma^*$, when

$$\delta^*(q_0, w) = q_x, o$$

1.26 Using the solution you gave to Exercise 1.25, given a formal description of the machines T1 and T2 depicted in Exercise 1.24.

T_1 is M = ({q_1, q_2}, {0, 1, 2}, {0, 1}, δ, q_1) where δ is defined as:

$\delta(q_1, 0) \rightarrow q_1, 0$
$\delta(q_1, 1) \rightarrow q_1, 0$
$\delta(q_1, 2) \rightarrow q_2, 1$
$\delta(q_2, 0) \rightarrow q_1, 0$
$\delta(q_2, 1) \rightarrow q_2, 1$
$\delta(q_2, 2) \rightarrow q_2, 1$

T_2 is M = ({q_1, q_2, q_3}, {a, b}, {0, 1}, δ, q_1) where δ is defined as:

$\delta(q_1, a) \rightarrow q_2, 1$
$\delta(q_1, b) \rightarrow q_3, 1$
$\delta(q_2, a) \rightarrow q_3, 1$
$\delta(q_2, b) \rightarrow q_1, 0$
$\delta(q_3, a) \rightarrow q_1, 0$
$\delta(q_3, b) \rightarrow q_2, 1$