

**Theory of Computation, CSCI 438 spring 2022**  
**Deterministic Context-Free Languages, pages 130-135, March 7**

Recall that:

- Deterministic and non-deterministic finite automaton are equivalent
- Deterministic and non-deterministic push down automaton are not equivalent

**Deterministic PDAs (DPDA)**

DPDAs are defined as expected so that at each step of its computation, the DPDA has at most one way to proceed. Since it must be possible to manipulate the stack,  $\epsilon$ -transitions need to be allowed, but in a restricted way.

Two types of  $\epsilon$ -moves:

- $\epsilon$ -input,  $\delta(q, \epsilon, x)$ , ignore input but read from the stack
- $\epsilon$ -stack,  $\delta(q, a, \epsilon)$ , consume input but ignore the stack

Say a machine is in state  $q$ , reading input 'a', and 'x' is on the top of the stack. The following transitions could be taken:

- $\delta(q, a, x)$
- $\delta(q, a, \epsilon)$
- $\delta(q, \epsilon, x)$
- $\delta(q, \epsilon, \epsilon)$

Only one of these should be possible, so the others must be  $\Phi$ .

A deterministic pushdown automaton (DPDA) is a 6-tuple  $M=(Q,\Sigma,\Gamma,\delta,q_0,F)$  where  $Q$ ,  $\Sigma$ ,  $\Gamma$  and  $F$  are all finite sets, and

1.  $Q$  is the set of states,
2.  $\Sigma$  is the input alphabet,
3.  $\Gamma$  is the stack alphabet,
4.  $\delta:Qx\Sigma_\epsilon x\Gamma_\epsilon \rightarrow(Qx\Gamma_\epsilon) \cup \{ \Phi \}$  is the transition function,
5.  $q_0 \in Q$  is the start state, and
6.  $F \subseteq Q$  is the set of accept states. (Definition 2.39, page 130)

The transition function  $\delta$  must satisfy the condition:

For every  $q \in Q$ ,  $a \in \Sigma$ , and  $x \in \Gamma$ , exactly one of the values

- $\delta(q, a, x)$ ,  $\delta(q, a, \epsilon)$ ,  $\delta(q, \epsilon, x)$ , and  $\delta(q, \epsilon, \epsilon)$

is not  $\Phi$ .

The language of a deterministic Push-Down Automaton (DPDA) is called a deterministic context-free language (DCFL).

Theorem 2.42 (page 133) The class of DCFLs is closed under complementation

The class of DCFL isn't closed under the operations of union, intersection, star or reversal. (Problem 2.53)