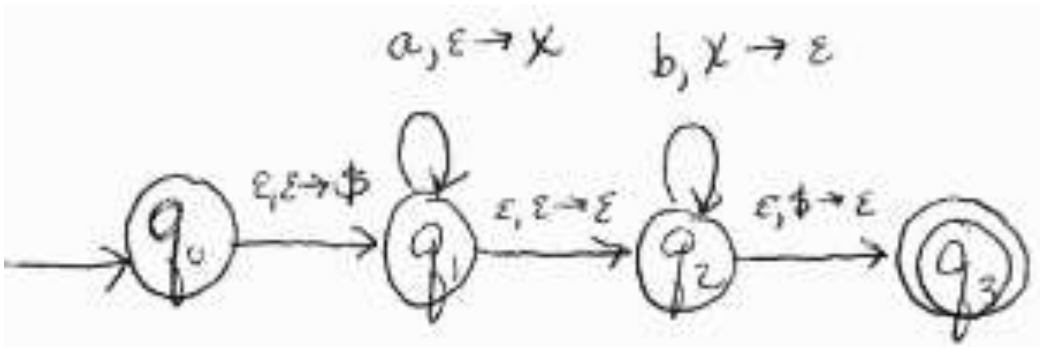
Theory of Computation, CSCI 438 spring 2022
Pushdown automaton, pages 111-116, Feb. 23

1. Create a PDA that recognizes the language L={$a^n b^n$ | n≥0}.

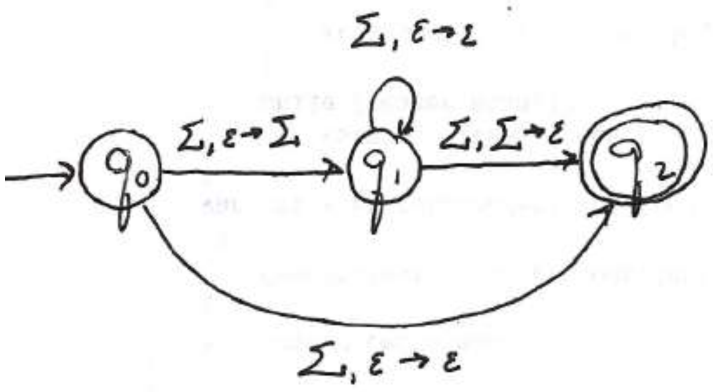
   Mark the bottom of the stack with a $. Consume a's pushing x's onto the stack. When encountering a b, begin consuming b's, popping x's from the stack. If a $ is on the stack when all of the b's were consumed, accept.



q_0
$\varepsilon, \varepsilon \to \$$
q_1
$a, \varepsilon \to x$
$\varepsilon, \varepsilon \to \varepsilon$
q_2
$b, x \to \varepsilon$
$\varepsilon, \$ \to \varepsilon$
q_3

2. Create a PDA that recognizes the language L={w | w∈{a,b}* such that w starts and ends with the same symbol}.

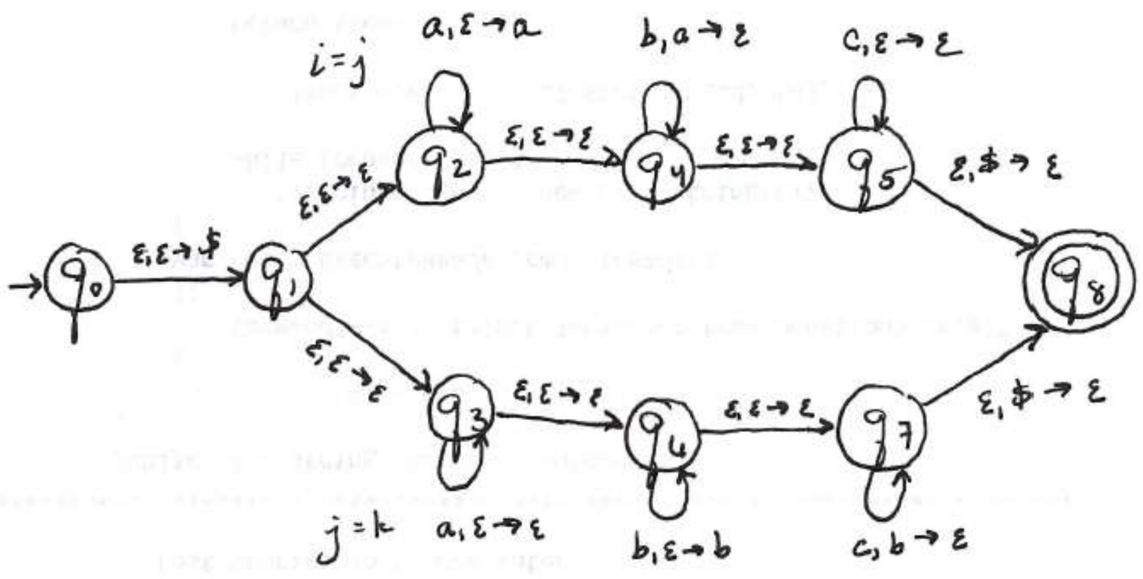
   Push the first symbol onto the stack, consume all middle symbols, non-deterministically determine when the last symbols is read and match that symbol with what is on the stack. Handle the case of single character strings.



$\Sigma, \varepsilon \to \varepsilon$
q_0
$\Sigma, \varepsilon \to \Sigma$
q_1
$\Sigma, \Sigma \to \varepsilon$
q_2
$\Sigma, \varepsilon \to \varepsilon$

3. Create a PDA that recognizes the language $L=\{a^i b^j c^k \mid i,j,k \geq 0 \text{ and } i=j \text{ or } j=k\}$.

   Mark the bottom of the stack. Non-deterministically go into the correct case, i=j or j=k. For i=j, consume a's, pushing them onto the stack, consume b's popping from the stack and then consume c's ignoring the stack. For j=k, consume a's ignoring the stack, consume b's pushing them onto the stack, consume c's popping from the stack. In both cases, accept if the marker on the bottom on the stack is found.



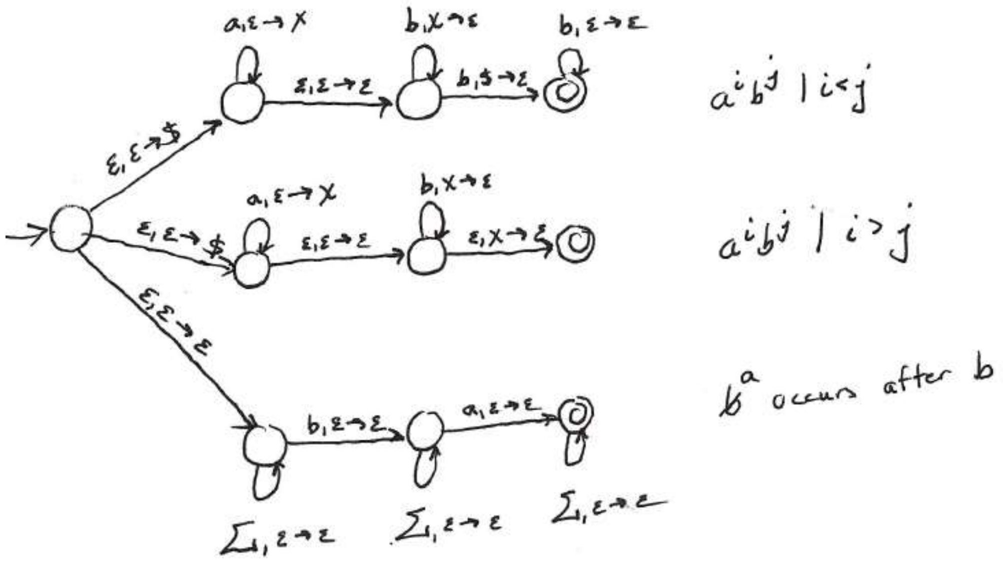
2.6 b Give a PDA for the complement of the language $\{a^n b^n \mid n \geq 0\}$

Notice that all strings in the complement of the language fall into one of the 3 disjoint sets:

1. The string is of the form a*b* and there are more b'a than a's
2. The string is of the form a*b* and there are more a's than b's
3. 'a' occurs after a 'b', which can be written $(a \cup b)^* b (a \cup b)^* a (a \cup b)^*$

Non-deterministically go into either case 1, 2 or 3.

- In case 1, mark the bottom of the stack. Consume a's pushing x's onto the stack. Consume b's, popping x's from the stack. When the bottom of stack symbol is found, consume the required extra 'b'. Freely consume any remaining 'b's.
- In case 2, mark the bottom of the stack. Consume a's pushing x's onto the stack. Consume b's, popping x's from the stack. Once the b's are consumed, there should still be an extra 'x' on the stack. Move into the accepting state when popping the extra 'x'.
- In case 3 the stack can be ignored. Consume any number of a's and b's, consume a 'b', then consume any number of a's and b's, then consume an 'a', finally consume any number of a's and b's and accept.

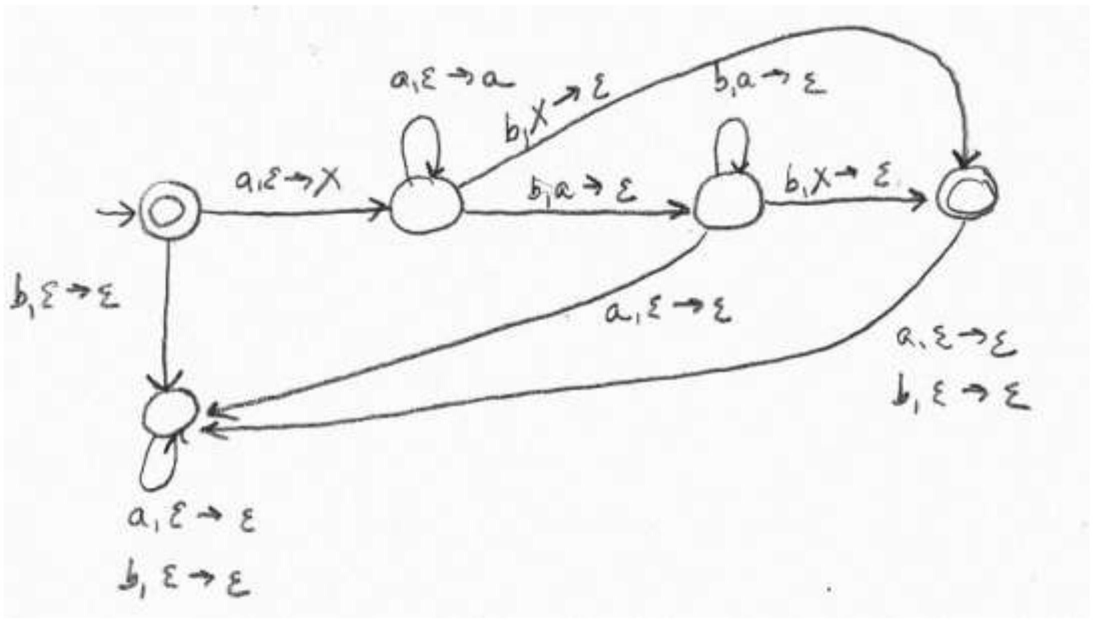


$a, \varepsilon \to x$
$b, x \to \varepsilon$
$b, \varepsilon \to \varepsilon$
$\varepsilon, \varepsilon \to \varepsilon$
$b, \$ \to \varepsilon$
$\varepsilon, \varepsilon \to \$$
$a^i b^j \mid i < j$
$a, \varepsilon \to x$
$b, x \to \varepsilon$
$\varepsilon, \varepsilon \to \$$
$\varepsilon, \varepsilon \to \varepsilon$
$\varepsilon, x \to \varepsilon$
$a^i b^j \mid i > j$
$\varepsilon, \varepsilon \to \varepsilon$
$b, \varepsilon \to \varepsilon$
$a, \varepsilon \to \varepsilon$
a occurs after b
$\Sigma, \varepsilon \to \varepsilon$
$\Sigma, \varepsilon \to \varepsilon$
$\Sigma, \varepsilon \to \varepsilon$

Alternate answer:
Fortunately a deterministic PDA can be developed for the language $\{a^nb^n \mid n\geq 0\}$. Therefore, a PDA for the complement of $\{a^nb^n \mid n\geq 0\}$ can be developed by starting with a PDA for $\{a^nb^n \mid n\geq 0\}$ and flipping the accepting and non-accepting states.

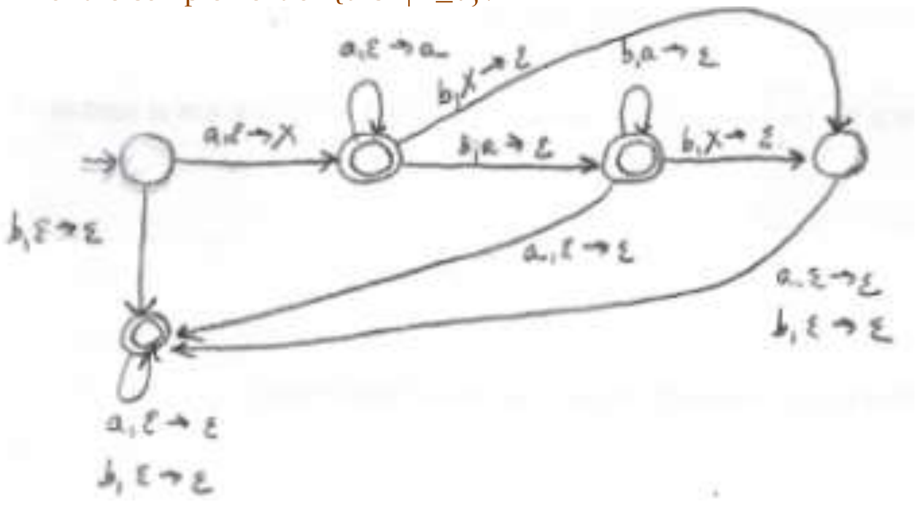
Deterministic PDA for $\{a^nb^n \mid n\geq 0\}$:

If there are no symbols to consume, accept. If an ‘a’ is found, mark bottom of the stack and push the ‘a’. If a ‘b’ is found, go to a dead state. Consume a’s, pushing them onto the stack until a ‘b’ is encountered. Consume ‘b’s while popping a’s from the stack. When the end of stack is found, accept. Since the PDA needs to be deterministic, include dead states.



$a,\varepsilon\to X$
$a,\varepsilon\to a$
$b,X\to\varepsilon$
$b,a\to\varepsilon$
$b,a\to\varepsilon$
$b,X\to\varepsilon$
$b,\varepsilon\to\varepsilon$
$a,\varepsilon\to\varepsilon$
$a,\varepsilon\to\varepsilon$
$b,\varepsilon\to\varepsilon$
$a,\varepsilon\to\varepsilon$
$b,\varepsilon\to\varepsilon$

PDA for the complement of $\{a^nb^n \mid n\geq 0\}$:



$a,\varepsilon\to X$
$a,\varepsilon\to a$
$b,X\to\varepsilon$
$b,a\to\varepsilon$
$b,a\to\varepsilon$
$b,X\to\varepsilon$
$b,\varepsilon\to\varepsilon$
$a,\varepsilon\to\varepsilon$
$a,\varepsilon\to\varepsilon$
$b,\varepsilon\to\varepsilon$
$a,\varepsilon\to\varepsilon$
$b,\varepsilon\to\varepsilon$