

Theory of Computation, CSCI 438 spring 2022
Pushdown automaton, pages 111-116, Feb. 23

Nondeterministic PDA is a 6-tuple $M=(Q, \Sigma, \Gamma, \delta, q_0, F)$ where Q, Σ, Γ and F are all finite sets, and

1. Q is the set of states
2. Σ is the input alphabet
3. Γ is the stack alphabet
4. $\delta: Q \times \Sigma \times \Gamma_\epsilon \rightarrow P(Q \times \Gamma_\epsilon)$ is the transition function
5. $q_0 \in Q$ is the start state
6. $F \subseteq Q$ is the set of accept states. (Definition 2.13, page 113)

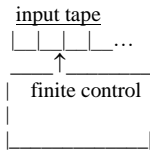
For the transition function $\delta: Q \times \Sigma \times \Gamma_\epsilon \rightarrow P(Q \times \Gamma_\epsilon)$

- Γ_ϵ in the domain - what is seen on top of the stack. ϵ indicates not to look at the stack. In order to look at the top of the stack the stack must be popped.
- Γ_ϵ in the range - what to push onto the stack. ϵ indicates not to push anything.
- The stack is just temporary storage. Once the string is consumed and the PDA is in the accept state, the string is accepted regardless of the state of the stack.
- Your program should never try to look at an empty stack.

Review:

Regular Languages:

Finite automata



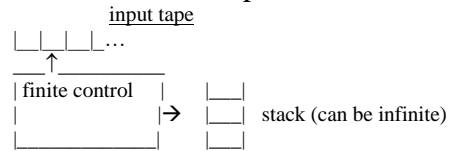
$M=(Q, \Sigma, \delta, q_0, F)$

DFA $\delta: Q \times \Sigma \rightarrow Q$

NFA $\delta: Q \times \Sigma \rightarrow P(Q)$

Context Free Languages:

Nondeterministic pushdown automata



$M=(Q, \Sigma, \Gamma, \delta, q_0, F)$

↑
Finite set of stack symbols

Deterministic PDA $\delta: Q \times \Sigma \times \Gamma_\epsilon \rightarrow Q \times \Gamma_\epsilon$

Nondeterministic PDA $\delta: Q \times \Sigma \times \Gamma_\epsilon \rightarrow P(Q \times \Gamma_\epsilon)$

Deterministic PDAs define a proper subset of context free languages. For now we'll discuss nondeterministic PDAs.

What it means for a machine to “accept” a string:

$M=(Q, \Sigma, \Gamma, \delta, q_0, F)$ accepts string w , where $w = w_1w_2\dots w_n$ and each $w_i \in \Sigma_\epsilon$ and sequences of states $r_0 r_1 r_2 \dots, r_m \in Q$ and strings $s_0, s_1, s_2, \dots, s_m \in \Gamma^*$ exists such that

1. $r_0 = q_0$ and $s_0 = \epsilon$ i.e. M starts out properly
2. For $i=0, 1, \dots, m-1$ $(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$ where $s_i = at$ and $s_{i+1} = bt$ for some $a, b \in \Gamma_\epsilon$ and $t \in \Gamma^*$
3. $r_m \in F$.

(I won't ask you to give this on an exam.)