

**Theory of Computation, CSCI 438 spring 2022**  
**Pushdown automaton, pages 111-116, Feb. 23**

Nondeterministic PDA is a 6-tuple  $M=(Q, \Sigma, \Gamma, \delta, q_0, F)$  where  $Q, \Sigma, \Gamma$  and  $F$  are all finite sets, and

1.  $Q$  is the set of states
2.  $\Sigma$  is the input alphabet
3.  $\Gamma$  is the stack alphabet
4.  $\delta: Q \times \Sigma \times \Gamma \rightarrow P(Q \times \Gamma)$  is the transition function
5.  $q_0 \in Q$  is the start state
6.  $F \subseteq Q$  is the set of accept states. (Definition 2.13, page 113)

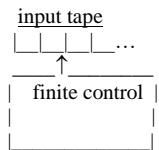
For the transition function  $\delta: Q \times \Sigma \times \Gamma \rightarrow P(Q \times \Gamma)$

- $\Gamma$  in the domain - what is seen on top of the stack.  $\epsilon$  indicates not to look at the stack. In order to look at the top of the stack the stack must be popped.
- $\Gamma$  in the range - what to push onto the stack.  $\epsilon$  indicates not to push anything.
- The stack is just temporary storage. Once the string is consumed and the PDA is in the accept state, the string is accepted regardless of the state of the stack.
- Your program should never try to look at an empty stack.

Review:

Regular Languages:

Finite automata



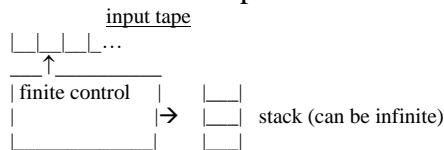
$$M = (Q, \Sigma, \delta, q_0, F)$$

$$\text{DFA } \delta: Q \times \Sigma \rightarrow Q$$

$$\text{NFA } \delta: Q \times \Sigma \rightarrow P(Q)$$

Context Free Languages:

Nondeterministic pushdown automata



$$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$$

↑

Finite set of stack symbols

$$\text{Deterministic PDA } \delta: Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma$$

$$\text{Nondeterministic PDA } \delta: Q \times \Sigma \times \Gamma \rightarrow P(Q \times \Gamma)$$

Deterministic PDAs define a proper subset of context free languages. For now we'll discuss nondeterministic PDAs.

What it means for a machine to “accept” a string:

$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$  accepts string  $w$ , where  $w = w_1w_2\dots w_n$  and each  $w_i \in \Sigma_\epsilon$  and sequences of states  $r_0 r_1 r_2 \dots, r_m \in Q$  and strings  $s_0, s_1, s_2, \dots, s_m \in \Gamma^*$  exists such that

1.  $r_0 = q_0$  and  $s_0 = \epsilon$  i.e.  $M$  starts out properly
2. For  $i=0, 1, \dots, m-1$   $(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$  where  $s_i = at$  and  $s_{i+1} = bt$  for some  $a, b \in \Gamma_\epsilon$  and  $t \in \Gamma^*$
3.  $r_m \in F$ .

(I won't ask you to give this on an exam.)