Theory of Computation, CSCI 438 spring 2022
Context-free grammars, pages 101-107, Feb. 9

1. Give a context free grammar for the language on $\sum=\{a,b\}$ defined by

   $L=\{w \mid w=w^R$ for $w\in\Sigma^*$, that is, w is a palindrome$\}$

   $S\rightarrow \varepsilon \mid a \mid b \mid aSa \mid bSb$

   The full grammar is:

   $G=(\{S\},\{a,b\},\{(S, \varepsilon), (S, a), (S, b), (S, aSa), (S, bSb)\}, S)$

2. Give a context free grammar for the language on $\sum=\{a,b\}$ defined by

   $L=\{a^nb^n : n \geq 0\}$

   $S\rightarrow aSb \mid \varepsilon$

3. Give a context free grammar for the language on $\sum=\{0,1\}$ defined by

   L= {w | w starts and ends with the same symbol}

   $S\rightarrow 0T0 \mid 1T1 \mid 0 \mid 1$

   $T\rightarrow 0T \mid 1T \mid \varepsilon$

4. Give a context free grammar for the language on $\sum=\{0,1\}$ defined by

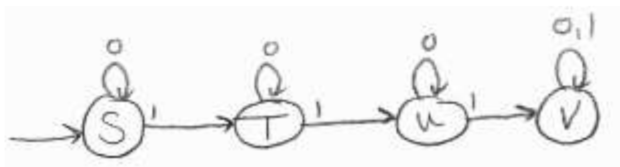
L= {w | w contains at least three 1's}

$S \rightarrow T1T1T1T$

$T \rightarrow 0T \mid 1T \mid \varepsilon$

Note that this language is regular. Thus, it can be done by starting with a DFA or a regular expression. (This relates to exercises 2.16 & 2.17)

This language is regular so I could begin with a DFA and get the grammar from that. (This relates to exercises 2.16 & 2.17)



$S \rightarrow 0S \mid 1T$

$T \rightarrow 0T \mid 1U$

$U \rightarrow 0U \mid 1V$

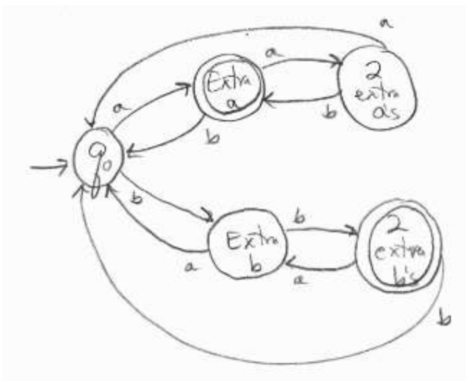
$V \rightarrow 0V \mid 1V \mid \varepsilon$

5. Define a context-free grammar for the following language

   $L=\{w \mid (n_a(w) - n_b(w)) \bmod 3 = 1\}$

   $w=ababaabbabaa \notin L$ since $(n_a(w) - n_b(w)) \bmod 3 = (7-5) \bmod 3 = 2$
   $w=bbabbbabbb \notin L$ since $(n_a(w) - n_b(w)) \bmod 3 = (2-8) \bmod 3 = (-6) \bmod 3 = 0$
   $w=bb \in L$ since $(n_a(w) - n_b(w)) \bmod 3 = (0-2) \bmod 3 = (-2) \bmod 3 = 1$

   Regular so create DFA:



$Q_0 \rightarrow$ a EA | b EB
EA $\rightarrow$ a EAA | b Q_0 | ε
EAA $\rightarrow$ a Q_0 | b EA
EB $\rightarrow$ b EBB | a Q_0
EBB $\rightarrow$ b Q_0 | a EB | ε

   It is also possible to create a smaller DFA for this language. This would result in a simple context-free grammar.

6. Create a context-free grammar for the language
   $L = \{w\#x \mid w^R \text{ is a substring of } x \text{ for } w,x \in \{0,1\}^*\}$
   (This is exercise 2.6c in the text.)

   Another way to write this is:
   $L = \{w\#\, x_1 w^R x_2 \mid x_1, x_2, w \in \{0,1\}^* \}$.

   $S \rightarrow TX$
   $T \rightarrow 0T0 \mid 1T1 \mid \#X$
   $X \rightarrow 0X \mid 1X \mid \varepsilon$

7. Give a context free grammar for the language on $\sum=\{a,b\}$ defined by
   $L=\{w \mid (n_a(w) > n_b(w)\}$.
   The set of strings over the alphabet {a,b} with more a's than b's
   $n_a(w)$ is the number of a's in the string w.
   $n_b(w)$ is the number of b's in the string w.

   $S \rightarrow MAM \mid SS$ // M for "match"
   $M \rightarrow aMb \mid bMa \mid \varepsilon$
   $A \rightarrow aA \mid a$

   Another answer:
   $S \rightarrow MaM \mid SS$
   $M \rightarrow aMb \mid bMa \mid a \mid \varepsilon$

   Another answer:
   $S \rightarrow MAM$
   $M \rightarrow MAMbM \mid MbMAM \mid \varepsilon$
   $A \rightarrow aA \mid a$

   Another answer:
   $S \rightarrow MS \mid aM \mid aS$
   $M \rightarrow MM \mid aMb \mid bMa \mid \varepsilon$