

**Theory of Computation, CSCI 438 spring 2023**  
**Class nondeterministic polynomial time, NP, pg. 292-298**  
**NP-Completeness, pg. 299-311**  
**April 27**

1. Prove that  $\text{HAMPATH} \in \text{NP}$

$\text{HAMPATH} = \{ \langle G, s, t \rangle \mid G \text{ is a directed graph that contains a directed path from source vertex } s \text{ to target vertex } t \text{ and the path passes through every vertex in } G \text{ exactly once} \}$

2. Prove that  $\text{CLIQUE} \in \text{NP}$

$\text{CLIQUE} = \{ \langle G, k \rangle \mid G \text{ is an undirected graph with a } k\text{-clique} \}$

Clique – all nodes in the clique are completely connected

$k$ -clique – a clique with  $k$  nodes.

3. Theorem 7.20 (page 294) A language is in NP iff it is decided by some nondeterministic polynomial time Turing machine.

Problems in NP:

HAMPATH = { $\langle G, s, t \rangle$  |  $G$  is a directed graph that contains a directed path from source vertex  $s$  to target vertex  $t$  and the path passes through every vertex in  $G$  exactly once}

COMPOSITES = { $x$  |  $x=pq$ , for integers  $p, q > 1$ }

CLIQUE = { $\langle G, k \rangle$  |  $G$  is an undirected graph with a  $k$ -clique}  
Clique – all nodes in the clique are completely connected  
 $k$ -clique – a clique with  $k$  nodes.

SUBSET-SUM = { $\langle S, t \rangle$  |  $S = \{x_1, \dots, x_k\}$ , and for some  
 $\{y_1, \dots, y_k\} \subseteq \{x_1, \dots, x_k\} \sum y_i = t$  }  
Note that  $S$  and its subset are multi-sets.