

**Theory of Computation, CSCI 438 spring 2022**  
**Introduction to Complexity Theory and Complexity Relationships among Models,**  
**pg. 275-284, April 22<sup>nd</sup>**

Let  $A = \{0^k 1^k \mid k \geq 0\}$

1. Find the time complexity of deciding  $A$  using a single tape Turing machine.

2. If your above algorithm runs in  $O(n^2)$ , define an algorithm that runs in  $O(n \log_2 n)$ .

3. Find the time complexity of deciding A using a 2-tape Turing machine.

4. Describe why the following theory makes sense:

Theorem 7.8 (page 282)

Let  $t(n)$  be a function, where  $t(n) \geq n$ . Then every  $t(n)$  time multi-tape Turing machine has an equivalent  $O(t^2(n))$  time single-tape Turing machine.

5. Describe why the following theory makes sense:

Theorem 7.11 (page 284)

Let  $t(n)$  be a function, where  $t(n) \geq n$ . Then every  $t(n)$  time nondeterministic single-tape Turing machine has an equivalent  $2^{O(t(n))}$  time deterministic Turing machine.

6. Exercise 7.5

Is the following formula satisfiable?

$$(x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$$

8. Using the fact that  $A_{TM}$  is not decidable, prove that  $HALT$  is not decidable.

$$HALT_{TM} = \{ \langle M, w \rangle \mid M \text{ encodes a TM, } w \text{ is a string in the alphabet of } M, \text{ and } M \text{ halts on } w \}$$