Theory of Computation, CSCI 438 spring 2022
Introduction to Complexity Theory and Complexity Relationships among Models, pg. 275-284, April 22nd

Computability theory versus complexity theory:

- Computability theory
  - What can and can't be computed.
  - Acceptors for non-Turing equivalent machines, are decidable. A_{TM} is not decidable, but is recognizable. The complement of A_{TM}, is not recognizable.
  - The Church-Turing thesis tells us that all "reasonable" models of computation are equivalent (decide and recognize the same class of languages).
- Complexity theory
  - Considers how difficult to compute (space or time). Only computable languages are considered.
  - The choice of model makes a difference.

Definition of the running time of a deterministic Turing-decider (page 276):
Let M be a deterministic Turing machine that halts on all inputs. The running time or time complexity of M is the function $f : N \rightarrow N$, where f(n) is the maximum number of steps that M uses on any input of length n.

If f(n) is the running time of M, we say that M is an f(n) time Turing machine.

Rather than specifying f exactly, use the Big-O/small-o categories for execution time (execution time, space, or some other resource):

$O(c)$, $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(n^3)$, … - P
$O(c^n)$, $O(c^{n*n})$,… - NP

In Big-O, it is only necessary that you find a particular multiplier k for which the inequality holds beyond some minimum x.

Relationships among computation models

Multi-tape Turing machine to single tape Turing machine:
Theorem 7.8 (page 282)
Let t(n) be a function, where t(n)≥n. Then every t(n) time multi-tape Turing machine has an equivalent $O(t^2(n))$ time single-tape Turing machine.

Recall from before:
Multitape Turing Machine: Like an ordinary Turing Machine but it has several tapes so several read/write heads. Say initially, input on tape 1 is as usual and the other tapes are blank.
Formally define the machine:

$M=(Q,\Sigma,\Gamma,\delta,q_0,q_{accept}, q_{reject})$

$\delta:Q x \Gamma^k \rightarrow Q x \Gamma^k x \{L,R,S\}^k$

$\delta:\underbrace{Q x \Gamma x \Gamma x \ldots x \Gamma}_{k \text{ times}} \rightarrow \underbrace{Q x \Gamma x \Gamma x \ldots x \Gamma}_{k \text{ times}}, \underbrace{\{L,R,S\} x \{L,R,S\} x \ldots x \{L,R,S\}}_{k \text{ times}}$

Nondeterministic Turing machine to deterministic Turing machine:
Theorem 7.11 (page 284)
Let t(n) be a function, where t(n)≥n. Then every t(n) time nondeterministic single-tape Turing machine has an equivalent $2^{O(t(n))}$ time deterministic Turing machine.

Recall from before:
Nondeterministic TM: Defined as expected. M is as usual,

$M=(Q,\Sigma,\Gamma,\delta,q_0,q_{accept}, q_{reject})$,

except that the signature of delta has changed:

deterministic: $Q x \Gamma \rightarrow Q x \Gamma x \{L,R\}$

nondeterministic: $Q x \Gamma \rightarrow \mathcal{P}(Q x \Gamma x \{L,R\})$

A string is accepted by a nondeterministic TM if there is some sequence of possible moves that will put the machine in an accept state.