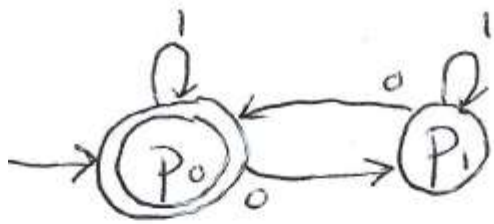
Theory of Computation, CSCI 438 spring 2022
Quiz 3, Jan. 31

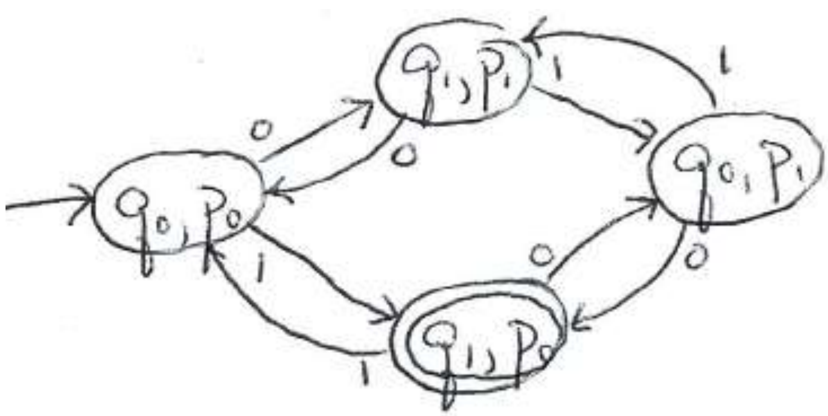
1. Following are two DFAs defined over the alphabet $\Sigma = \{0, 1\}$.

DFA that recognizes the language {w | w has an odd length}:

DFA that recognizes the language {w | w contains an even number of 0's}



Use the method described in the text and in class to create a DFA for language
{w | w has an odd length and an even number of 0's } (10 pts.)



2. Prove that regular languages are closed under intersection. (10 pts.)

Say that A and B are regular languages. Then we know that there are DFAs

$M_1 = (Q_1, \Sigma, \delta_1, q_{1,0}, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_{2,0}, F_2)$

where $\mathcal{L}(M_1)$=A and $\mathcal{L}(M_2)$=B.

Consider a new DFA, $M_{intersection}$, defined as follows:

$M_{intersection}$, $= (Q_1 x Q_2, \Sigma, \delta`, (q_{1,0}, q_{2,0}), F_1 x F_2)$

where $\delta`$ is defined by:

$$\delta`((q_i, q_j),a) = (\delta_1(q_i,a), \delta_2(q_j,a))$$

With some thought, it can be see that $M_{intersection}$ accepts A∩B. Thus A∩B is regular and regular languages are closed under intersection.

3. The proof that regular languages are closed under union is very similar to the proof that regular languages are closed under intersection. In fact, the proofs are identical everywhere except for the final states in the new machine.

Copy your answer to the first question here, except change the final state(s) to give a machine for:
{w | w has an odd length or an even number of 0's } (5 pts.)