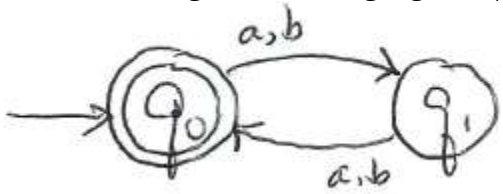
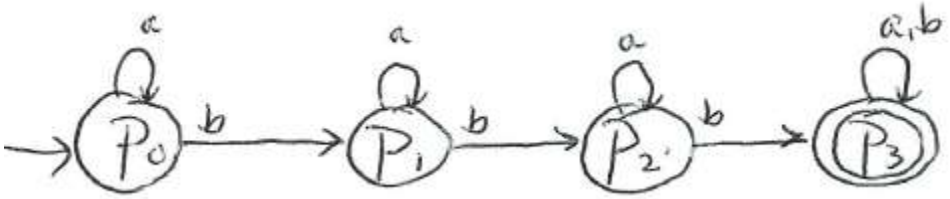
Theory of Computation, CSCI 438 spring 2022
Quiz 2, Jan. 26

1. Following are two DFAs defined over the alphabet Σ = {a, b}.

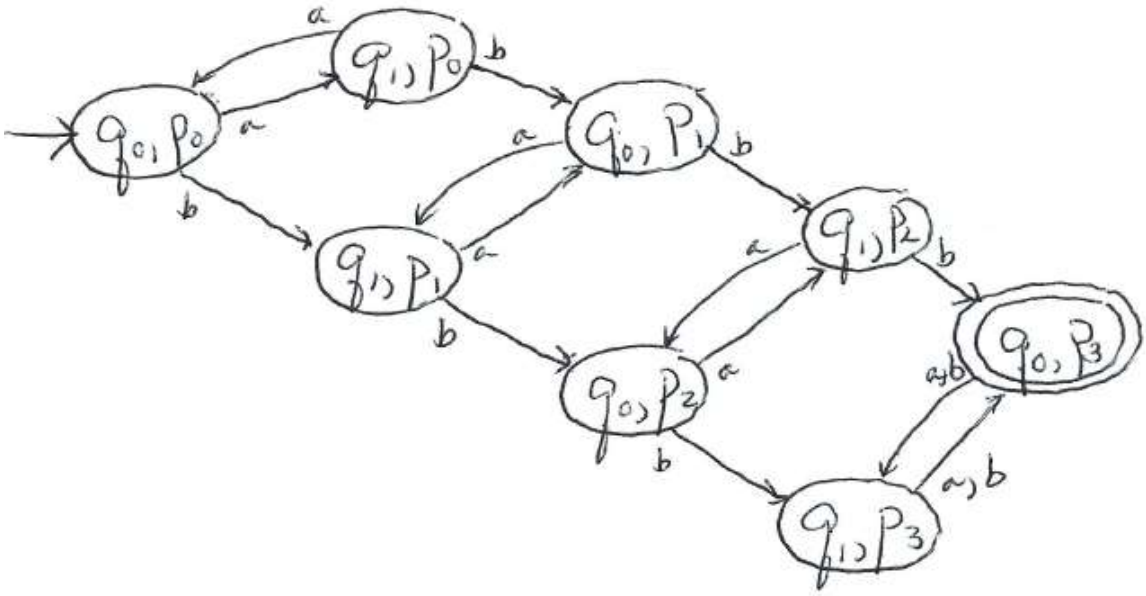
DFA that recognizes the language {w | w has an even length}:



DFA that recognizes the language {w | w contains at least three b's}



Use the method described in the text and in class to create a DFA for language
~~{w | w has an even length and an odd number of a's }~~ (10 pts.)
{w | w has an even length and contains at least three b's }



2. Prove that regular languages are closed under intersection. That is, show that if the languages A and B are regular, then the language A ∩ B is regular. (Note, this is proving that the construction which you used in the previous question, works.) (10 pts.)

Say that A and B are regular languages. Then we know that there are DFAs

$M_1=(Q_1,\Sigma,\delta_1,q_{1,0},F_1)$ and $M_2=(Q_2,\Sigma,\delta_2,q_{2,0},F_2)$

where $\mathcal{L}(M_1)=A$ and $\mathcal{L}(M_2)=B$.

Consider a new DFA, $M_{intersection}$, defined as follows:

$M_{intersection}$, $= (QxP, \Sigma, \delta`, (q_0, p_0), F_{intersection})$

where $\delta`$ is defined by:

$$\delta`((q_i, p_j),a) = (\delta_1(q_i,a), \delta_2(p_j,a))$$

and F $M_{intersection}$ is defined by

$$F_{intersection}, = \{(q_i, p_j)\}\ q_i \in F_q \text{ and } p_j \in F_p\}$$

With some thought, it can be see that $M_{intersection}$, accepts $L_1 \cap L_2$. Thus $L_1 \cap L_2$ is regular and regular languages are closed under intersection.