

Theory of Computation, CSCI 438, Spring 2022
Exam 3, April 13

Name _____

Essay Questions

(15 pts.)

Definitions

1. State the Church-Turing Thesis.

(5 pts.)

2. Give the format in which we will be writing algorithms in this class **and** what restrictions exist for this formalism. (We started completely defining machines, regular expressions and grammars. However, from now on we will be giving formalisms at a higher level.)

(10 pts.)

Problem Solving

1. Can a Turing Machine ever write the blank symbol onto its tape? (3 pts.)

2. Can the tape alphabet Γ be the same as the input alphabet Σ ? Explain why or why not. (3 pts.)

3. Can a Turing Machine's head ever be in the same location in two successive steps? Explain why or why not. (3 pts.)

4. Can a Turing Machine contain only a single state? Explain why or why not. (3 pts.)

5. Draw a sample tape, containing symbols, and state transitions, that demonstrates that a Turing Machine can loop forever. (3 pts.)

6. Define a Turing machine that places a \$ onto the front of a tape, shifting all non-blank symbols one position to the right, returning the read/write head to the position one place to the right of the \$.
- Let the language alphabet be $\Sigma = \{a, b, c\}$ and the tape alphabet be $\Gamma = \{a, b, c, \$, _ \}$.
 - Define this machine completely. That is, give a high-level and detailed plan, and the TM itself.

High-level plan: (5 pts.)

Detailed plan: (5 pts.)

Turing machine: (5 pts.)

7. Consider a machine that is similar to a TM but has three possible moves, left, right or stay. Call such a machine a TM_{stay} . Prove that a language is Turing-recognizable iff some Turing machine with stays recognizes it. (10 pts.)

8. Give a context-free grammar for the set of strings over the alphabet $\{a, b\}$ that has twice as many a's as b's. (10 pts.)

9. Define a push-down automaton (PDA) for the language L below, on $\Sigma=\{0,1\}$:
 $L = \{ 0^n 0^{2n} \mid n \geq 0 \}$. (10 pts.)

Plan:

Machine (PDA):

10. Consider the language $L = \{w \mid n_a(w) \leq n_b(w) \leq n_c(w)\}$. Prove that L is context-free by creating a PDA or context-free grammar for it, or use the pumping lemma for context-free languages to prove that L is not context free. (10 pts.)