

Theory of Computation, CSCI 438, Spring 2022
Exam 2, March 11

Name _____

My phone number is **406-498-4884**. Feel free to call with any questions that you have.

There is one question per page.

Essay Questions

(20 pts.)

Definitions

1. Given a context-free grammar $G = (V, \Sigma, R, S)$, give the definition of the language of that grammar. (10 pts.)

Language Placement

2. Fill out the two columns, telling whether or not the language is regular and/or context free (10 pts.)

	Language	Is the language regular?	Is the language context free?
1	Language consisting of all strings on $\{0,1\}^*$ with the same number of 0's and 1's.		
2	$L = \{w \mid n_a(w) \geq 2 \text{ and } n_b(w) > 2\}$ where $n_a(w)$ is the number of a's in w and $n_b(w)$ is the number of b's in w .		
3	$L = \{a^n b^n a^n : n \geq 0\}$		
4	$L = \{w \mid (n_a(w) - n_b(w)) \bmod 3 = 1\}$		

Problem Solving

3. Give a context-free grammar for the language L, on $\Sigma=\{0,1\}$, defined as:
 $L = \{ 0^n 1^* 0^{2n} \mid n \geq 0 \}$ (10 pts.)

4. Create a PDA for the language L , defined on $\{a,b,\#\}$, as follows:

$L = \{x_1\#x_2\#\dots\#x_k \mid k \geq 2, x_1, x_2, \dots, x_k \in \{a,b\}^* \text{ and for some } i, x_i = x_{i+1}^R\}$.
(10 pts.)

5. Using the method described in class and the text, convert the following context-free grammar into a PDA.

$S \rightarrow abAB \mid ab$

$A \rightarrow aA \mid \varepsilon$

$B \rightarrow Bab \mid \varepsilon$

(10 pts.)

6. Convert the following grammar to Chomsky normal form.

$G = (\{S, T, U\}, \{0, \#\}, R, S)$ where R is defined by:

$S \rightarrow TT \mid U$

$T \rightarrow 0T \mid T0 \mid \#$

$U \rightarrow 0U00 \mid \#$

(10 pts.)

7. Consider the language $L = \{ ww \mid w \in \{a, b\}^* \}$. Prove that L is context-free by giving a grammar for it, or use a pumping lemma to prove L is not a context-free language.
(20 pts.)

Extra Credit

Consider the language $L = \{a^n b^j \mid n = j^2\}$. Prove that L is context-free by giving a grammar for it or use the pumping lemma for context-free languages to prove that there is no context-free grammar for L .