

Theory of Computation, CSCI 438, Spring 2021
Exam 1, Feb. 14
Happy Valentine's Day

Name _____

Essay Questions

(20 pts.)

Definitions

1. Give the complete definition of a non-deterministic finite automaton.

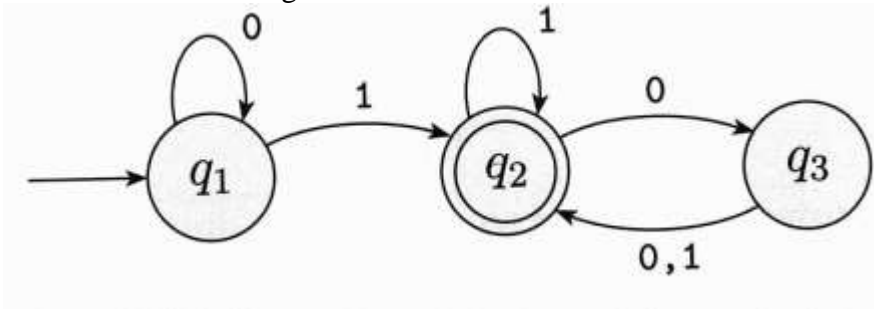
(5 pts.)

2. Describe, without using formulas, what it means for a string to be accepted by an NFA. (5 pts.)

3. Give the formal definition of what it means for a string to be accepted by an NFA. (5 pts.)

Problem Solving

4. Consider the following finite automaton.



Write the definition of this machine using the linear format.

(5 pts.)

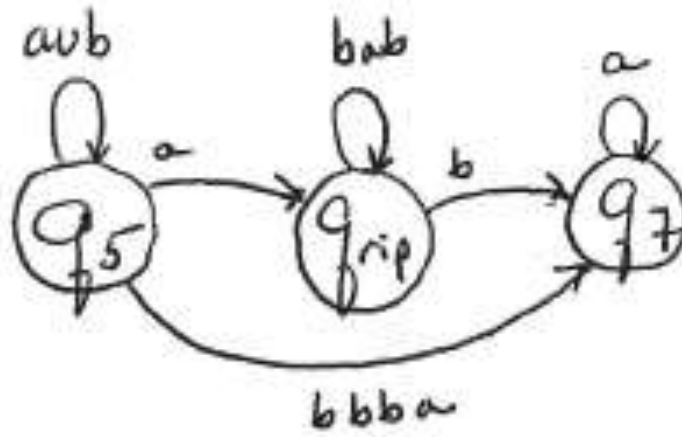
$M = (\dots\dots)$

5. Give an NFA for the language L on $\Sigma=\{a, b\}$ where
 $L = \{w \mid w \text{ starts with an } a \text{ or has at most one } b\}$ (5 pts.)

6. Give a regular expression for all strings on the alphabet $\Sigma=\{a,b\}$ which end with three a's, or contain an even number of a's. (5 pts.)

7. Show that regular languages are closed under union by beginning with a DFA for each of the regular languages, and constructing an NFA for the union of the languages. Show this by drawing a picture of the construction. You do NOT need to formally write the construction. (You will need to do this later in the exam.)
(10 pts.)

8. Give the result of ripping the state q_{rip} from the following Generalized NFA using the algorithm discussed in class and given in the text. (10 pts.)



Proofs

9. Prove that regular languages are closed under union by beginning with a DFA for each of the regular languages, and constructing an NFA for the union of the languages. (This is the same question as problem 6, only there you only needed to show what to do. Here you need to prove the statement.) (10 pts.)

10. Prove that L is regular by giving an NFA or regular expression for L , or prove that it is not regular using the pumping lemma for regular languages. (10 pts.)

$$L = \{w \mid (n_a(w) = n_b(w))\}$$

where $n_a(w)$ is the number of a's in w and $n_b(w)$ is the number of b's in w .

11. Consider the language $F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i=1 \text{ then } j=k\}$. Note that this language is not regular, because any string beginning with a single 'a', must have an equal number of b's and c's.
- a. Attempt to show that F is not regular using the pumping lemma, and explain why this can't be done. (5 pts.)

b. Prove that F is not regular.

(5 pts.)

Hint: It can be helpful to remember that regular languages are closed under the operations of complementation, concatenation, union, intersection, and Kleene closure.