Theory of Computation, CSCI 438, Spring 2021
Final, April 22nd

Name ______________________

Definitions (10 pts.)

1. Give the definition of a deterministic Turing Machine which we used in class and is given in the text. Be sure to define each of the elements. (5 pts.)

2. Give the definition of what it means for a context-free grammar to be in Chomsky Normal which we used in class and is given in the text. Be sure to define each of the elements. (5 pts.)

Short Answer (10 pts.)

3. Give a regular expression for all strings on the alphabet Σ={a,b} which end with three a's, or contain an even number of a's. (5 pts.)

4. Give an NFA for all strings on the alphabet Σ={a,b} which end with three a's, or contain an even number of a's. (5 pts.)

Assimilation (20 pts.)

5. Describe what is meant by a Universal Turing machine, along with the difference between deciders, recognizers, and a machine that is neither a decider nor a recognizer. Relate these concepts by telling if a Universal Turing machine is a decider, recognizer, or a machine that is neither a decider nor a recognizer.
(20 pts.)

Problem Solving /Proofs (60 pts.)

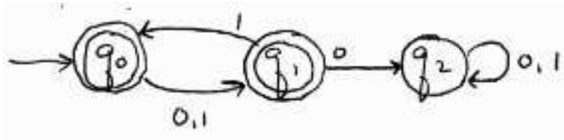
6. Prove that regular languages are closed under union by beginning with a DFA for each of the regular languages, and constructing an NFA for the union of the languages.

    Begin by drawing a picture of what you plan to do. (5 pts.)

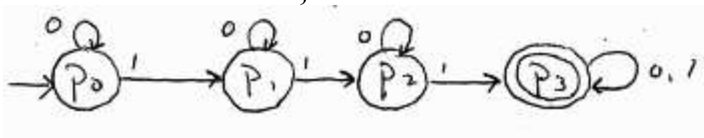
    Write a formal definition of the construction. (5 pts.)

7. DFA's are defined below for the languages L_1 and L_2 on $\Sigma = \{0,1\}$.

$L_1 = \{w \mid \text{every other symbol in } w\text{, beginning with the second symbol, is a } 1\}$



$L_2 = \{w \mid w \text{ contains at least three 1's}\}$



Using the method shown in class and in Theorem 1.25, create a DFA for the union of L_1 and L_2.

(15 pts.)

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8. Give an algorithm that shows that the acceptance problem for regular expressions is Turing-decidable.
   A_{RegExp} = {<E, w> | E is a regular expression, w is a string in the language of E, and $w \in \mathcal{L}(E)$}

(10 pts.)

9. Prove that A_{TM} is not decidable. (15 pts.)

A_{TM} = {<M,w> | M is a TM, w is a string in the language of the TM and $w \in L(M)$}

10. Prove the “if” direction ($\Leftarrow$) of the following theorem.

A language is decidable iff it is Turing-recognizable and co-Turing-recognizable.
(10 pts.)