Theory of Computation, CSCI 438, Spring 2021
Exam 1, Jan 29 **Name _________________**

Definitions

1. What is the signature (i.e. inputs and outputs) of the transition function, δ, in the following definition. (5 pts.)

   A deterministic finite automaton, DFA, is a machine $M = (Q, \Sigma, \delta, q_0, F)$ where
   - Q - finite set of states
   - Σ - finite set of symbols, input alphabet
   - δ - a transition function
   - $q_0 \in Q$, initial state
   - $F \subseteq Q$, set of accept states

   $\delta: Q \times \Sigma \rightarrow Q$

2. Give the definition of regular expressions which is given in the text and which we have been using in class. (5 pts.)

   Regular expressions are defined as follows:
   Basis:
   - a where $a \in \Sigma$ is a regular expression
   - ε is a regular expression
   - Φ is a regular expression

   Given regular expressions R_1 and R_2
   - $R_1 \cup R_2$ is a regular expression
   - $R_1 \circ R_2$ is a regular expression (this is often written $R_1 R_2$)
   - R_1^* is a regular expression

Problem Solving

3. Define a regular expression for
   L={w | w ∈ {0,1}*, it at least three characters long and its third symbol is 0 }

   (6 pts.)

   $(0 \cup 1) \circ (0 \cup 1) \circ 0 \circ (0 \cup 1)^*$

4. For the following, circle either True or False (9 pts.)

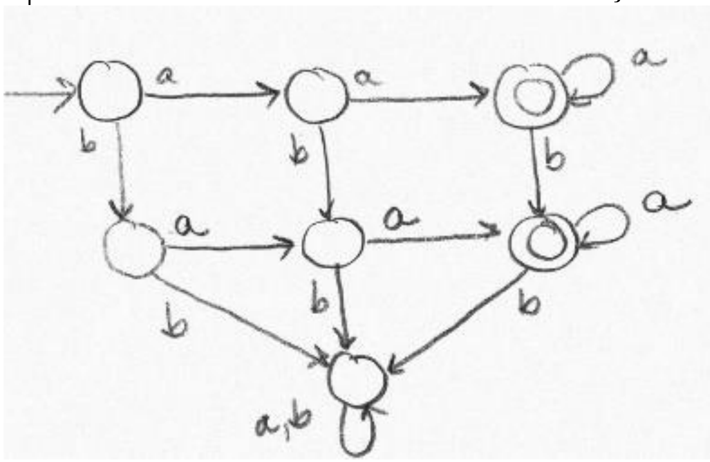
   A language is recognized by exactly one machine. True False

   A machine recognizes exactly one language. True False

   Given the alphabet $\Sigma=\{0,1\}$ all languages on this alphabet are subsets of Σ^*.

   True False

5. Using the alphabet Σ = {a, b} create a DFA's for the language defined by
{w | w contains at least two a's and at most one b} (10 pts.)

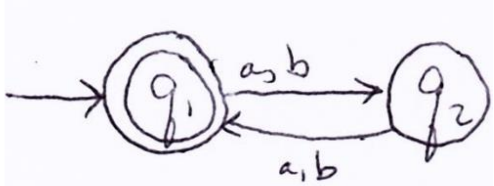


6. Give a regular expression for the language above. You are not required to use the mechanical method described in class. (10 pts.)

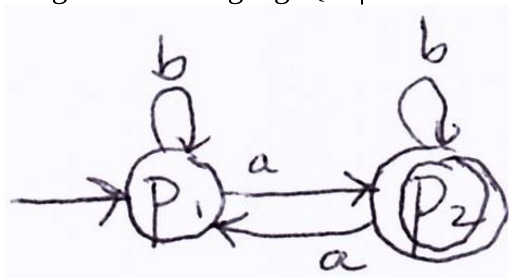
a* (aaa* ∪ baaa* ∪ abaa* ∪ aaba*)

7. Following are two DFAs defined over the alphabet Σ = {a, b}.

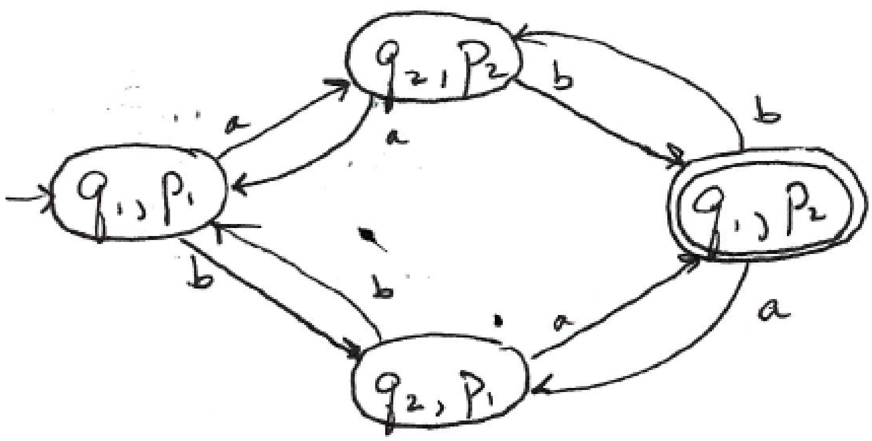
DFA that recognizes the language {w | w has an even length}:



DFA that recognizes the language {w | w has an odd number of a’s}



Use the method described in the text and in class to create a DFA for language
{w | w has an even length and an odd number of a’s } (15 pts.)



8. Prove that regular languages are closed under intersection. That is, show that if the languages A and B are regular, then the language A ∩ B is regular. (Note, this is proving that the construction which you used in the previous question, works.)
(20 pts)

Say that A and B are regular languages. Then we know that there are DFAs

$M_1=(Q_1,\Sigma,\delta_1,q_{1,0},F_1)$ and $M_2=(Q_2,\Sigma,\delta_2,q_{2,0},F_2)$

where $\mathcal{L}(M_1)=A$ and $\mathcal{L}(M_2)=B$.

Consider a new DFA, $M_{intersection}$, defined as follows:

$M_{intersection}$, $= (QxP, \Sigma, \delta`, (q_0, p_0), F_{intersection})$

where $\delta`$ is defined by:

$\delta`((q_i, p_j),a) = (\delta_1(q_i,a), \delta_2(p_j,a))$

and F $M_{intersection}$ is defined by

$F_{intersection}$, $= \{(q_i, p_j)\}$ $q_i \in F_q$ and $p_j \in F_p\}$

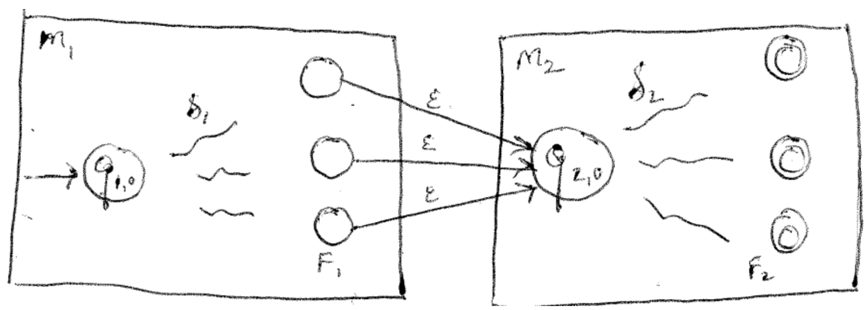
With some thought, it can be see that $M_{intersection}$, accepts $L_1 \cap L_2$. Thus $L_1 \cap L_2$ is regular and regular languages are closed under intersection.

9. Consider the proof of the following statement:

If a regular expression describes a language, the language is regular.

A portion of this proof includes proving that, given regular expressions R_1 and R_2 which are recognized by NFAs, $M_1 = (Q_1, \sum, \delta_1, q_{1,0}, F_1)$ and $M_2 = (Q_2, \sum, \delta_2, q_{2.0}, F_2)$, respectively, there is an NFA that recognizes $R_1 \circ R_2$ (R_1 concatenate R_2).

Draw a picture of the NFA which recognizes $R_1 \circ R_2$. (10 pts.)



Write a formal definition of this construction. (10 pts.)

$R_1 \circ R_2$ is a regular expression
can be described by the NFA $M_{concat} = (Q_1 \cup Q_2, \sum, \delta, q_{1,0}, F_2)$ where
δ (q,x) is equal to the following:
δ_1 (q,x) for $q \in Q_1 - F_1$ or $x \in \sum$
$\delta_1 (q,x) \cup \{q_{2,0}\}$ for $q \in F_1$ and $x = \varepsilon$
δ_2 (q,x) for $q \in Q_2$

Extra Credit:
For any string $w=w_1w_2\ldots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n\ldots w_2w_1$. For any language A, let $A^R=\{w^R \mid w\in A\}$. Show that if A is regular, so is A^R. (5 pts.)

This question is asking if regular languages are closed under the reverse operation. Regular languages are closed under the reverse operation.

Proof: Suppose that A is regular. By the definition of what it means for a language to be regular, there is a DFA which recognizes A. To show A^R is regular, we need to define a DFA that recognizes A^R.

Since A is recognized by a DFA, there is also an NFA that recognizes A. Using the results of Exercise 1.11 from last time, there is also an NFA with only one accepting state that recognizes A. Call this $M = (Q, \Sigma, \delta, q_0, \{q_{accept}\})$. Thus $\mathcal{L}(M)=A$.

Consider the NFA $M^R = (Q, \Sigma, \delta^R, q_{accept}, \{q_0\})$ where

$$p\in\delta^R(q,a) \text{ whenever } q\in\delta(p,a).$$

That is, consider the new NFA, M^R, whose start state is the accept state of M, where all transitions are reversed, and that has the single accept state, that was the start state of M. With some thought it can be seen that $\mathcal{L}(M^R)=A^R$.

We have defined an NFA that recognizes A^R. Using the theorem that says that for every NFA, there is an equivalent DFA, there is a DFA which recognizes A^R, and A^R is regular.