Theory of Computation, CSCI 438, Spring 2021
Exam 1, Jan 29 **Name _________________**

Definitions

1. What is the signature (i.e. inputs and outputs) of the transition function, δ, in the following definition. (5 pts.)

   A deterministic finite automaton, DFA, is a machine $M = (Q, \Sigma, \delta, q_0, F)$ where
   - Q - finite set of states
   - Σ - finite set of symbols, input alphabet
   - δ - a transition function
   - $q_0 \in Q$, initial state
   - $F \subseteq Q$, set of accept states

2. Give the definition of regular expressions which is given in the text and which we have been using in class. (5 pts.)

Problem Solving

3. Define a regular expression for
   L={w | w ∈ {0,1}*, it at least three characters long and its third symbol is 0 }
   (6 pts.)

4. For the following, circle either True or False (9 pts.)

   A language is recognized by exactly one machine. True False

   A machine recognizes exactly one language. True False

   Given the alphabet Σ={0,1} all languages on this alphabet are subsets of Σ^*.

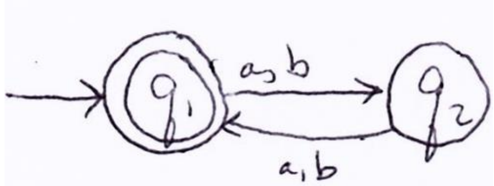
   True False

5. Using the alphabet $\Sigma = \{a, b\}$ create a DFA's for the language defined by
   {w | w contains at least two a's and at most one b} (10 pts.)

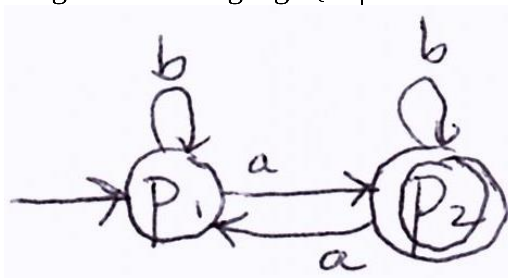
6. Give a regular expression for the language above. You are not required to use the mechanical method described in class. (10 pts.)

7. Following are two DFAs defined over the alphabet Σ = {a, b}.

DFA that recognizes the language {w | w has an even length}:



DFA that recognizes the language {w | w has an odd number of a's}



Use the method described in the text and in class to create a DFA for language
{w | w has an even length and an odd number of a's } (15 pts.)

Proofs

8. Prove that regular languages are closed under intersection. That is, show that if the languages A and B are regular, then the language $A \cap B$ is regular. (Note, this is proving that the construction which you used in the previous question, works.)

(20 pts)

9. Consider the proof of the following statement:

If a regular expression describes a language, the language is regular.

A portion of this proof includes proving that, given regular expressions R_1 and R_2 which are recognized by NFAs, $M_1 = (Q_1, \sum, \delta_1, q_{1,0}, F_1)$ and $M_2 = (Q_2, \sum, \delta_2, q_{2.0}, F_2)$, respectively, there is an NFA that recognizes $R_1 \circ R_2$ (R_1 concatenate R_2).

Draw a picture of the NFA which recognizes $R_1 \circ R_2$. (10 pts.)

Write a formal definition of this construction. (10 pts.)

Extra Credit:

For any string $w=w_1w_2\ldots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n\ldots w_2w_1$. For any language A, let $A^R=\{w^R \mid w\in A\}$. Show that if A is regular, so is A^R. (5 pts.)