Discrete Structures, CSCI 246, Fall 2017
Review for Final, Dec. 11

Logic

Propositional logic:

- Know the truth table for the logical connectives: $\wedge$, $\vee$, $\neg$, $\oplus$, $\rightarrow$, $\leftrightarrow$
- Be able to convert:

Conversion
Given English and propositions, be able to the express the English mathematically using the propositions
Given English, be able to define propositions and express the English mathematically using the propositions that you defined
Given a compound proposition and the meaning of the propositions, be able to express the compound proposition in English

- Given a compound proposition be able to create a truth table for it.
- Know the definition of tautology, contradiction and contingency
- Know the difference between $\equiv$, also written $\Leftrightarrow$, and the biconditional ($\leftrightarrow$)
- Using the tables of logical equivalences, logical equivalences involving conditional statements, and logical equivalences involving biconditional statements, be able to prove the compound propositions are equivalent.

Predicate logic:

- Know the difference between propositional and predicate logic.
- Be able to convert:

Conversion
Given English and predicates, be able to the express the English mathematically, possibly using quantifiers ($\forall$ and $\exists$)
Given English, be able to define predicates and express the English mathematically, possibly using quantifiers ($\forall$ and $\exists$)
Given a compound predicate and the meaning of the predicates, be able to express the compound predicate in English

- Know when an argument is valid.
- Know and be able to use DeMorgan's Law for quantifiers.
- Be able to explain the meaning of quantification on more than one variable.
- Using the tables of rules of inference, rules of inference for quantified statements, universal modus ponens and modus tollens, and DeMorgan's Laws for quantifiers, be able to prove logical statements
- Know the difference between direct proofs (including exhaustive proofs), proof by contrapositive, and proof by contradiction and be able to prove things using each of the techniques, and be able to state which proof technique you are using.
- Know the difference between a theorem, lemma, corollary, axiom
- Be able to prove things by breaking them into cases

Sets

- Be able to describe sets using set builder notation
- Know the special sets: N, Z, Z^+, Z^-, Q, R
- Know the meaning of the set operations $\subseteq$, $\subset$, $\cup$, $\cap$, $\setminus$, complement of A, $\overline{A}$, cardinality of A, |A|, power set of A, $\mathscr{P}$(A), Cartesian product of two sets, AxB
- Be able to show combinations of sets using Venn diagrams
- Know what is meant by two sets being disjoint
- Know what is meant by a partition of a set and be able to answer questions pertaining to partitions
- Using the table of set identifies be able to prove statements about sets
- Using set builder notation be able to prove set identifies, possibly those given in the table

Functions

- Know the definition of a function
- Know the meaning of the domain, target (or co-domain), and range of a function
- Know the meaning of a 1-1 function (injection), onto function (surjection), and 1-1 and onto (bijection, text calls a 1-1 correspondence)
- Know the definition of the floor and ceiling functions
- Know what it means for a function to be invertible and be able to find the inverse of an invertible function
- Know what is meant by the composition of functions and be able to determine the result of composing functions on a value
- Know the meaning of the terms increasing, strictly increasing, decreasing, strictly decreasing, as they apply to functions
- Be able to show that two finite sets are the same size
- Be able to perform conversions

Induction and Recursion

- Know the definition of a sequence (functions from N to a set)
- Be able to define a sequence by describing its nth element
- Know the definition of an arithmetic progression (also called an arithmetic sequence) and a geometric progression (also called a geometric sequence)
- Know the meaning of and be able to use summation notation
- Know, or be able to discover, the laws of arithmetic that hold for Σ
- Know the Principle of Mathematical Induction (also known as the well-ordering principle) and be able prove statements using this principle
- Know the Principle of Strong Mathematical Induction and be able prove statements using this principle

- Know when structural induction can be used and be able to use it to prove statements about recursively defined structures such as well –formed compound propositions, tree, rooted trees, and others

Counting

- Know the Sum Rule, Subtraction Principle, Product Rule and the Bijection Rule.
- Know what permutations are and be able to use the idea of permutations to count items.
- Know what combinations are and be able to use the idea of combinations to count items.
- Be able to use the idea of functions to count items.
- Be able to use the idea of permutations with repetitions to count items.
- Know the Pigeon Hole Principle and be able to use it in proofs.
- Know the Binomial Theorem and be able to use it to give the coefficient of terms in the expansion of a binomial.
- Know Pascal's Identify and that the nth row gives the coefficients in the expansion of $(x+y)^n$.

Relations

- Know that a binary relation is a relationship between two sets and that elements (a,b) of a binary relation R can be written aRb, $(a,b) \in R$, in an arrow diagram, or a matrix representation.
- Know what is meant for a relation on a set to be reflexive, transitive and symmetric, and know how each of these can be represented in a digraph.
- Know what is meant by an equivalence relation.
- Know that an equivalence relation partitions a set into disjoint equivalence classes.
- Know how to denote an equivalence class and how to show that two elements belong to the same equivalence class.
- Know what is meant by the in-degree and the out-degree of a vertex in a digraph. (repeated this again under graphs.)
- Know when the composition of two relations can be computed and be able to compute the composition when it is possible.
- Know what is meant by the kth closure of a relation.
- Know what is meant by the transitive closure of a relation.

Graphs

- Know that graphs can be written as G=(V,E) where V is a nonempty set of vertices and E are edges on the vertices.
- Know that edges in a digraph edges are ordered pairs, while edges in an undirected graph edges are sets of pairs.
- Know the meaning of endpoints of an edge and that an edge {a, b} is incident to vertices a and b, making a and b neighbors, also stated as a and b are adjacent.
- Know what is meant by the in-degree and the out-degree of a vertex in a digraph.
- Know what is meant by the degree of a vertex in an undirected graph.
- Know what is meant by the total degree of a graph.
- Know what is meant by a subgraph of a graph and be able to count the subgraphs of a given graph.
- Know what is meant by parallel edges, also called multi-edges, and self-loops in a graph.
- Know that a simple graph has no parallel edges and no self-loops, and that a graph containing parallel edges and/or self-loops is called a multi-graph/multi-digraph.
- Know what is meant by an isolated vertex and a pendant vertex.
- Be able to state and to prove the Handshaking Theorem.
- Know what is meant by the special graphs: K_n, C_n, Q_n and $K_{n,m}$.
- Know what is meant by a walk, path, circuit, cycle, and length of a walk in a graph.
- Know what is meant by a connected component of a graph
- Know what is meant by k-vertex-connected and k-edge-connected, and given a graph, be able to determine its k-vertex connection and k-edge connection
- Know the upper bound for vertex and edge connectivity

Trees

- Know a definition for a tree.
- Know a definition for rooted trees.
- Know the vocabulary of trees: parent, ancestor, child, descendant, leaf, siblings and subtree.

Vocabulary

Logical Terms

Tautology
Contradiction
Contingency
Bi-conditional
Proposition
Predicate
Valid argument
DeMorgan's law
Theorem
Lemma
Corollary
Axiom

Set terminology

N
Z (Z^+, Z^-)
Q
R
Cardinality
Cartesian product
Power set

Function terminology

Function
Domain
Co-domain or target
Range
Injective function
Surjective function
Bijection (or 1-1 correspondence)
Invertible
Composition

Sequence terminology

Sequence
Arithmetic progression
Geometric sequence
Common difference
Ratio
Closed formula
Summation

Recursion terminology

Principle of Mathematical Induction
Principle of Strong Mathematical Induction

Counting terminology

Bijection Rule
Product Rule
Sum Rule
Subtraction Principle
Permutation
Combination
Multi-sets
Binomial Theorem
Pascal's Identify

Terminology of Relations

Binary relation
Reflexive
Symmetric
Transitive
Equivalence relation
Equivalence class
Partition
Composition of two relations
k^{th} closure of a relation
Transitive closure of a relation

Terminology of Graphs

Di-graph
Undirected graph
Endpoints of an edge
Incident to
Adjacent to
In-degree
Out-degree
Degree
Total degree
Subgraph
Parallel edges

Multi-edges
Self-loops
Multi-graph
Simple graph
Isolated vertex
Pendant vertex
Handshaking Theorem
K_n
C_n
Q_n
$K_{n,m}$
Walk
Path
aCircuit
Cycle
Length of a walk
Connected

Terminology of Trees

Tree
Rooted tree
Parent
Ancestor
Child
Descendant
Leaf
Siblings
Subtree