

Data Mining, CSCI 347, Fall 2019
Homework 1, Naïve Bayes, due Sept. 23

Consider the following database with the class value being “buys_computer”.
(In this image the open parenthesis represents a less-than sign, so the “(<=30” value for age represents “<=30”.)

Relation: Customers					
No.	buys_computer Nominal	age Nominal	income Nominal	student Nominal	credit_rating Nominal
1	no	(<=30	high	no	fair
2	no	(<=30	high	no	excellent
3	yes	31..40	high	no	fair
4	yes)40	medium	no	fair
5	yes)40	low	yes	fair
6	no)40	low	yes	excellent
7	yes	31..40	low	yes	excellent
8	no	(<=30	medium	no	fair
9	yes	(<=30	low	yes	fair
10	yes)40	medium	yes	fair
11	yes	(<=30	medium	yes	excellent
12	yes	31..40	medium	no	excellent
13	yes	31..40	high	yes	fair
14	no)40	medium	no	excellent
15	yes)40	high	no	excellent

- Write the formula to predict if a 25 year old student with a low income and a fair credit rating is likely to purchase a computer using Naïve Bayes Theorem. That is, write formulas for the following.

$$\Pr[\text{buys_computer}=\text{'yes'} \mid \text{age is } \leq 30 \ \& \ \text{income}=\text{'low'} \ \& \ \text{student}=\text{'yes'} \ \& \ \text{credit_rating}=\text{'fair'}]$$

$$= \frac{\Pr[\text{age is } \leq 30 \mid \text{buys_computer}=\text{'yes'}] * \Pr[\text{income}=\text{'low'} \mid \text{buys_computer}=\text{'yes'}] * \Pr[\text{student}=\text{'yes'} \mid \text{buys_computer}=\text{'yes'}] * \Pr[\text{credit_rating}=\text{'fair'} \mid \text{buys_computer}=\text{'yes'}] * \Pr[\text{buys_computer}=\text{'yes'}]}{\Pr[\text{age is } \leq 30 \ \& \ \text{income}=\text{'low'} \ \& \ \text{student}=\text{'yes'} \ \& \ \text{credit_rating}=\text{'fair'}]}$$

$$\Pr[\text{buys_computer}=\text{'no'} \mid \text{age is } \leq 30 \ \& \ \text{income}=\text{'low'} \ \& \ \text{student}=\text{'yes'} \ \& \ \text{credit_rating}=\text{'fair'}]$$

$$= \frac{\Pr[\text{age is } \leq 30 \mid \text{buys_computer}=\text{'no'}] * \Pr[\text{income}=\text{'low'} \mid \text{buys_computer}=\text{'no'}] * \Pr[\text{student}=\text{'yes'} \mid \text{buys_computer}=\text{'no'}] * \Pr[\text{credit_rating}=\text{'fair'} \mid \text{buys_computer}=\text{'no'}] * \Pr[\text{buys_computer}=\text{'no'}]}{\Pr[\text{age is } \leq 30 \ \& \ \text{income}=\text{'low'} \ \& \ \text{student}=\text{'yes'} \ \& \ \text{credit_rating}=\text{'fair'}]}$$

2. Since this dataset is small, determine the values to use in the above formulas, counting instance by hand. Apply a Laplace estimator of 1, to avoid probabilities of 0.

age			income			student			credit_rating			buys_computer	
	yes	no		yes	no		yes	no		yes	no	Yes	no
<=30	2/3	3/4	high	3/4	2/3	yes	6/7	1/2	fair	6/7	2/3	10/11	5/6
31..40	4/5	0/1	medium	4/5	2/3	no	4/5	4/5	excellent	4/5	3/4		
>40	4/5	2/3	low	3/4	1/2								
<=30	3/13	4/8	high	4/13	3/8	yes	7/12	2/7	fair	7/12	3/7	11/17	6/17
31..40	5/13	1/8	medium	5/13	3/8	no	5/12	5/7	excellent	5/12	4/7		
>40	5/13	3/8	low	4/13	2/8								

3. Calculate the values, ignoring the denominators.

$\Pr[\text{buys_computer}=\text{'yes'} \mid \text{age is} \leq 30 \ \& \ \text{income}=\text{'low'} \ \& \ \text{student}=\text{'yes'} \ \& \ \text{credit_rating}=\text{'fair'}]$

$$\begin{aligned}
 & \Pr[\text{buys_computer}=\text{'yes'} \mid E] \\
 &= (\Pr[\text{age is} \leq 30 \mid \text{buys_computer}=\text{'yes'}] * \\
 & \quad \Pr[\text{income}=\text{'low'} \mid \text{buys_computer}=\text{'yes'}] * \\
 & \quad \Pr[\text{student}=\text{'yes'} \mid \text{buys_computer}=\text{'yes'}] * \\
 & \quad \Pr[\text{credit_rating}=\text{'fair'} \mid \text{buys_computer}=\text{'yes'}] * \\
 & \quad \Pr[\text{buys_computer}=\text{'yes'}]) / \Pr[E] \\
 &= (3/13 * 4/13 * 7/12 * 7/12 * 11/17) / \Pr[E] \\
 &\sim 0.016 / \Pr[E]
 \end{aligned}$$

$\Pr[\text{buys_computer}=\text{'no'} \mid \text{age is} \leq 30 \ \& \ \text{income}=\text{'low'} \ \& \ \text{student}=\text{'yes'} \ \& \ \text{credit_rating}=\text{'fair'}]$

$$\begin{aligned}
 & \Pr[\text{buys_computer}=\text{'no'} \mid E] \\
 &= (\Pr[\text{age is} \leq 30 \mid \text{buys_computer}=\text{'no'}] * \\
 & \quad \Pr[\text{income}=\text{'low'} \mid \text{buys_computer}=\text{'no'}] * \\
 & \quad \Pr[\text{student}=\text{'yes'} \mid \text{buys_computer}=\text{'no'}] * \\
 & \quad \Pr[\text{credit_rating}=\text{'fair'} \mid \text{buys_computer}=\text{'no'}] * \\
 & \quad \Pr[\text{buys_computer}=\text{'no'}]) / \Pr[E] \\
 &= (4/8 * 2/8 * 2/7 * 3/7 * 6/17) / \Pr[E] \\
 &\sim 0.005 / \Pr[E]
 \end{aligned}$$

4. Normalize the results.

$$\Pr[\text{buys_computer}=\text{'yes'} \mid E] = 0.016 / (0.016 + 0.005) = 76\%$$

$$\Pr[\text{buys_computer}=\text{'no'} \mid E] = 0.005 / (0.016 + 0.005) = 24\%$$

The actual numbers give 74.3% and 25.7%

5. What would be predicted?

Yes would be predicted.

6. Use Weka to check your results.
 - a. Go to the classify tab and make “buys_computer” the class attribute (can be set using the drop-down box that appears above the “Start” button)
 - b. Run the classifier weka.classifiers.bayes,NaiveBayes on the dataset (you can use cross-validation or use the training set for testing because the classifier model uses the full training set)
 - c. Compare Weka’s statistical values with yours.

Weka results match:

=== Classifier model (full training set) ===

Naive Bayes Classifier

Attribute	Class	
	yes	no
	(0.65)	(0.35)
=====		
age		
<=30	3.0	4.0
31..40	5.0	1.0
>40	5.0	3.0
[total]	13.0	8.0
income		
high	4.0	3.0
medium	5.0	3.0
low	4.0	2.0
[total]	13.0	8.0
student		
yes	7.0	2.0
no	5.0	5.0
[total]	12.0	7.0
credit_rating		
fair	7.0	3.0
excellent	5.0	4.0
[total]	12.0	7.0

6. Use Weka to classify the above instance.

buys_computer	age	income	student	credit_rating
?	<=30	low	yes	fair

In order to do this do the following:

- Create a new dataset which is like the original, only it just contains the above instance.

Your dataset can be:

```
% Test dataset consisting of a single instance

@relation Customers

@attribute buys_computer {yes, no}
@attribute age {<=30, 31..40, >40}
@attribute income {high, medium, low}
@attribute student {yes, no}
@attribute credit_rating {fair excellent}

@data
?,<=30, low, yes, fair
```

- On the Classify tab, in the Test options area, chose “Supplied test set” and select the new dataset with the single instance.
- Click the “more options...” button and check the “Output predictions” box.
- Run the Naïve Bayes classifier to see how Weka would classify this instance.
- Do the results match what you expected.

Yes.