

**Concepts of Programming Languages, CSCI 305, Fall 2021**  
**Context-Free Grammars, Section 2.1.2**  
**Precedence & Associativity in Context-Free Grammars, Sept. 22**

Context-Free Grammars

1. Give a context free grammar for the language on  $\Sigma = \{a,b\}$  defined by  
 $L = \{\text{all strings with no more than three as}\}$

$S \rightarrow \text{ZERO}a \mid \text{ONE}a \mid \text{TWO}a \mid \text{THREE}a$   
 $\text{ZERO}a \rightarrow B$   
 $B \rightarrow b B \mid \varepsilon$   
 $\text{ONE}a \rightarrow B a B$   
 $\text{TWO}a \rightarrow B a B a B$   
 $\text{THREE}a \rightarrow B a B a B a B$

2. Give a context free grammar for the language on  $\Sigma = \{a,b\}$  defined by  
 $L = \{a^n b^n : n \geq 0\}$

$S \rightarrow aSb \mid \varepsilon$

3. Give a context-free grammar that generates the language  
 $A = \{a^i b^j c^k : i=j \text{ or } j=k \text{ where } i,j,k \geq 0\}$

$S \rightarrow IJ C \mid A JK$   
 $C \rightarrow c C \mid \varepsilon$   
 $A \rightarrow a A \mid \varepsilon$   
 $IJ \rightarrow a IJ b \mid \varepsilon$   
 $JK \rightarrow b JK c \mid \varepsilon$

4. Create a context-free grammar for the language  
 $L = \{w\#x \mid w^R \text{ is a substring of } x \text{ for } w,x \in \{0,1\}^*\}$

Hint: another way to write this is:

$L = \{w\# x_1 w^R x_2 \mid x_1, x_2, w \in \{0,1\}^*\}$

$S \rightarrow TX$   
 $T \rightarrow 0T0 \mid 1T1 \mid \#X$   
 $X \rightarrow 0X \mid 1X \mid \varepsilon$