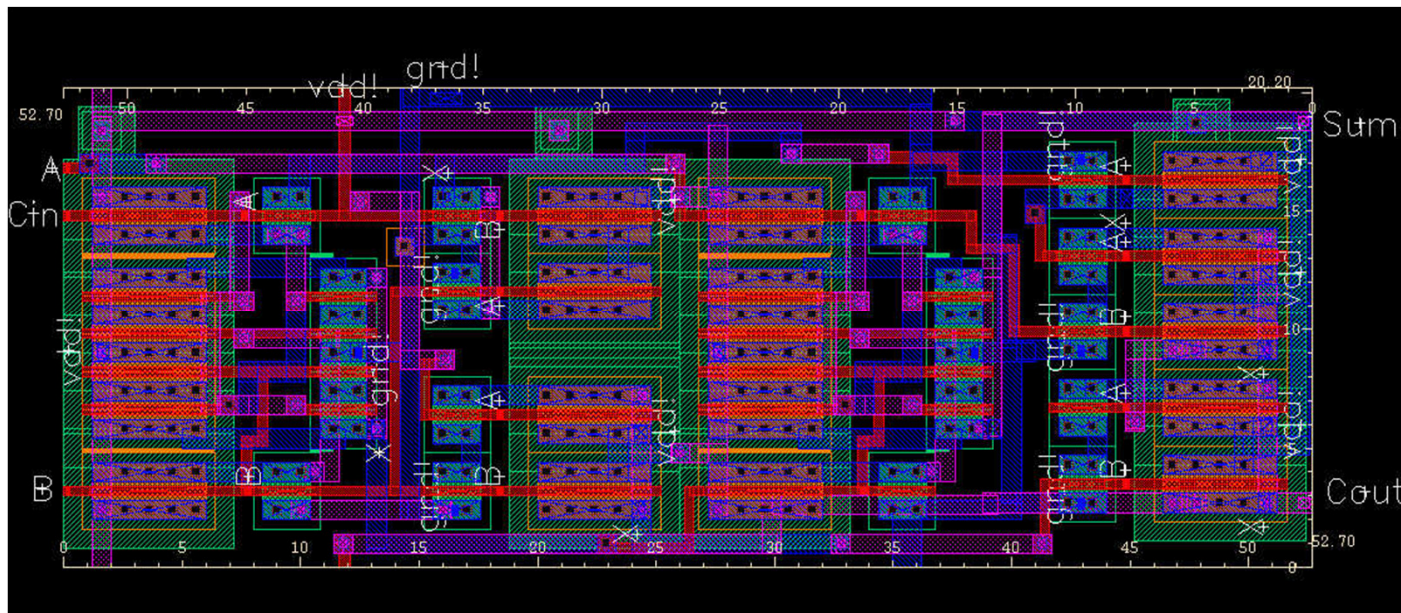


Bit Adder/Subtractor

All of this for a... $1 + 1$???

CSCI 255

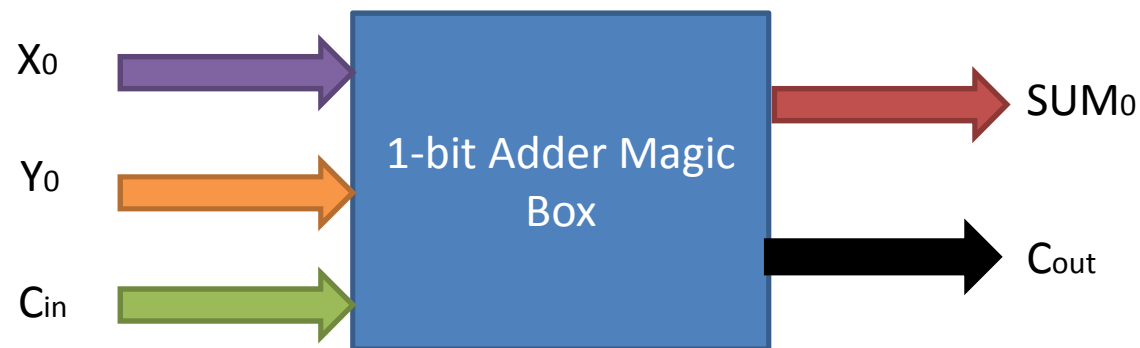


<http://web.eecs.utk.edu/courses/fall2011/ece651/adder-slice-layout.jpg>

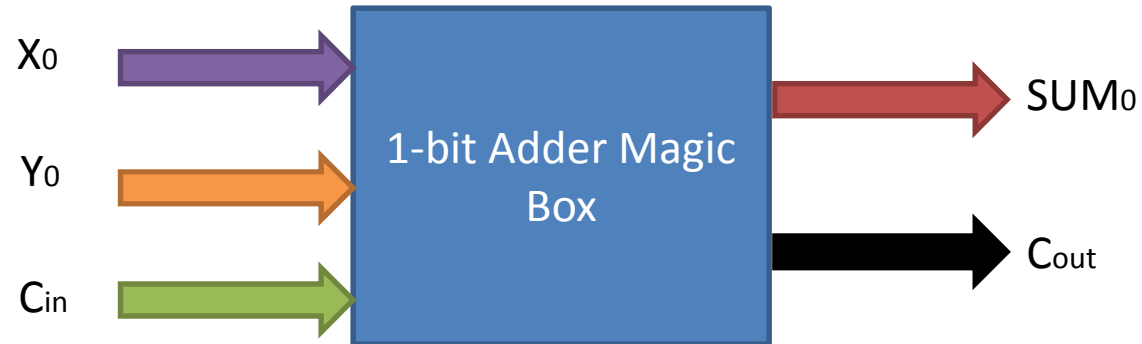


Bit Adder/Subtractor

- When adding a single bit to another bit, there's always designed metal behind it.
- In order to design the hardware for the simple logical addition: you must know what you need to consider as INPUTS & OUTPUTS
- What do you need to add two bits?



Bit Adder/Subtractor



INPUTS:

- X_0 – Bit 0 from X-value
- Y_0 – Bit 0 from Y-value
- C_{in} – Carry In value (...comes from other 1-bit adder or R2 register)

OUTPUTS:

- SUM_0 – Sum result - bit 0 value
- C_{out} – Carry Out value (...goes as a status bit on the R2 register or other adder)



Bit Adder/Subtractor



- Magic Box => $X_0 + Y_0 + C_{in} = C_{out}:SUM_0$
- Mathematical Model -> Truth table
- Truth table -> K-map outputs
- K-map -> Actual circuit schematic



Bit Adder/Subtractor

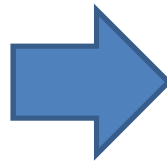
- 1-bit Adder Truth Table:

C_{in}	X_0	Y_0	C_{out}	SUM_0
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Bit Adder/Subtractor

C _{in}	X ₀	Y ₀	C _{out}
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



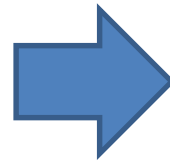
C _{in} X ₀	Y ₀ =0	Y ₀ =1
00	0	0
01	0	1
11	1	1
10	0	1

$$C_{out} = C_{in} X_0 + X_0 Y_0 + C_{in} Y_0$$



Bit Adder/Subtractor

C _{in}	X ₀	Y ₀	SUM ₀
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



C _{in} X ₀	Y ₀ =0	Y ₀ =1
00	0	1
01	1	0
11	0	1
10	1	0

$$SUM_0 = C_{in}' X_0' Y_0 + C_{in} X_0' Y_0' + C_{in} X_0 Y_0 + C_{in}' X_0 Y_0$$



Bit Adder/Subtractor

$$C_{in} \oplus X_0 \oplus Y_0 = C_{in}' X_0' Y_0 + C_{in} X_0' Y_0' + C_{in} X_0 Y_0 + C_{in}' X_0 Y_0$$

if: $a \oplus b = ab' + a'b$

$$a \oplus b \oplus c = (ab' + a'b) c' + (ab' + a'b)' c$$

$$= [ab'c' + a'bc'] + [(a'+b)(a+b')] c$$

$$= [ab'c' + a'bc'] + [a'a + a'b' + ba + bb'] c$$

$$= [ab'c' + a'bc'] + [a'b'c + abc]$$

If $a=C_{in}$, $b=X_0$ & $c=Y_0$, then:

$$C_{in} \oplus X_0 \oplus Y_0 = C_{in}' X_0' Y_0 + C_{in} X_0' Y_0' + C_{in} X_0 Y_0 + C_{in}' X_0 Y_0$$

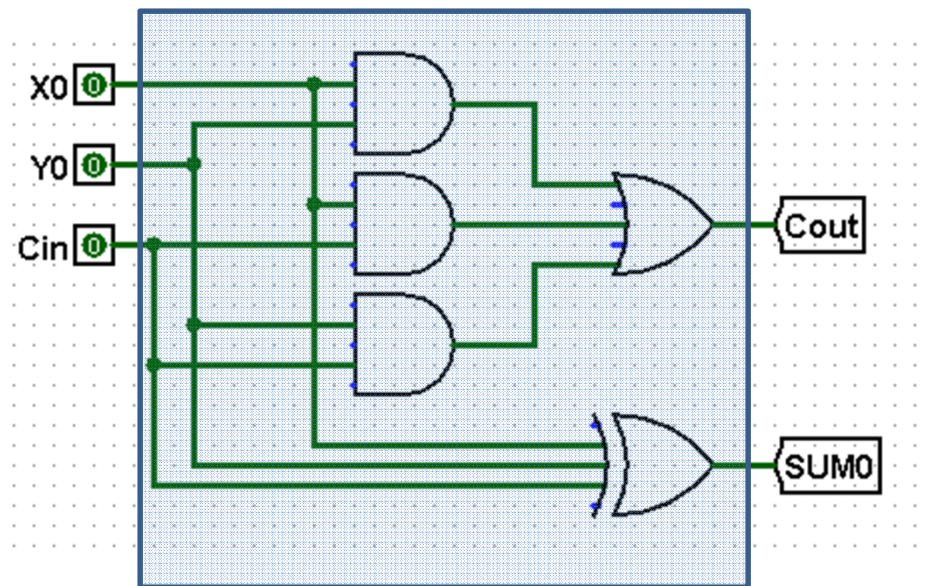
Always show your work.....



Bit Adder/Subtractor

$$\text{SUM}_0 = C_{in} \oplus X_0 \oplus Y_0$$

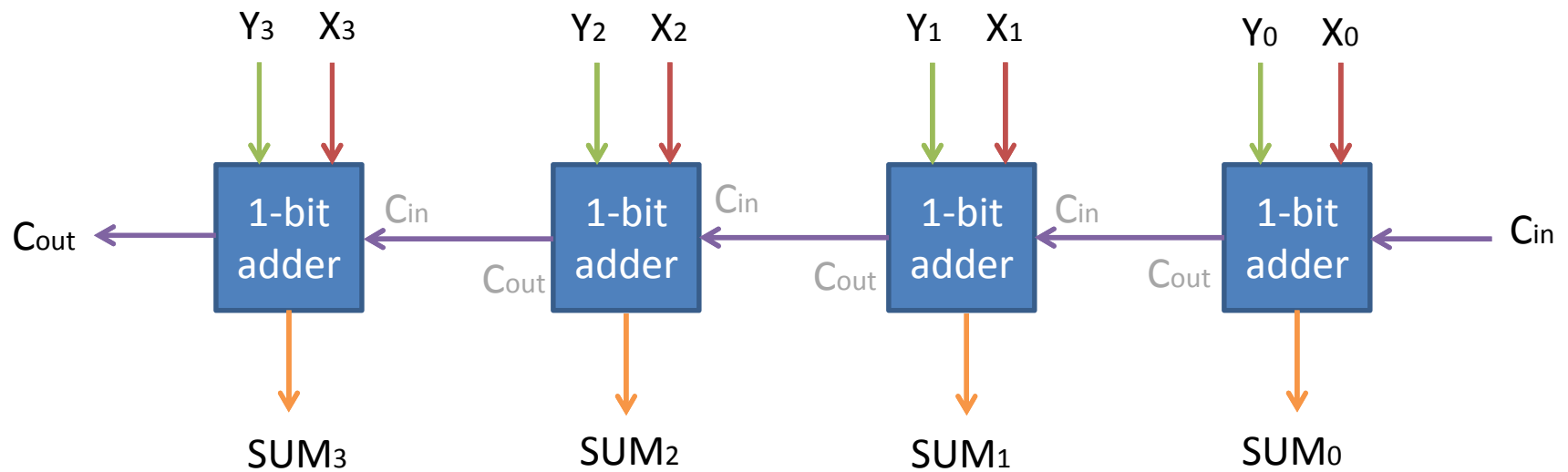
$$\text{Cout} = C_{in} X_0 + X_0 Y_0 + C_{in} Y_0$$



1 – bit Adder



Bit Adder/Subtractor



4 – bit Adder

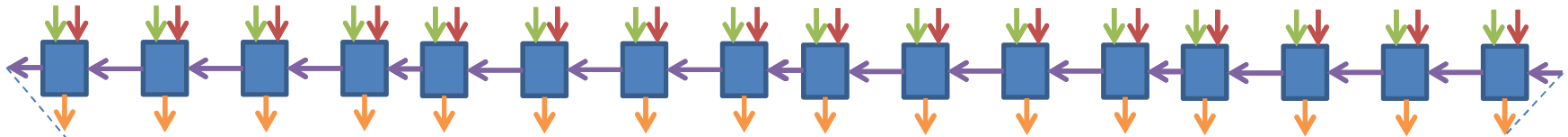
$$X = X_3X_2X_1X_0, Y = Y_3Y_2Y_1Y_0$$

$$SUM = X + Y$$

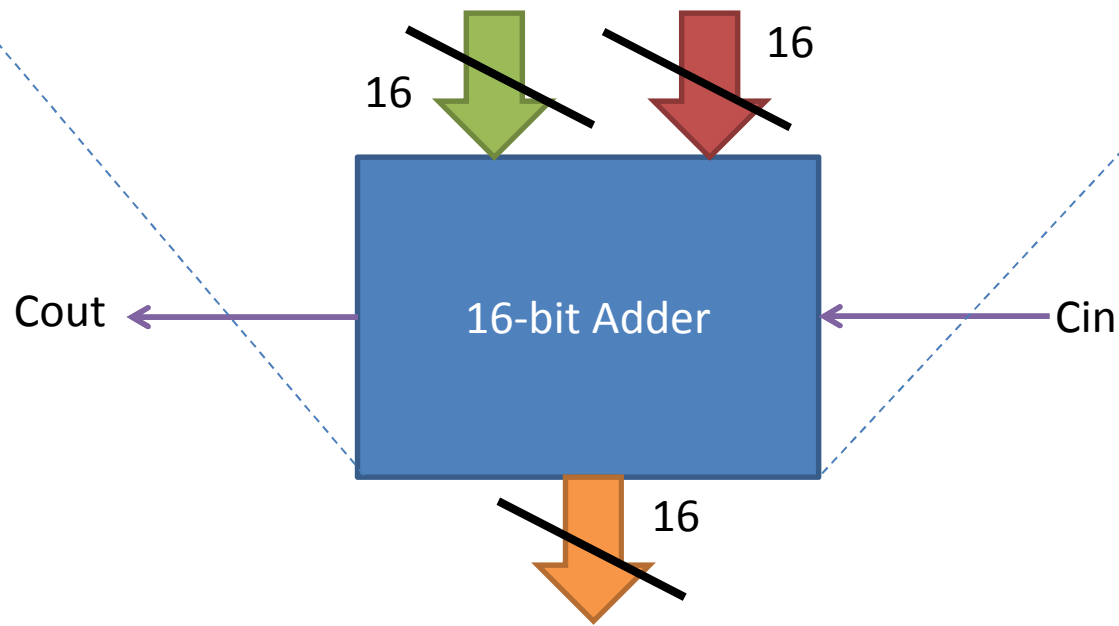


Bit Adder/Subtractor

16 – bit Adder



This is what is present on the MSP430 CPU



Bit Adder/Subtractor

- On paper, it is possible to “**Subtract by Addition**”
- The same thing must be done on the hardware in order to perform the **SUB.(b)** instruction.

SUB.(b) \Rightarrow sub.(b) src,dst : dst + not.src + 1 \rightarrow dst

- Then, we must go back to the Truth Table
- Assume: $C_{in} - X_0 - Y_0$, C_{in} is the borrow bit



Bit Adder/Subtractor

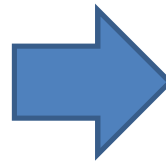
- Subtraction:

X_0	Y_0	Borrow ₀	Borrow ₁	Diff ₀
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



Bit Adder/Subtractor

X ₀	Y ₀	Borrow ₀	Borrow ₁
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



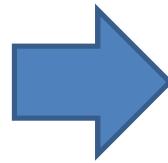
X ₀ Y ₀	B ₀ =0	B ₀ =1
00	0	1
01	1	1
11	0	1
10	0	0

$$B_1 = X_0' Y_0 + X_0' B_0 + Y_0 B_0$$



Bit Adder/Subtractor

X_0	Y_0	Borrow ₀	Diff ₀
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



$X_0 Y_0$	$B_0=0$	$B_0=1$
00	0	1
01	1	0
11	0	1
10	1	0

$$\text{Diff}_0 = X_0 Y_0' B_0' + X_0 Y_0 B_0 + X_0' Y_0 B_0' + X_0' Y_0' B_0$$

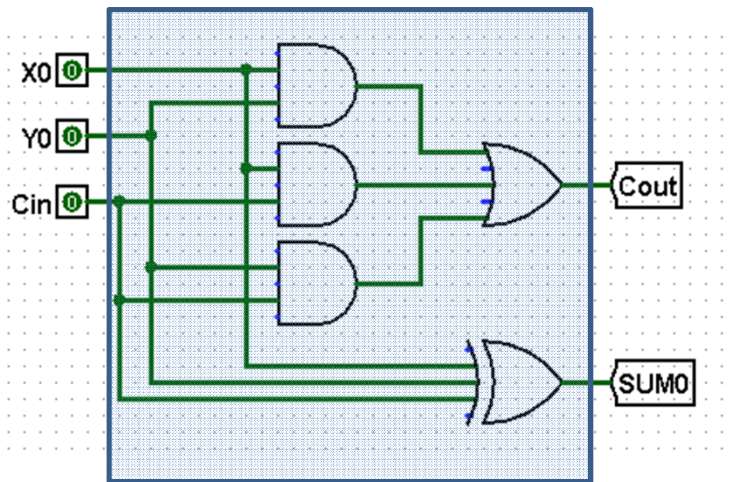
$$\text{Diff}_0 = B_0 \oplus X_0 \oplus Y_0$$



Bit Adder/Subtractor

$$\text{SUM}_0 = C_{in} \oplus X_0 \oplus Y_0$$

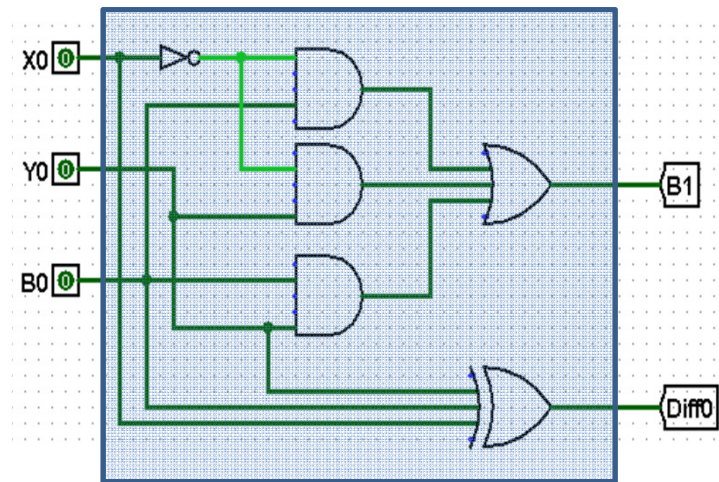
$$C_{out} = C_{in} X_0 + X_0 Y_0 + C_{in} Y_0$$



1 – bit Adder

$$\text{Diff}_0 = B_0 \oplus X_0 \oplus Y_0$$

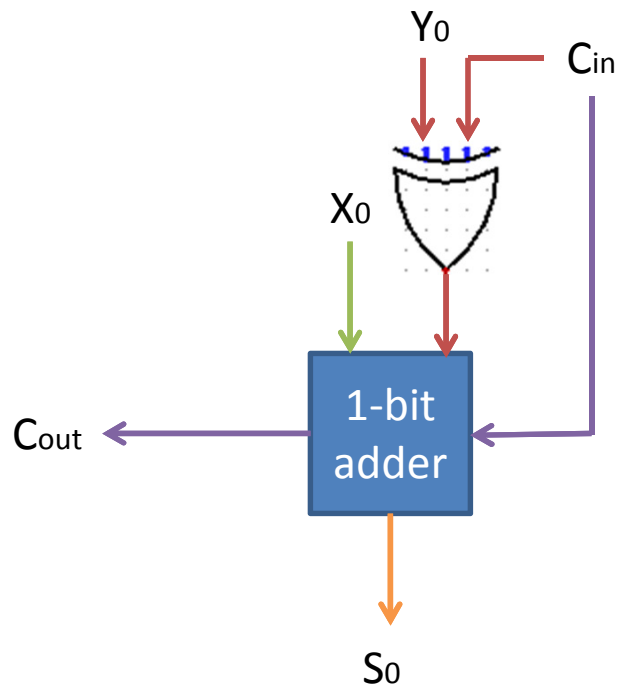
$$B_1 = X_0' Y_0 + X_0' B_0 + Y_0 B_0$$



1 – bit Subtractor



Bit Adder/Subtractor



Cin	Y0	XOR	Y-value
0	0	0	Y0
0	1	1	Y0
1	0	1	Y0'
1	1	0	Y0'

- What happens when: Cin = 0?

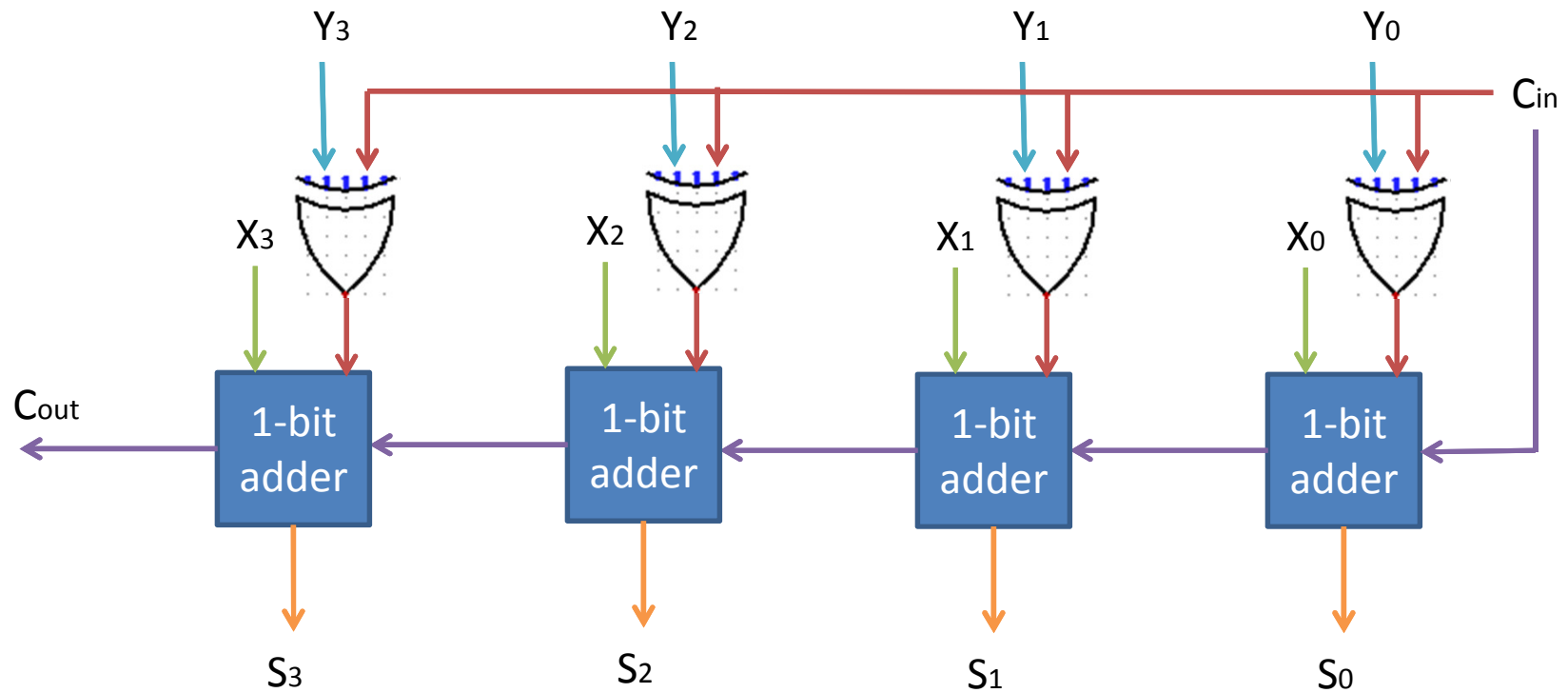
$$C_{in} = 0 + Y_0 + X_0 = C_{out} : S_0$$

- What happens when: Cin = 1?

$$C_{in} = 1 + Y_0' + X_0 = C_{out} : S_0$$



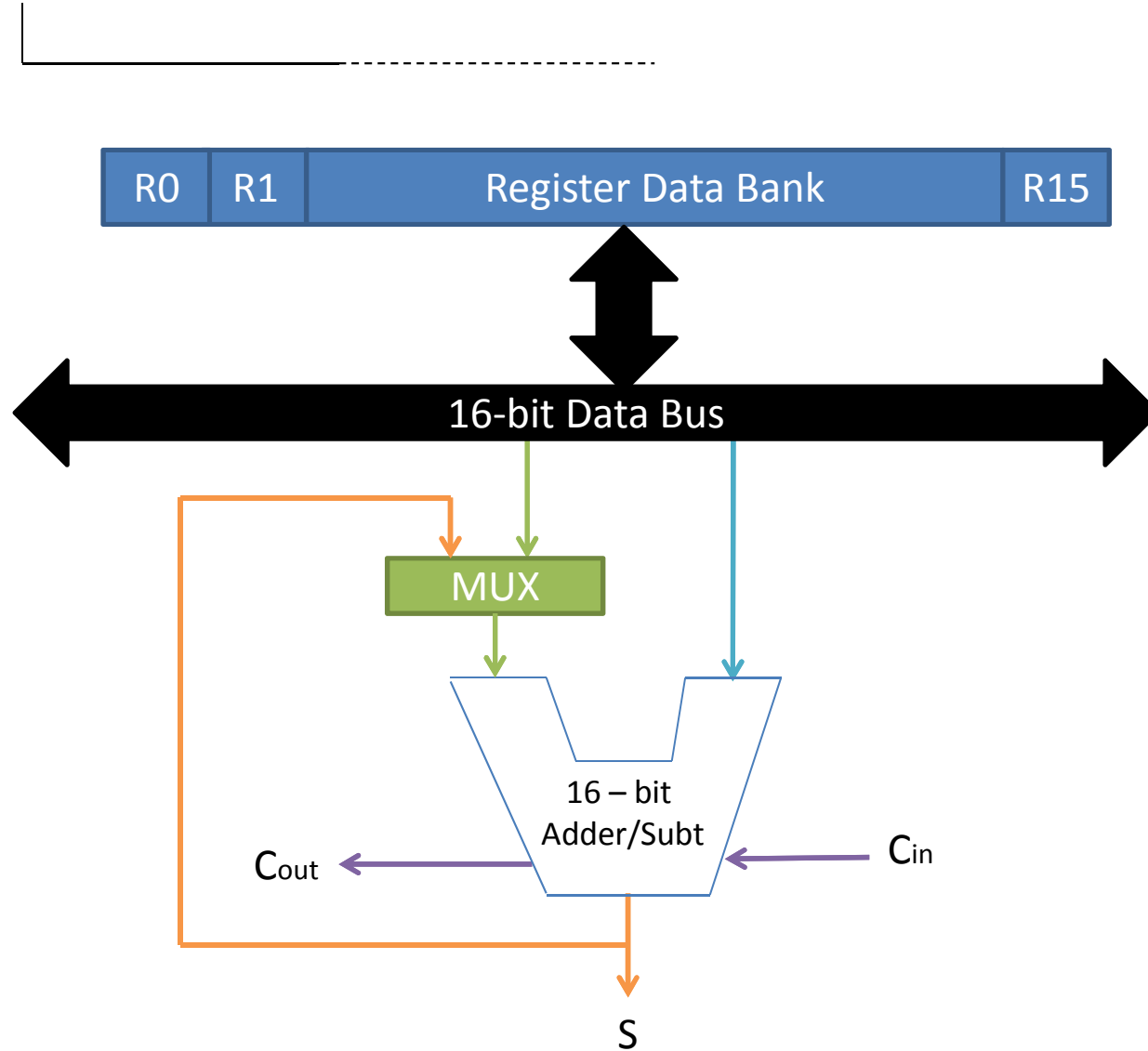
Bit Adder/Subtractor



4 – bit Adder/Subtractor

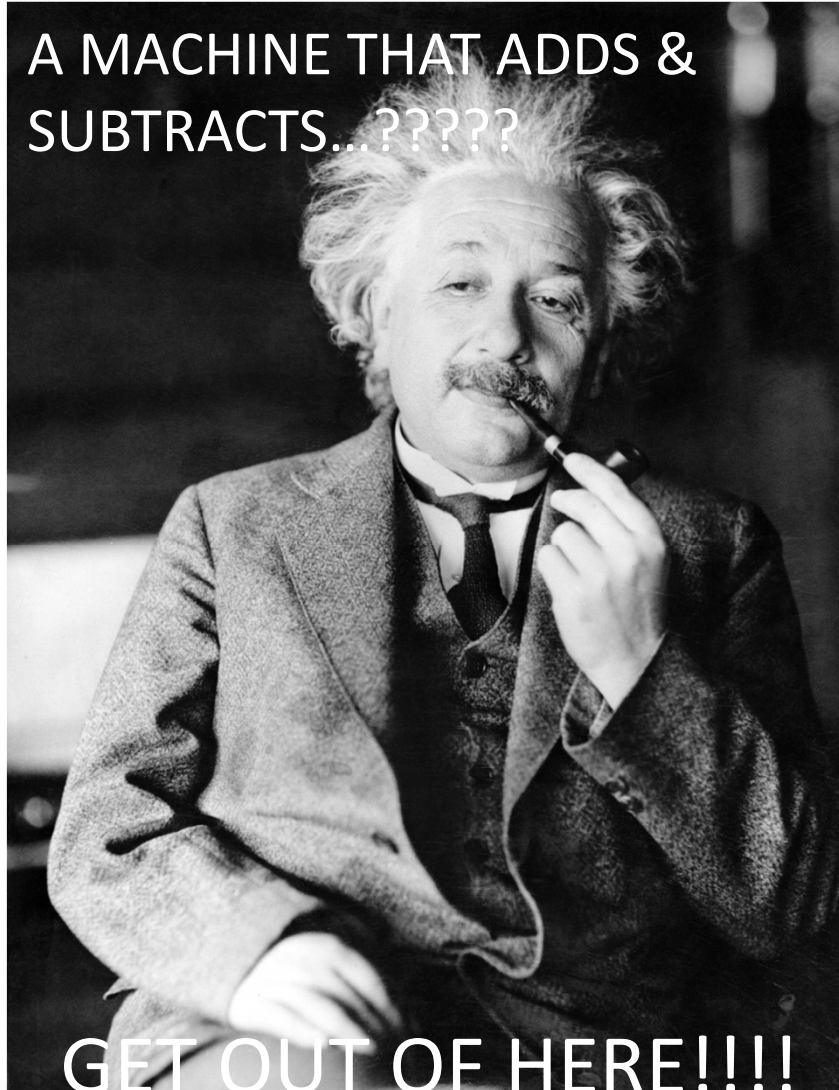


Bit Adder/Subtractor



Bit Adder/Subtractor

A MACHINE THAT ADDS &
SUBTRACTS...?????



GET OUT OF HERE!!!!

