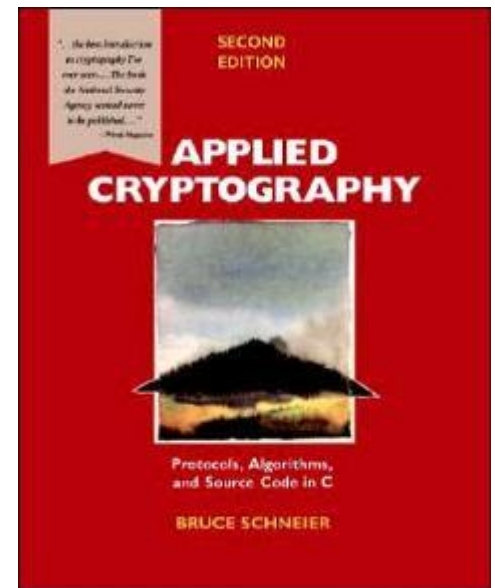
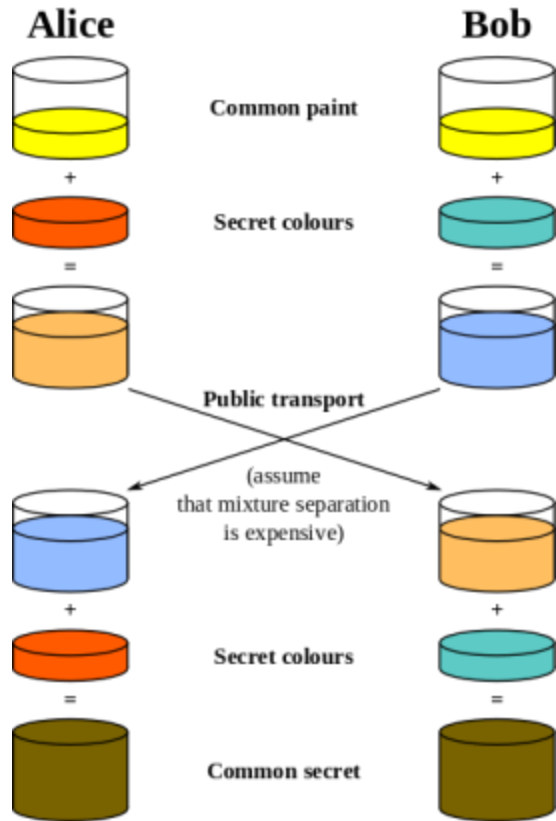


Modern Cryptography

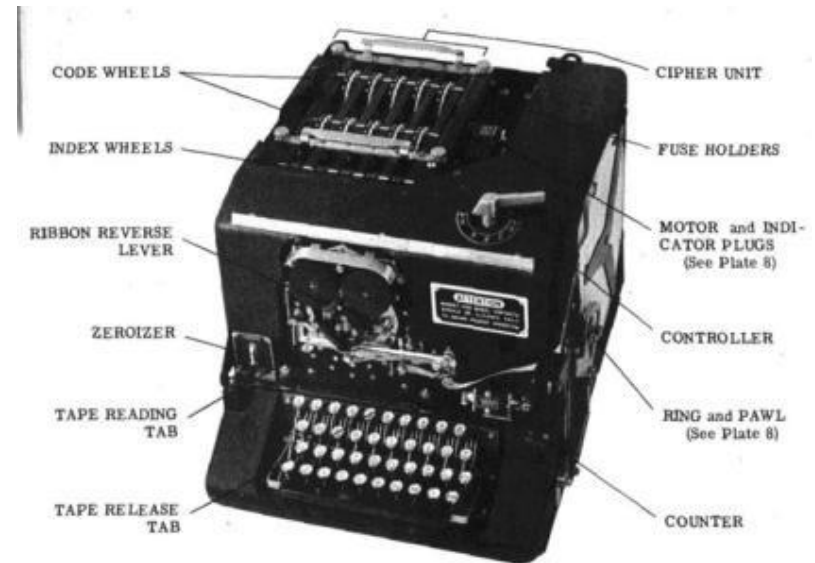


Overview

- Historical cryptography
 - WWII allied encryption
- Modern cryptography
 - Symmetric cryptography
 - DES/3DES
 - AES
 - Asymmetric cryptography
 - Diffie-Hellman key exchange
 - RSA

Allied encryption

- **Typex**
 - British army and air force
 - 5 rotors
- **ECM Mark II (SIGABA)**
 - United States
 - 15 rotors
 - No known successful cryptanalysis during service lifetime
 - But big, expensive, fragile



Navy Department, Office of Chief of Naval Operations, Washington, D.C.

CLASSIFICATION: CONFIDENTIAL Date: 27 Dec 1943

CARELESS COMMUNICATIONS COST LIVES

The following is a list of some of common violations of security principles:

DRAFTING:

Unnecessary word repetition

Unnecessary or improper punctuation

Plain language reply to encrypted dispatch

Classification too high

Precedence too high

Cancellation in plain language of an encrypted dispatch

ENCRYPTION:

"XYX" or "X"'s for nulls

"XX" & "KK" to separate padding from text

Same letters at both ends to separate padding from text

Continuity of padding

Seasonal and stereotyped padding

Repetition of generatrices (Ed. Note: CSP-845)

Systematic selection of generatrices (Ed. Note: CSP-845)

Using plain text column for encryption (Ed. Note: CSP-845)

Proper strips not eliminated as prescribed by internal indicator (Ed. Note: CSP- 845)

Improper set-up according to date

Using system not held by all addressees

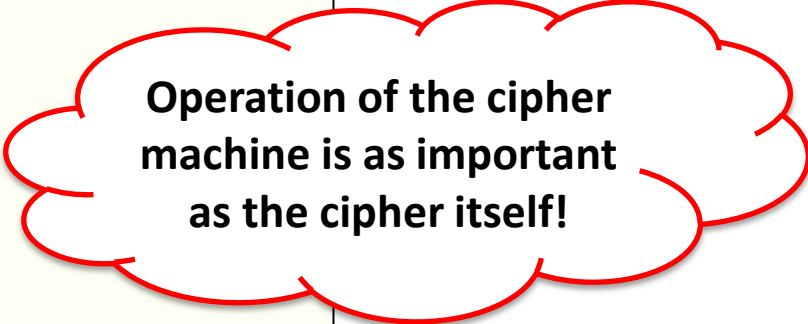
Failing to use system of narrowest distribution

CALLS:

Enciphering indefinite call sign

Enciphering call signs of shore activities

CODRESS might have been used



Operation of the cipher machine is as important as the cipher itself!

Code talkers

- Machine based encryption
 - Heavy equipment
 - Slow to perform
- Code talking
 - Use Native American languages
 - Started in WWI with Choctaw
 - Improvise phrases for out-of-vocabulary words
 - "big gun" = artillery
 - "little gun shoot fast" = machine gun



Code talkers

- Navajo code talkers

- WW II
- Few outsiders had learned the unwritten language
- 3 line message, 20 seconds vs. machine 30 min
- Lexicon of 274 words + phonetic alphabet



<http://library.thinkquest.org/28005/flashed/timemachine/courseofhistory/navajo-dic.shtml>

Modern cryptography

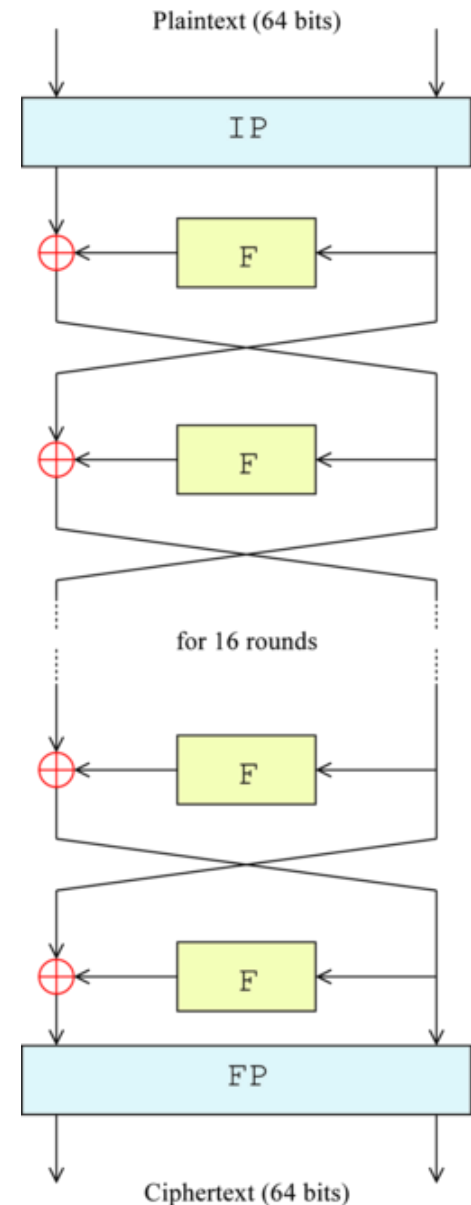
- Moving into computer age
 - Not limited to physical engineering constraints
 - 100's of rotors instead of 3, changing in complex ways
 - Much faster
 - Scrambling at the bit level
- Symmetric encryption (what we've seen thus far)
 - Encrypting message M with key K : $E_K(M) = C$
 - Decrypting ciphertext C with key K : $D_K(C) = M$
 - $D_K(E_K(M)) = M$
 - Stream cipher (bit level) vs. Block cipher (multiple bytes)

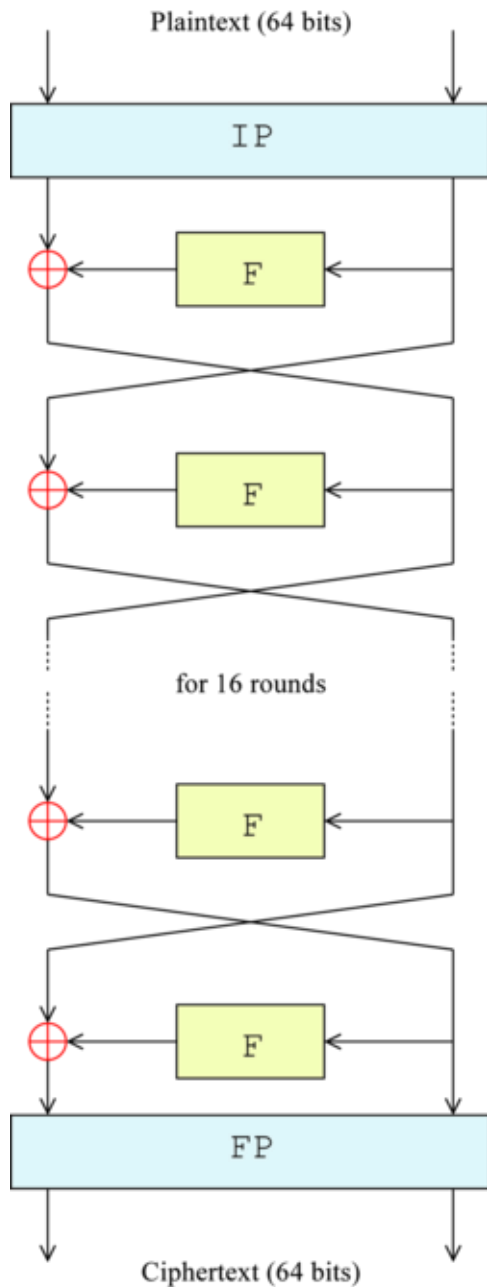
DES

- Data Encryption Standard (DES)
 - NIST wanted a government standard
 - Developed from IBM's **Lucifer** cipher
 - With "**cooperation**" from NSA
 - Improved S-boxes
 - Reduced key length from 64 to 48 bits
 - 1976 approved as a standard
 - Same hardware can encrypt/decrypt

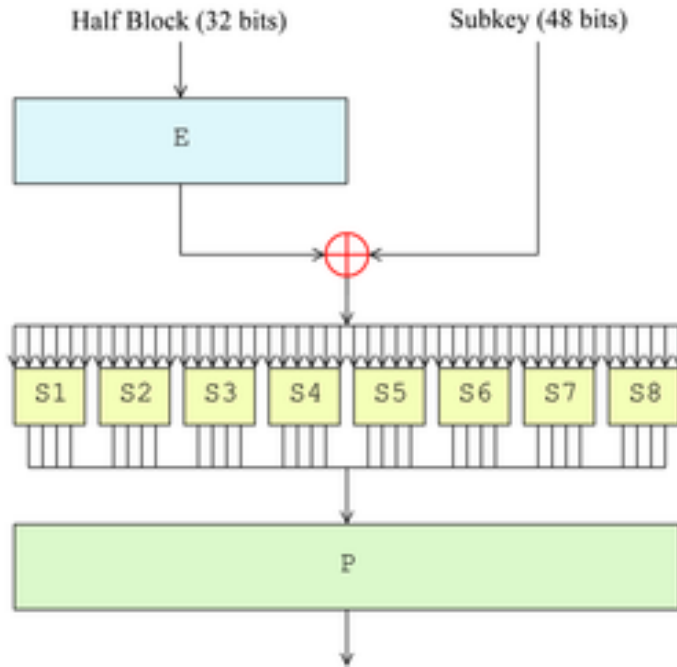
"DES did more to galvanize the field of cryptanalysis than anything else. Now there was an algorithm to study: one that the NSA said was secure"

-Bruce Schneier

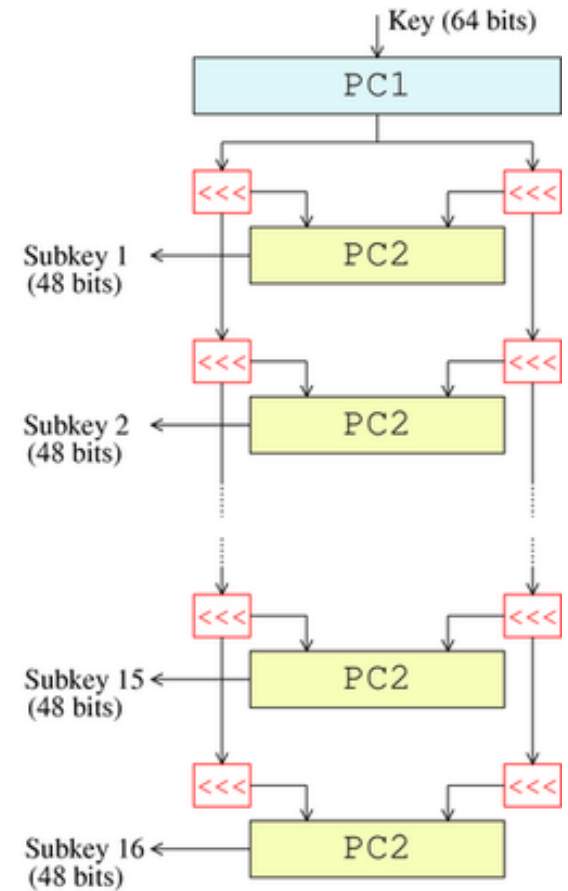




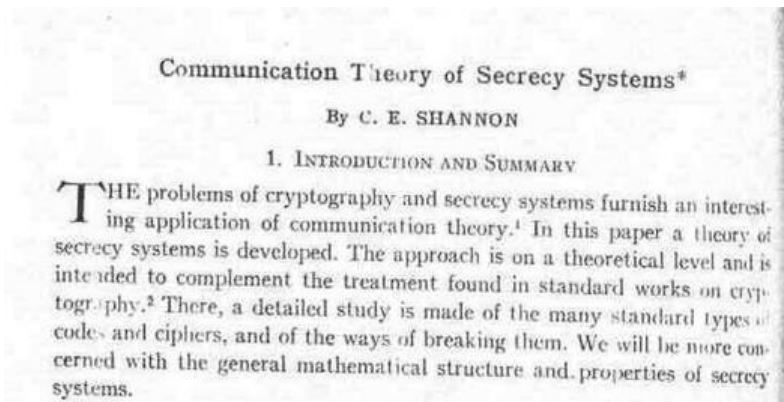
Overall structure



The Feistel function (F-function)



Key schedule



"Confusion and diffusion"

Breaking DES

- Key size, 72 quadrillion

- $2^{56} = 72,057,594,037,927,936$

- DES Challenges (brute force)

- Sponsored by RSA Security

- Challenge I: 96 days, Internet users

- Challenge II: 41 days, distributed.net

- Challenge II-2: 56 hours, EFF deep crack

- \$250,000 to develop, \$10,000 prize

- 90 billion keys/second

- Challenge III: 22 hours, EFF+distributed.net

- 2008, FPGA, 1 day



Stronger symmetric schemes

- Triple DES (3DES)

- Ciphertext: $E_{K3}(D_{K2}(E_{K1}(\text{plaintext})))$

- Plaintext: $D_{K1}(E_{K2}(D_{K3}(\text{ciphertext})))$

- Keying option 1: $K1 \neq K2 \neq K3$

- 168-bits instead of 56-bits

- Advantages:

- Uses DES, most analyzed crypto algorithm
 - No known effective attack besides brute-force


- Disadvantages:

- Slow in software, DES designed for 1970's hardware
 - Small block size of 64-bits

AES

- Advanced Encryption Standard (AES)
 - 2001 new NIST standard, Rijndael
 - Symmetric block cipher
 - Key lengths of 128, 192, and 256 bits
 - Approved by NSA for top secret information

Mix Columns is the hardest. I treat each column as a polynomial. I then use our new multiply method to multiply it by a specially crafted polynomial and then take the remainder after dividing by x^4+1 . This all simplifies to a matrix multiply:



$$b(x) = c(x) \cdot a(x) \pmod{x^4+1}$$

$$= (03x^3+01x^2+01x+02) \cdot (a_3x^3+a_2x^2+a_1x+a_0) \pmod{x^4+1}$$

special polynomial the column

$$= \frac{03a_3x^6+03a_2x^5+03a_1x^4+03a_0x^3+01a_3x^5+01a_2x^4+01a_1x^3+01a_0x^2+01a_3x^4+01a_2x^3+01a_1x^2+01a_0x+02a_3x^4+02a_2x^3+02a_1x^2+02a_0x+02a_0}{x^4+1}$$

$$\oplus \frac{03a_3x^2+03a_2x^2}{3a_2x^3+3a_1x^2+3a_0x^2+a_3x^2+a_2x^2+a_1x^2+a_0x^2+a_3x^2+a_2x^2+a_1x^2+a_0x^2+2a_3x^3+2a_2x^2+2a_1x+2a_0}$$

$$\oplus \frac{3a_3x^2+a_3x^2+3a_2x^2+a_2x^2}{3ax^3+3a_0x^3+a_3x^2+a_2x^2+a_1x^2+a_0x^2+a_3x^2+a_2x^2+a_1x^2+a_0x^2+2a_3x^2+2a_2x^2+2a_1x+2a_0}$$

$$\oplus \frac{(3a_1+a_2+a_3)x^2+(3a_1+a_2+a_3)}{(2a_3+a_2+a_1+3a_0)x^3+(3a_3+2a_2+a_1+a_0)x^2+(a_3+3a_2+2a_1+a_0)x+(a_3+a_2+3a_1+2a_0)}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix}$$

Plaintext in 4x4 grid

AES Crib Sheet
(Handy for memorizing)

Initial Round

General Math
1.9B = AES Polynomial: $m(x)$
 $x^4+x^3+x^2+x+1$ Fast Multiply
 $x \cdot ax^3 = (a \ll 1) \oplus (a_2 = 1) ? 1B : 00$
 $\log(x \cdot y) = \log(x) + \log(y)$
Use $(x+1) = 03$ for log base

S-Box (SRD) \oplus
SRD[a] = f(g(a))
 $g(a) = a^{-1} \pmod{m(x)}$
5 is 5, 3 is 3 (0110 0011)
01111000
01111100
00111110
00011111
10001111
10001111
11000111
11100011
11100011

Key Expansion: Round Constants: 01, 02, 04, 08, ...

Other Columns: S, B, E, I, R, T, Y, L, O, M, F, C, A, G, H, K, J, P, Q, V, W, X, Z

Final Round

Intermediate Rounds

#	Key
9	128
11	192
13	256

Ciphertext

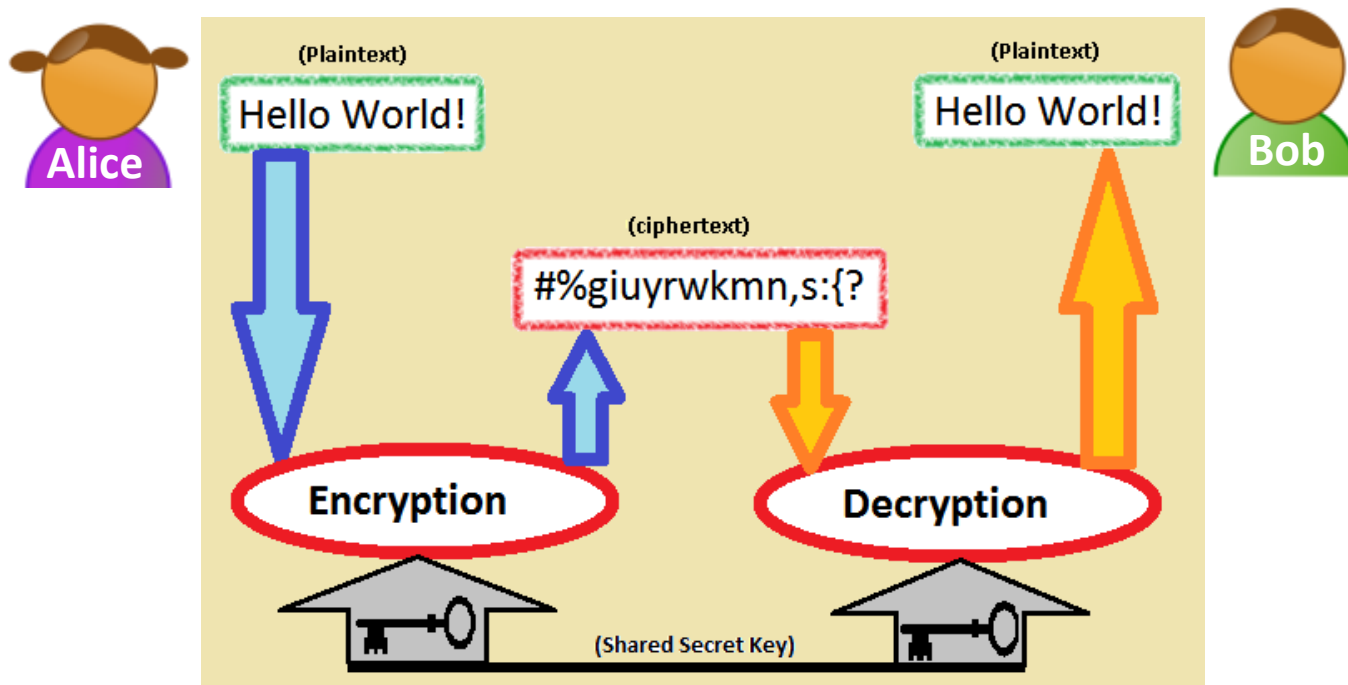
Mix Columns: 2113, 2113, 3211, 1321, 1132

Inverse Mix EDD9, 9EB8, 09E8, B098

Prev Col @ Col from Previous round Key

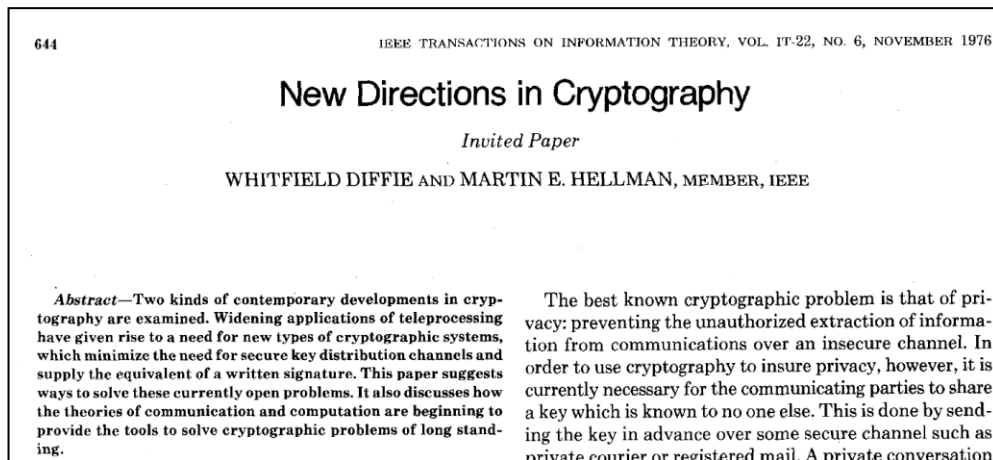
Key exchange

- Thus far: **symmetric encryption**
 - Alice and Bob need to have shared secret
 - But how do you distribute?
 - Doesn't scale



Diffie-Hellman

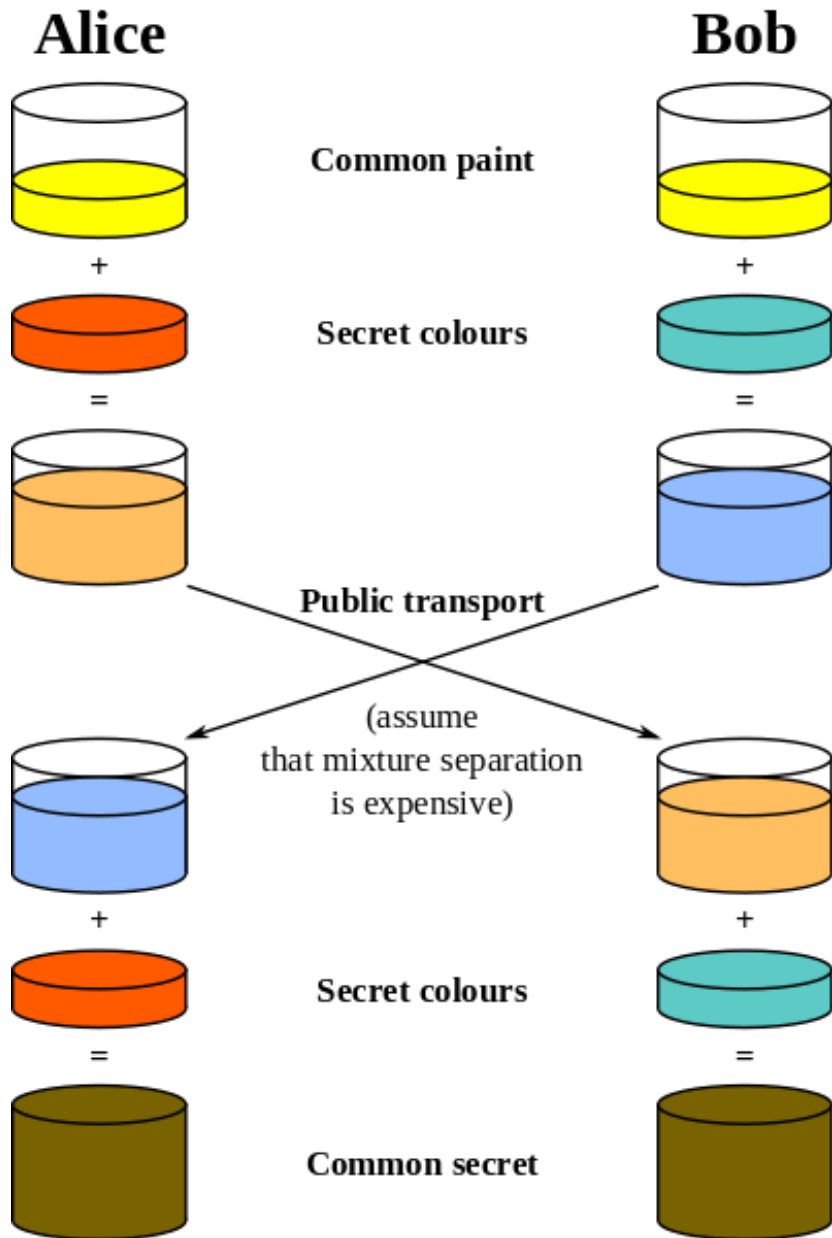
- Diffie-Hellman (DH) key exchange
 - 1976, Whitfield Diffie & Martin Hellman
 - Alice and Bob **agree on a private secret:**
 - On a public channel <http://www.youtube.com/watch?v=3QnD2c4Xovk>
 - Where Eve hears all the traffic
 - Only Alice and Bob end up knowing the secret
 - Relies on **one-way function**
 - Function must be easy to do, but difficult to undo



Whitfield Diffie



Martin Hellman



Alice	Bob
Alice and Bob agree publically on values for Y and P for the one-way function: $Y^x \pmod{P}$, e.g. $Y=7, P=11$	
Alice chooses secret number: $A = 3$	Bob chooses secret number: $B = 6$
$\alpha = 7^A \pmod{11}$ $= 7^3 \pmod{11}$ $= 343 \pmod{11}$ $= 2$	$\beta = 7^B \pmod{11}$ $= 7^6 \pmod{11}$ $= 117649 \pmod{11}$ $= 4$
Sends $\alpha = 2$ to Bob	Sends $\beta = 4$ to Alice
Using Bob's result: $\beta^A \pmod{11}$ $4^3 \pmod{11} = 9$	Using Alice's result $\alpha^B \pmod{11}$ $2^6 \pmod{11} = 9$
$7^{B \cdot A} \pmod{11}$	$7^{A \cdot B} \pmod{11}$

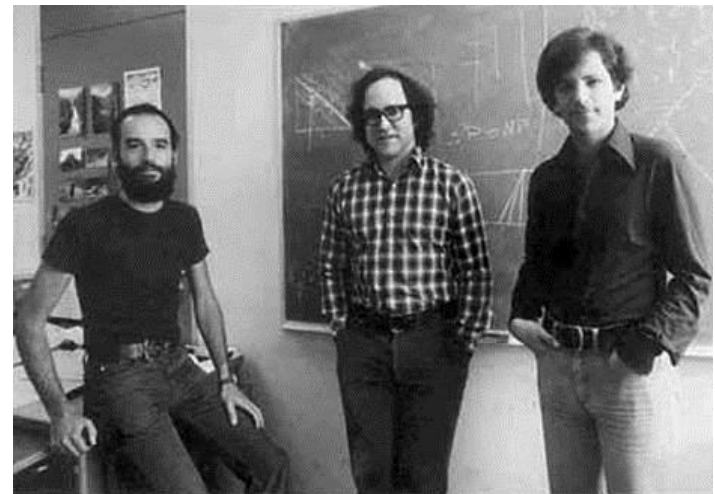
Public key cryptography

- Diffie-Hellman key exchange
 - Both parties had to be around to negotiate secret
- Symmetric encryption
 - Encrypting message M with key K : $E_K(M) = C$
 - Decrypting ciphertext C with key K : $D_K(C) = M$
- Asymmetric encryption
 - 1975, Diffie conceives of idea
 - Users have a private key and a public key
 - Alice encrypts plaintext with Bob's public key
 - Only Bob can (tractably) decrypt using his private key
 - Special one-way function
 - Hard to reverse unless you know something special

RSA

- RSA public key encryption
 - 1977, Rivest, Shamir, Adleman
 - Choose two prime numbers, p and q
 - Public key: $N = pq$
 - Private key: p and q
 - If N is product of two large primes, factoring is "hard"
 - 1973, equivalent algorithm, Clifford Cocks (GCHQ)

http://www.youtube.com/watch?v=wXB-V_Keiu8



RSA example



Alice	Bob
<p>Alice picks two giant primes, p and q e.g. $p = 61$, $q = 53$</p> <p>$N = p * q = 61 * 53 = 3233$</p> <p>$(p - 1) * (q - 1) = 60 * 52 = 3120$ Find number $1 < e < 3120$, e is relatively prime with 3120, say $e = 17$</p> <p>Alice's public key: $N = 3233$, $e = 17$</p>	
	<p>Bob wants to send message 65 to Alice, looks up her public key.</p> <p>$C = M^e \pmod{N}$ $C = 65^{17} \pmod{3233} = 2790$</p>

RSA example



Alice	Bob
	<p>Bob wants to send message 65 to Alice, looks up her public key.</p> $C = M^e \pmod{N}$ $C = 65^{17} \pmod{3233} = 2790$
<p>Compute special number d</p> $e * d = 1 \pmod{(p - 1) * (q - 1)}$ $17 * d = 1 \pmod{3120}$ $d = 2753 \text{ (using Euclid's algorithm)}$ <p>Alice's private key $d = 2753$, or p and q</p> <p>Decrypt message:</p> $M = C^d \pmod{N}$ $M = 2790^{2753} \pmod{3233} = 65$	

RSA security

- Attacks on RSA

$$O\left(\exp\left(\left(\frac{64}{9}b\right)^{\frac{1}{3}}(\log b)^{\frac{2}{3}}\right)\right)$$

General number field sieve, b-bit number

- Brute force

- Try all possible private keys
 - Use a large key space, but large keys slows things down
- RSA is not as fast as symmetric crypto
 - ~1000 times slower in hardware than DES

- Mathematical

- Factoring the product of two large primes

- Timing

- Keep track of how long it takes to decipher messages

- Chosen ciphertext

2009:
768-bit RSA factored using
hundreds of machines in 2 years

Unsolved problems in computer science

*Can integer factorization be done in
polynomial time?*



Summary

- Historical cryptography
 - Code talkers
- Modern cryptography
 - Computer-based symmetric ciphers
 - DES, 3DES, AES
 - Rise of asymmetric cryptography
 - Diffie-Hellman
 - RSA