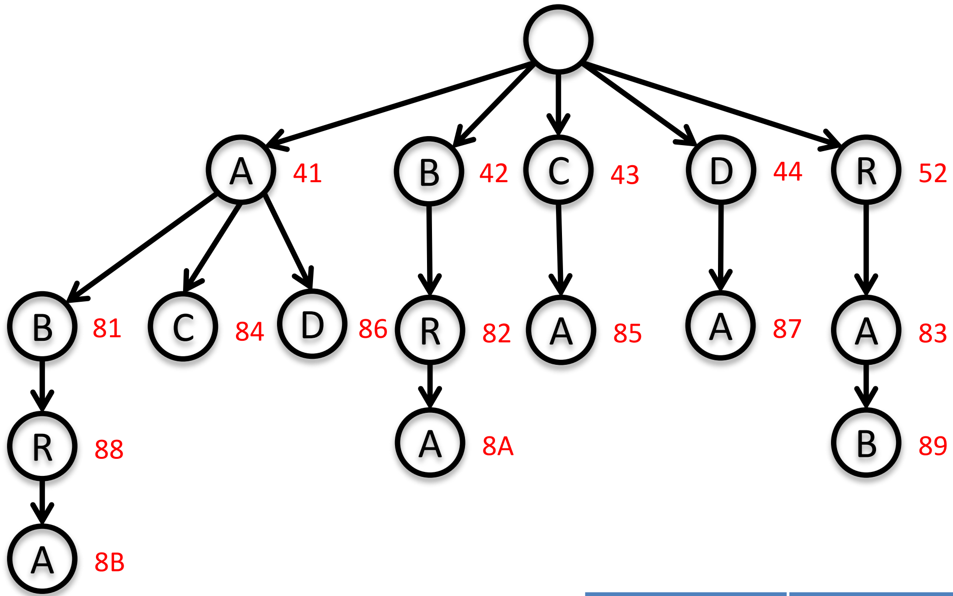
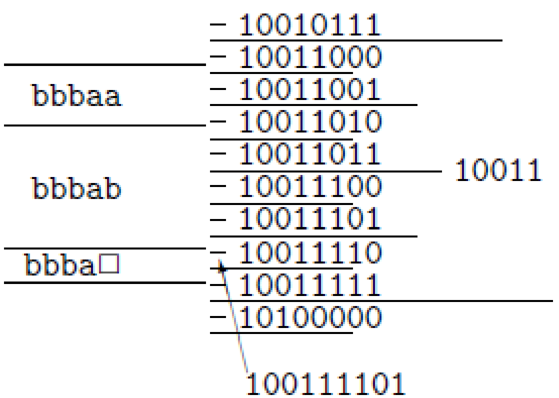
# Lossless compression II

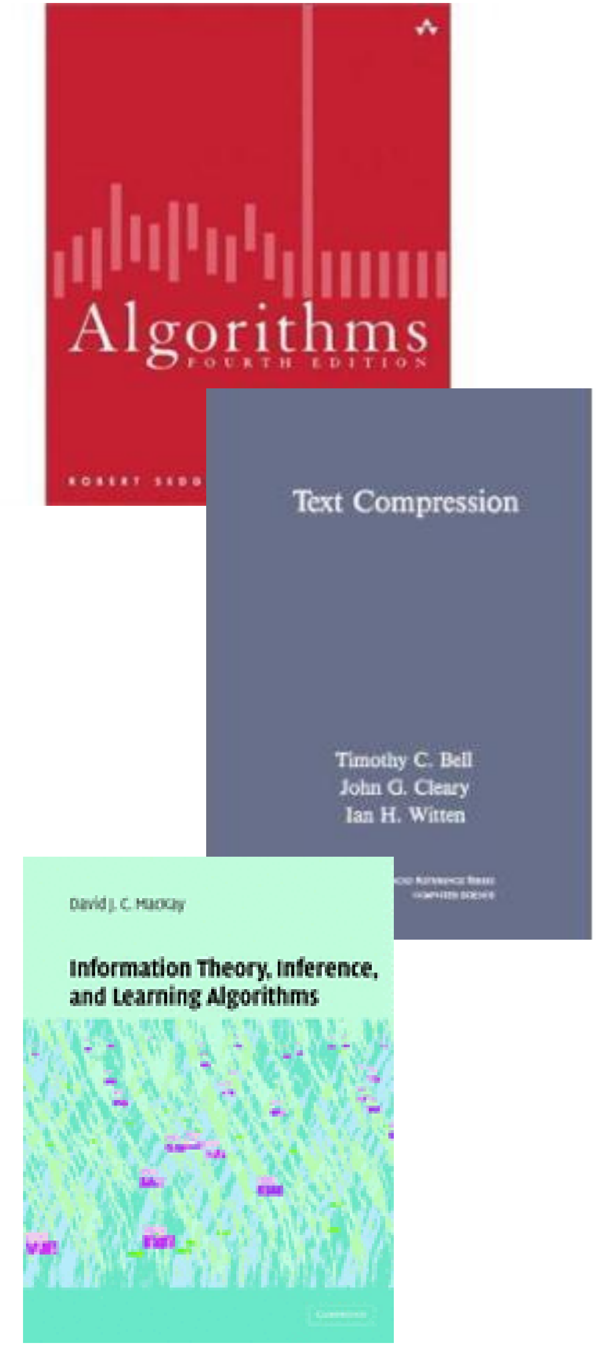



| Symbol | Probability | Range |
|---|---|---|
| a | 0.2 | [0.0, 0.2) |
| e | 0.3 | [0.2, 0.5) |
| i | 0.1 | [0.5, 0.6) |
| o | 0.2 | [0.6, 0.8) |
| u | 0.1 | [0.8, 0.9) |
| ! | 0.1 | [0.9, 1.0) |

# Overview

- Flavors of compression
  - Static vs. Dynamic vs. Adaptive
- Lempel-Ziv-Welch (LZW)
  - Fixed-length codeword
  - Variable-length pattern
- Statistical coding
  - Arithmetic coding
  - Prediction by Partial Match (PPM)



# Compression models

- Static models
  - Predefined map for all text, e.g. ASCII, Morse Code
  - Not optimal, different texts = different statistics
- Dynamic model
  - Generate model based on text
  - Requires initial pass before compression can start
  - Must transmit the model (e.g. Huffman coding)
- Adaptive model
  - More accurate modeling = better compression
  - Decoding must start from beginning (e.g. LZW)

# LZW

- Lempel-Ziv-Welch compression (LZW)
  - Basis for many popular compression formats
    - e.g. PNG, 7zip, gzip, jar, PDF, compress, pkzip, GIF, TIFF
- Algorithm basics
  - Maintain a table of fixed-length codewords for variable-length patterns in the input
  - Table does not need to be transmitted!
    - Built progressively by both compressor and expander
  - Table is of finite size
    - Holds entry for all single characters in alphabet
    - Plus entries for longer substrings encountered

# LZW compression example

- Details:
  - 7-bit ASCII characters, 8-bit codeword
    - For real use, 8-bit → ~12-bit codeword
  - Store alphabet: 128 characters + 128 longer strings
  - Codeword is 2-digit hex value:
    - 00-79 = single characters, e.g. 41 = A, 52 = R
    - 80 = end of file
    - 81-FF = longer strings, e.g. 81 = AB, 88 = ABR
  - Employs a lookahead character to add codewords

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | | | | | | | | | | | | | | | | | |
| codeword | | | | | | | | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| | |
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*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | | | | | | | | | | | | | | | | |
| codeword | 41 | | | | | | | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| | |
| | |
| | |
| | |
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*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | B | | | | | | | | | | | | | | | |
| codeword | 41 | 42 | | | | | | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| | |
| | |
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| | |
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| | |
| | |

*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | B | R | | | | | | | | | | | | | | |
| codeword | 41 | 42 | 52 | | | | | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| | |
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| | |

*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | B | R | A | | | | | | | | | | | | | |
| codeword | 41 | 42 | 52 | 41 | | | | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| | |
| | |
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| | |
| | |

*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | B | R | A | C | A | D | | | | | | | | | | |
| codeword | 41 | 42 | 52 | 41 | 43 | 41 | 44 | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| | |
| | |
| | |
| | |

*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **matches** | A | B | R | A | C | A | D | AB | | | | | | | | | |
| **codeword** | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| | |
| | |
| | |

*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | B | R | A | C | A | D | AB | | RA | | | | | | | |
| codeword | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | | 83 | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| | |
| | |

*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | B | R | A | C | A | D | AB | | RA | | BR | | | | | |
| codeword | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | | 83 | | 82 | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| | |

*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | B | R | A | C | A | D | AB | | RA | | BR | | ABR | | | |
| codeword | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | | 83 | | 82 | | 88 | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| ABRA | 8B |

*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

| input | A | B | R | A | C | A | D | A | B | R | A | B | R | A | B | R | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| matches | A | B | R | A | C | A | D | AB | | RA | | BR | | ABR | | | A |
| codeword | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | | 83 | | 82 | | 88 | | | 41 |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| ABRA | 8B |

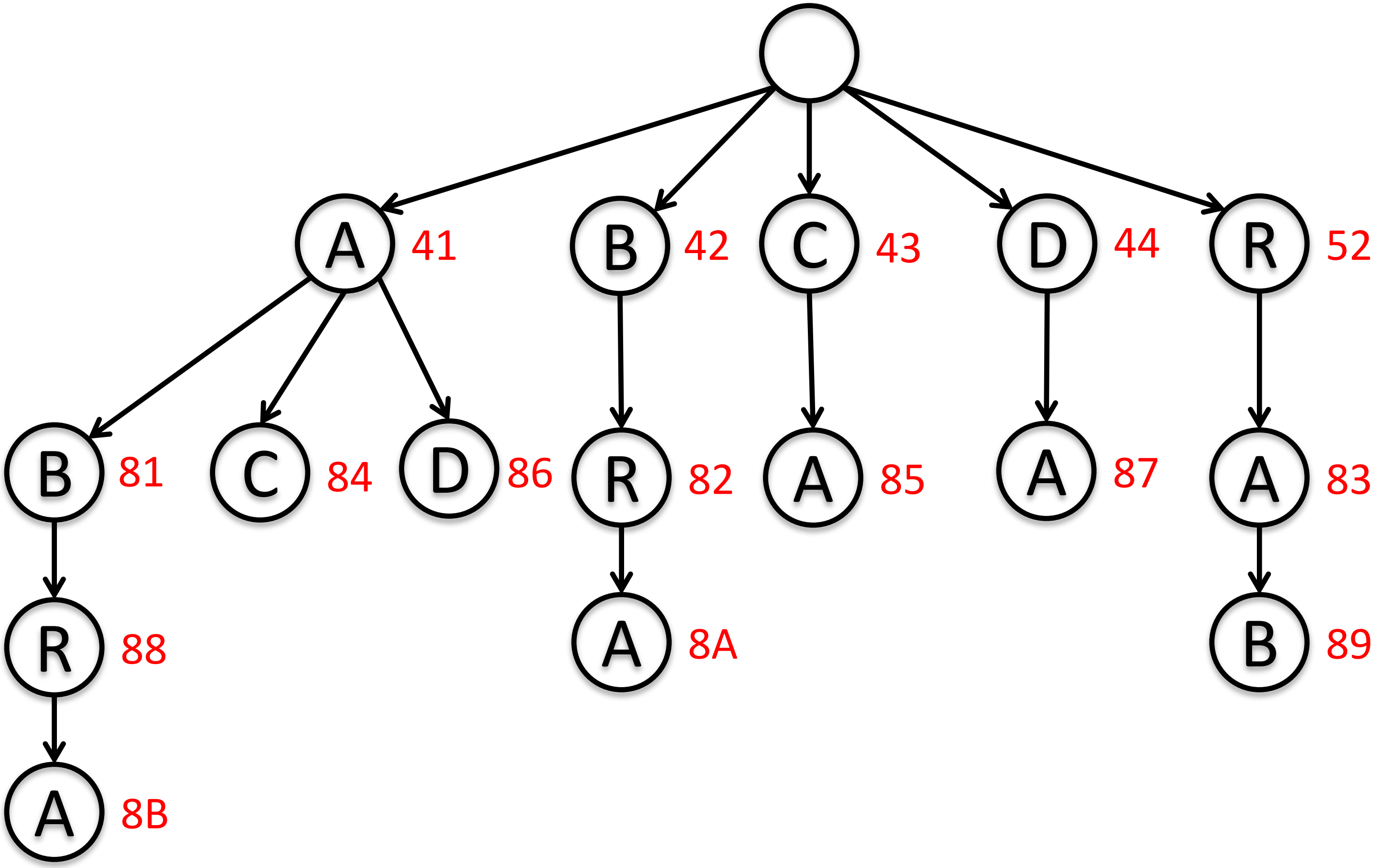
*Mappings found during compression*

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c

# Compression data structure

- LZW compression uses two table operations:
  - Find longest-prefix match from current position
  - Add entry with character added to longest match
- Trie data structure
  - Linked tree structure
  - Path in tree defines string

- Compressing
  - Find longest string s in table that is prefix of unscanned input
  - Write codeword of string s
  - Scan one character c ahead
  - Associate next free codeword with s + c




| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |

| string | codeword |
|---|---|
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |

```java
private static final int R = 256;             // number of input chars
private static final int L = 4096;            // number of codewords = 2^W
private static final int W = 12;              // codeword width

public static void compress()
{
   String input = BinaryStdIn.readString();  // read input as string
   TST<Integer> st = new TST<Integer>();     // trie data structure

   for (int i = 0; i < R; i++)               // codewords for single chars
      st.put("" + (char) i, i);
   int code = R+1;                           // R is codeword for EOF

   while (input.length() > 0)
   {
      String s = st.longestPrefixOf(input);
      BinaryStdOut.write(st.get(s), W);      // write W-bit codeword for s
      int t = s.length();
      if (t < input.length() && code < L)
         st.put(input.substring(0, t + 1), code++);  // add new codeword
      input = input.substring(t);
   }
   BinaryStdOut.write(R, W);                 // write last codeword
   BinaryStdOut.close();
}
```

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | | | | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = A
x = 42
s = B
c = B
$next_{code}$ = 81
$val_{new}$ = B

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | | | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = B
x = 52
s = R
c = R
$next_{code}$ = 82
$val_{new}$ = R

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| | |
| | |
| | |
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| | |
| | |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = R
x = 41
s = A
c = A
$next_{code}$ = 83
$val_{new}$ = A

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| | |
| | |
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| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = A
x = 43
s = C
c = C
$next_{code}$ = 84
$val_{new}$ = C

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = C
x = 41
s = A
c = A
$next_{code}$ = 85
$val_{new}$ = A

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | C | | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| | |
| | |
| | |
| | |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = A
x = 44
s = D
c = D
$next_{code}$ = 86
$val_{new}$ = D

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | C | A | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| **AB** | **81** |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| | |
| | |
| | |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

```
val_old   = D
x         = 81
s         = AB
c         = A
next_code = 87
val_new   = AB
```

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | C | A | D | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| | |
| | |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = AB
x = 83
s = RA
c = R
$next_{code}$ = 88
$val_{new}$ = RA

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | C | A | D | AB | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| **BR** | **82** |
| **RA** | **83** |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| | |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = RA
x = 82
s = BR
c = B
$next_{code}$ = 89
$val_{new}$ = BR

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | C | A | D | AB | RA | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| **BR** | **82** |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| **ABR** | **88** |
| RAB | 89 |
| | |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = BR
x = 88
s = ABR
c = A
$next_{code}$ = 8A
$val_{new}$ = RA

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | C | A | D | AB | RA | BR | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| | |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ = ABR
x = 41
s = A
c = A
$next_{code}$ = 8B
$val_{new}$ = RA

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | C | A | D | AB | RA | BR | ABR | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| ABRA | 8B |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

```
val_old   = A
x         = 80
s         =
c         =
next_code =
val_new   =
```

| input | 41 | 42 | 52 | 41 | 43 | 41 | 44 | 81 | 83 | 82 | 88 | 41 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| output | A | B | R | A | C | A | D | AB | RA | BR | ABR | A | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

*Initial set of character codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| ABRA | 8B |

*Mappings found during compression*

- Expansion
  - Write string for val codeword
  - Read codeword x from input
  - Set s to string for codeword x
  - Set next unassigned codeword to val + c where c is 1st char in s
  - Set val = s

$val_{old}$ =
x =
s =
c =
$next_{code}$ =
$val_{new}$ =

# Expansion data structure

- For a W-bit codeword, look up string value
  – An array of size $2^W$

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| D | 44 |
| … | … |
| R | 52 |
| … | … |

| string | codeword |
|---|---|
| AB | 81 |
| BR | 82 |
| RA | 83 |
| AC | 84 |
| CA | 85 |
| AD | 86 |
| DA | 87 |
| ABR | 88 |
| RAB | 89 |
| BRA | 8A |
| ABRA | 8B |

- Expansion
  – Write string for val codeword
  – Read codeword x from input
  – Set s to string for codeword x
  – Set next unassigned codeword to val + c where c is 1st char in s
  – Set val = s

# A tricky case: compression

**ABABABA**

| input | A | B | A | B | A | B | A |
|---|---|---|---|---|---|---|---|
| matches | | | | | | | |
| codeword | | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| | |
| | |
| | |
| | |
| | |

*Mappings found during compression*

# A tricky case: compression

**ABABABA**

| input | A | B | A | B | A | B | A |
|---|---|---|---|---|---|---|---|
| matches | A | | | | | | |
| codeword | 41 | | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| AB | 81 |
| | |
| | |
| | |
| | |

*Mappings found during compression*

# A tricky case: compression

**ABABABA**

| input | A | B | A | B | A | B | A |
|---|---|---|---|---|---|---|---|
| matches | A | B | | | | | |
| codeword | 41 | 42 | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BA | 82 |
| | |
| | |
| | |

*Mappings found during compression*

# A tricky case: compression

**ABABABA**

| input | A | B | A | B | A | B | A |
|---|---|---|---|---|---|---|---|
| matches | A | B | AB | | | | |
| codeword | 41 | 42 | 81 | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BA | 82 |
| ABA | 83 |
| | |
| | |

*Mappings found during compression*

# A tricky case: compression

**ABABABA**

| input | A | B | A | B | A | B | A |
|---|---|---|---|---|---|---|---|
| matches | A | B | AB | | ABA | | |
| codeword | 41 | 42 | 81 | | 83 | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BA | 82 |
| ABA | 83 |
| | |
| | |

*Mappings found during compression*

# A tricky case: expansion

**ABABABA**

| input | 41 | 42 | 81 | 83 | 80 |
|---|---|---|---|---|---|
| output | | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| | |
| | |
| | |
| | |
| | |

*Mappings found during compression*

# A tricky case: expansion

**ABABABA**

| input | 41 | 42 | 81 | 83 | 80 |
|---|---|---|---|---|---|
| output | A | | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| AB | 81 |
| | |
| | |
| | |
| | |

*Mappings found during compression*

# A tricky case: expansion

**ABABABA**

| input | 41 | 42 | 81 | 83 | 80 |
|---|---|---|---|---|---|
| output | A | B | | | |

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BA | 82 |
| | |
| | |
| | |

*Mappings found during compression*

# A tricky case: expansion

**ABABABA**

| input | 41 | 42 | 81 | 83 | 80 |
|---|---|---|---|---|---|
| output | A | B | AB | | |

We need to know the first character of 83 in order to enter 83 into the table!

| string | codeword |
|---|---|
| … | … |
| A | 41 |
| B | 42 |
| C | 43 |
| … | … |

*Initial set of codewords*

| string | codeword |
|---|---|
| AB | 81 |
| BA | 82 |
| | |
| | |
| | |

*Mappings found during compression*

# LZW decisions

- How big of a symbol table?
  - How long is message?
  - Whole message similar model?
  - Many variations…
- What to do when symbol table fills up?
  - Throw away and start over (e.g. GIF)
  - Throw away when not effective (e.g. Unix compress)
  - Many variations…
- Put longer substrings in symbol table?
  - Many variations…

# LZW variants

- Lempel-Ziv variants
  - LZ77, published Lempel & Ziv, 1977, not patented
    - PNG
  - LZ78, published Lempel & Ziv in 1978, patented
  - LZW, Welch extension to LZ78, patented (expired 2003)
    - GIF, TIFF, Pkzip
  - Deflate = LZ77 variant + Huffman coding
    - 7zip, gzip, jar, PDF

# Some experiments

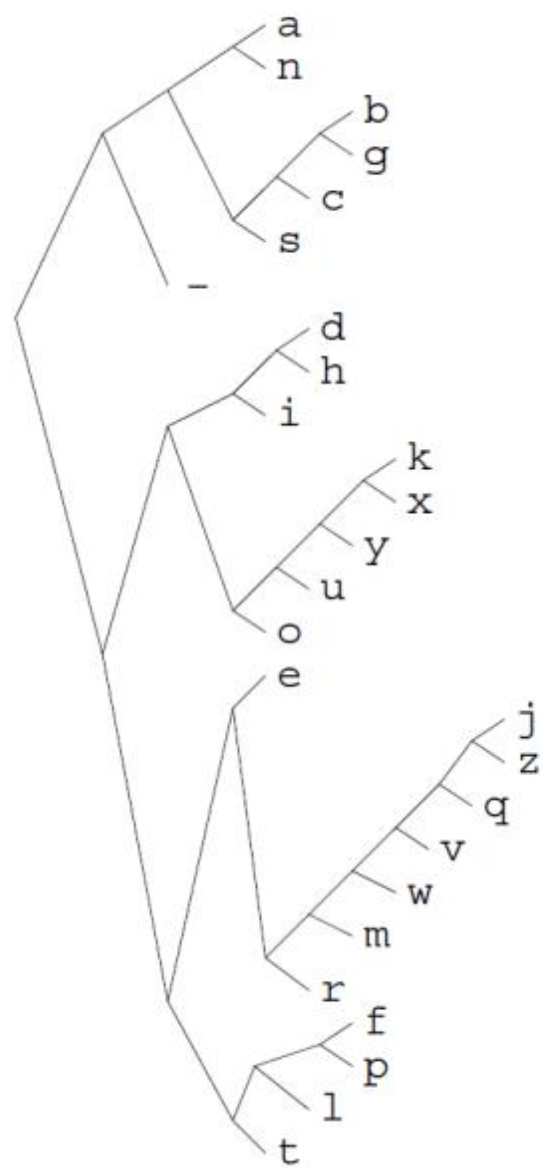
- Compressing Moby Dick
  - Using Java programs for LZW and Huffman
    - Different dictionary sizes for LZW or resetting when full
  - Compared to zip and gzip

```
09/30/2011  09:13 AM        1,191,463 mobydick.txt
04/22/2012  10:33 PM          485,561 mobydick.gz
04/22/2012  10:31 PM          485,790 mobydick.zip

04/22/2012  10:32 PM          667,651 mobydick.huff
04/22/2012  10:31 PM          597,060 mobydick.lzw
04/22/2012  10:32 PM          814,578 mobydick.huff.lzw
04/22/2012  10:31 PM          592,830 mobydick.lzw.huff
04/22/2012  11:06 PM          682,400 mobydick.lzw.reset

04/22/2012  10:36 PM          541,261 mobydick.lzw14
04/22/2012  10:38 PM          521,700 mobydick.lzw15
04/22/2012  10:34 PM          503,050 mobydick.lzw16
04/22/2012  10:37 PM          501,193 mobydick.lzw16.huff
04/22/2012  10:38 PM          521,700 mobydick.lzw17
04/22/2012  10:36 PM          514,393 mobydick.lzw18
04/22/2012  10:35 PM          571,548 mobydick.lzw20
```

# Enter statistical coding…

- Natural language quite predictable
  - ~ 1 bit of entropy per symbol
  - Huffman coding still requires 1-bit min per symbol
    - We're forced to use an integral number of bits
  - Dictionary-based (LZW and friends)
    - Just memorizes sequences
- Statistical coding
  - Use long, specific context for prediction
    - P(A | The_United_State_of_) = ?
  - Blend knowledge using contexts of different lengths
  - Model can update and change as text seen
    - Often after every letter!

| $a_i$ | $p_i$ | $\log_2 \frac{1}{p_i}$ | $l_i$ | $c(a_i)$ |
|---|---|---|---|---|
| a | 0.0575 | 4.1 | 4 | 0000 |
| b | 0.0128 | 6.3 | 6 | 001000 |
| c | 0.0263 | 5.2 | 5 | 00101 |
| d | 0.0285 | 5.1 | 5 | 10000 |
| e | 0.0913 | 3.5 | 4 | 1100 |
| f | 0.0173 | 5.9 | 6 | 111000 |
| g | 0.0133 | 6.2 | 6 | 001001 |
| h | 0.0313 | 5.0 | 5 | 10001 |
| i | 0.0599 | 4.1 | 4 | 1001 |
| j | 0.0006 | 10.7 | 10 | 1101000000 |
| k | 0.0084 | 6.9 | 7 | 1010000 |
| l | 0.0335 | 4.9 | 5 | 11101 |
| m | 0.0235 | 5.4 | 6 | 110101 |
| n | 0.0596 | 4.1 | 4 | 0001 |
| o | 0.0689 | 3.9 | 4 | 1011 |
| p | 0.0192 | 5.7 | 6 | 111001 |
| q | 0.0008 | 10.3 | 9 | 110100001 |
| r | 0.0508 | 4.3 | 5 | 11011 |
| s | 0.0567 | 4.1 | 4 | 0011 |
| t | 0.0706 | 3.8 | 4 | 1111 |
| u | 0.0334 | 4.9 | 5 | 10101 |
| v | 0.0069 | 7.2 | 8 | 11010001 |
| w | 0.0119 | 6.4 | 7 | 1101001 |
| x | 0.0073 | 7.1 | 7 | 1010001 |
| y | 0.0164 | 5.9 | 6 | 101001 |
| z | 0.0007 | 10.4 | 10 | 1101000001 |
| – | 0.1928 | 2.4 | 2 | 01 |




*MacKay, D.J.C. Information Theory, Inference, and Learning Algorithms.*

# Guess the phrase

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |

# A simple unigram model of English

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **t** | **h** | **e** | **r** | **e** | **_** | **i** | **s** | **_** | **n** | **o** | **_** | **s** | **p** | **o** | **o** | **n** |

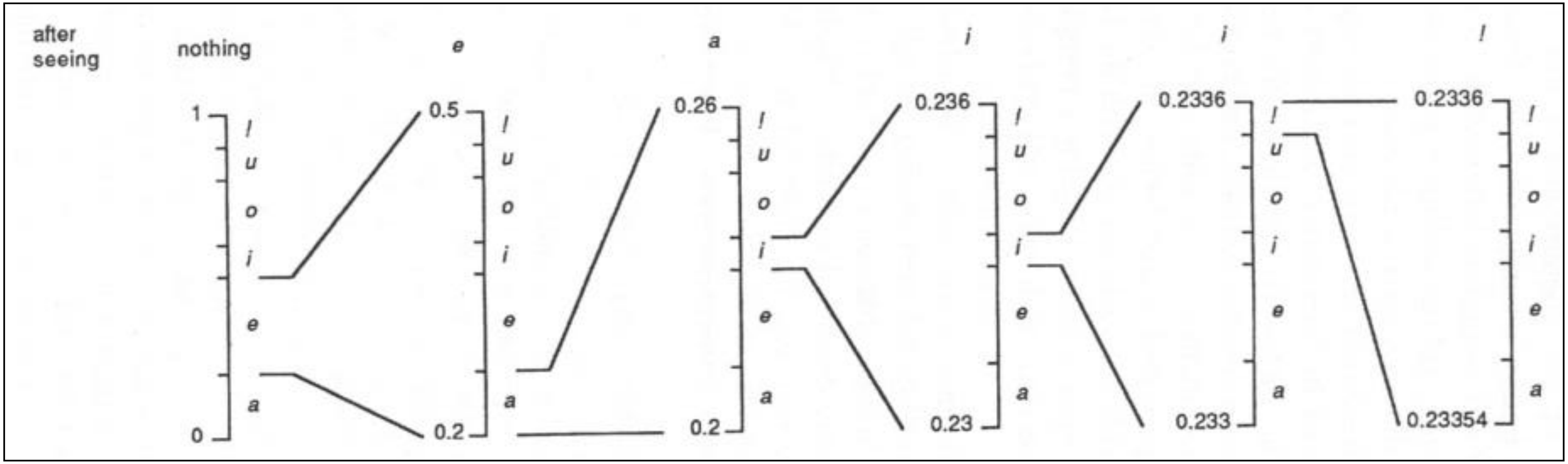
| | |
|---|---|
| ' | 0.0071063 |
| - | 0.0000001 |
| . | 0.0000384 |
| </s> | 0.0368291 |
| <sp> | 0.1653810 |
| a | 0.0595248 |
| b | 0.0112864 |
| c | 0.0174441 |
| d | 0.0282733 |
| e | 0.0890307 |
| f | 0.0127512 |
| g | 0.0213974 |
| h | 0.0403836 |
| i | 0.0586443 |
| j | 0.0018080 |
| k | 0.0117826 |

| | |
|---|---|
| l | 0.0343399 |
| m | 0.0247835 |
| n | 0.0490316 |
| o | 0.0762119 |
| p | 0.0134453 |
| q | 0.0003078 |
| r | 0.0408972 |
| s | 0.0433802 |
| t | 0.0680194 |
| u | 0.0273347 |
| v | 0.0083669 |
| w | 0.0210079 |
| x | 0.0010829 |
| y | 0.0295698 |
| z | 0.0005395 |

# From text to a real number

- Arithmetic coding
  - Message represented by real interval in [0, 1)
  - More precise interval = species more bits
    - e.g. [0.28272722, 0.28272724) = "it was the best of times"
    - Or any number in that interval, 0.28272723
- Example
  - Alphabet = {a, e, i, o, u, !}
  - Transmission = eaii!

| Symbol | Probability | Range |
|---|---|---|
| a | 0.2 | [0.0, 0.2) |
| e | 0.3 | [0.2, 0.5) |
| i | 0.1 | [0.5, 0.6) |
| o | 0.2 | [0.6, 0.8) |
| u | 0.1 | [0.8, 0.9) |
| ! | 0.1 | [0.9, 1.0) |




*Bell, Cleary, Witten. Text Compression.*

Transmission = eaii!

| After seeing | Range |
|---|---|
| | [0.0, 1.0] |
| e | [0.2, 0.5) |
| a | [0.2, 0.26) |
| i | [0.23, 0.236) |
| i | [0.233, 0.2336) |
| ! | [0.23354, 0.2336) |

| Symbol | Probability | Range |
|---|---|---|
| a | 0.2 | [0.0, 0.2) |
| e | 0.3 | [0.2, 0.5) |
| i | 0.1 | [0.5, 0.6) |
| o | 0.2 | [0.6, 0.8) |
| u | 0.1 | [0.8, 0.9) |
| ! | 0.1 | [0.9, 1.0) |

*Note:* Encoder/decoder need to agree on symbol to terminate message, here we use !

| Context (sequence thus far) | Probability of next symbol | | |
|---|---|---|---|
| | $P(\mathtt{a}) = 0.425$ | $P(\mathtt{b}) = 0.425$ | $P(\square) = 0.15$ |
| b | $P(\mathtt{a} \mid \mathtt{b}) = 0.28$ | $P(\mathtt{b} \mid \mathtt{b}) = 0.57$ | $P(\square \mid \mathtt{b}) = 0.15$ |
| bb | $P(\mathtt{a} \mid \mathtt{bb}) = 0.21$ | $P(\mathtt{b} \mid \mathtt{bb}) = 0.64$ | $P(\square \mid \mathtt{bb}) = 0.15$ |
| bbb | $P(\mathtt{a} \mid \mathtt{bbb}) = 0.17$ | $P(\mathtt{b} \mid \mathtt{bbb}) = 0.68$ | $P(\square \mid \mathtt{bbb}) = 0.15$ |
| bbba | $P(\mathtt{a} \mid \mathtt{bbba}) = 0.28$ | $P(\mathtt{b} \mid \mathtt{bbba}) = 0.57$ | $P(\square \mid \mathtt{bbba}) = 0.15$ |

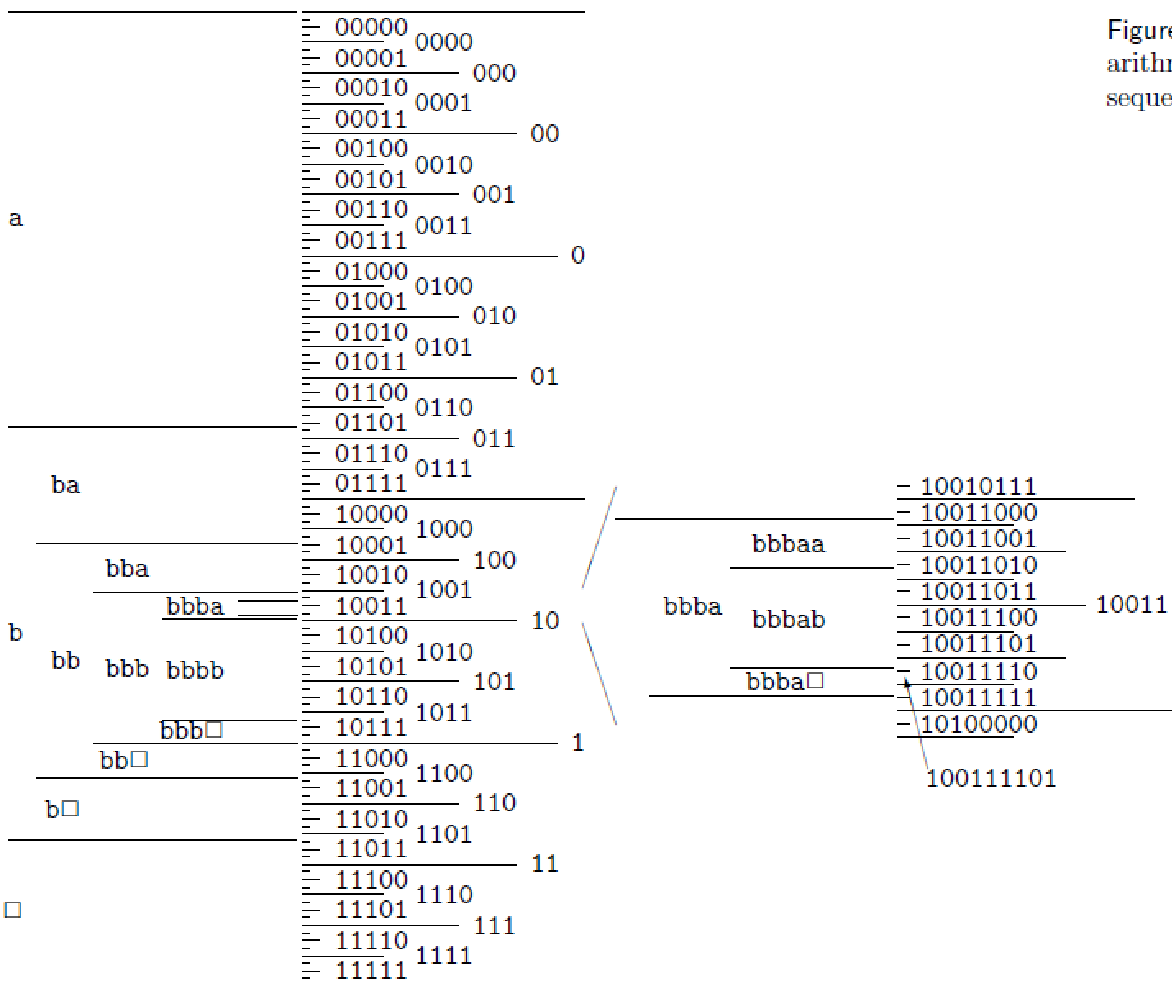



Figure 6.4. Illustration of the arithmetic coding process as the sequence bbba□ is transmitted.

*MacKay, D.J.C. Information Theory, Inference, and Learning Algorithms.*

# Context modeling

- Prediction by Partial Match (PPM)
  - Probability of next symbol depends on previous symbol(s)
  - Blending strategy for dealing with 0-frequency problem
  - Easy to build as adaptive
    - Learn from the text as you go
    - Not trie to send like Huffman coding
  - 1984, but still competitive for text compression
    - Various variants, PPM-A/B/C/D/Z/*
    - Implemented in 7-zip, open source packages, PPM for XML, PPM for executables, …

# PPM

- An alphabet A with q symbols
- Order = How many previous symbols to use

  …

  2 = condition on 2 previous symbols, $P(x_n \mid x_{n-1}, x_{n-2})$

  1 = condition on 1 previous symbol, $P(x_n \mid x_{n-1})$

  0 = condition on current symbol, $P(x_n)$

  -1 = uniform over the alphabet, $P(x_n) = 1/q$

- For a given order:
  - Probability of next symbol based on counting occurrences given prior context

$$p_o(\phi) = \frac{c_o(\phi)}{C_o}.$$

**TABLE 6-1** CALCULATION OF BLENDED PROBABILITIES (SIMPLE SCHEME AND ESCAPE METHOD A) FOR THE MESSAGE "cacbcaabca"

| | Context | Counts | Predictions — $p_o(\phi)$ $c_o(\phi)$ | | | | | | | | | | Weight | Escape |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $o$ | | $C_o$ | a | | b | | c | | d | | e | | $w_o$ | $e_o$ |
| 4 | abca: | 0 | — | 0 | — | 0 | — | 0 | — | 0 | — | 0 | 0 | — |
| 3 | bca: | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 2 | ca: | 2 | $\frac{1}{2}$ | 1 | 0 | 0 | $\frac{1}{2}$ | 1 | 0 | 0 | 0 | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ |
| 1 | a: | 3 | $\frac{1}{3}$ | 1 | $\frac{1}{3}$ | 1 | $\frac{1}{3}$ | 1 | 0 | 0 | 0 | 0 | $\frac{1}{8}$ | $\frac{1}{4}$ |
| 0 | : | 10 | $\frac{4}{10}$ | 4 | $\frac{2}{10}$ | 2 | $\frac{4}{10}$ | 4 | 0 | 0 | 0 | 0 | $\frac{5}{132}$ | $\frac{1}{11}$ |
| −1 | — | — | $\frac{1}{5}$ | — | $\frac{1}{5}$ | — | $\frac{1}{5}$ | — | $\frac{1}{5}$ | — | $\frac{1}{5}$ | — | $\frac{1}{264}$ | 0 |
| Blended probabilities | | | $\frac{956}{1320}$ | | $\frac{66}{1320}$ | | $\frac{296}{1320}$ | | $\frac{1}{1320}$ | | $\frac{1}{1320}$ | | | |

*Bell, Cleary, Witten. Text Compression.*

$$p(\phi) = \sum_{o=-1}^{m} w_o p_o(\phi).$$

# Escape probabilities

- Need weights to blend probabilities
  - Compute based on "escape" probability
  - Allocate some mass in each model order for when a lower-order model should make prediction instead
  - Method A:
    - Add one to the count of characters seen in a context
    - $e_o = 1 / (C_0 + 1)$
  - Method D:
    - u = number of unique characters seen
    - $e_o = (u / 2) / C_0$

# Lossless compression benchmarks

| year | scheme | bits /char |
|---|---|---|
| 1967 | ASCII | 7.00 |
| 1950 | Huffman | 4.70 |
| 1977 | LZ77 | 3.94 |
| 1984 | LZMW | 3.32 |
| 1987 | LZH | 3.30 |
| 1987 | move-to-front | 3.24 |
| 1987 | LZB | 3.18 |
| 1987 | gzip | 2.71 |
| 1988 | PPMC | 2.48 |
| 1994 | PPM | 2.34 |
| 1995 | Burrows-Wheeler | 2.29 |
| 1997 | BOA | 1.99 |
| 1999 | RK | 1.89 |

*Data compression using Calgary corpus*

# My benchmark

```
09/30/2011  09:13 AM         1,191,463 mobydick.txt
04/22/2012  10:33 PM           485,561 mobydick.gz
04/22/2012  10:31 PM           485,790 mobydick.zip
04/24/2012  01:03 PM           371,394 mobydick.bz2

04/22/2012  10:32 PM           667,651 mobydick.huff
04/22/2012  10:31 PM           597,060 mobydick.lzw
04/22/2012  10:32 PM           814,578 mobydick.huff.lzw
04/22/2012  10:31 PM           592,830 mobydick.lzw.huff
04/22/2012  11:06 PM           682,400 mobydick.lzw.reset

04/22/2012  10:36 PM           541,261 mobydick.lzw14
04/22/2012  10:38 PM           521,700 mobydick.lzw15
04/22/2012  10:34 PM           503,050 mobydick.lzw16
04/22/2012  10:37 PM           501,193 mobydick.lzw16.huff
04/22/2012  10:38 PM           521,700 mobydick.lzw17
04/22/2012  10:36 PM           514,393 mobydick.lzw18
04/22/2012  10:35 PM           571,548 mobydick.lzw20

04/24/2012  01:02 PM           325,378 mobydick.ppmd
```

# Summary

- Dictionary-based compression
  - LZW and variants
  - Memorizes sequences in the data
- Statistical coding
  - Language model produces probabilities
  - Probability sequence defines point on [0, 1)
  - Use arithmetic coding to convert to bits
  - Prediction by Partial Match (PPM)