## Sampling Examples



| $A$ | $P(A)$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0.200 |  |
| 1 | 0.800 |  |
| 0 | $A$ | $P(B \mid A)$ |
| 1 | 0 | 0.400 |
|  | 0 | 0.600 |
| 0 | 1 | 0.200 |
| 1 | 1 | 0.800 |$\quad$| $C$ | $B$ | $P(C \mid B)$ |
| :--- | :--- | :--- |
| 0 | 0 | 0.600 |
| 1 | 0 | 0.400 |
| 0 | 1 | 0.600 |
| 1 | 1 | 0.400 |$\quad$| $D$ | $B$ | $P(D \mid B)$ |
| :--- | :--- | :--- |
| 0 | 0 | 0.800 |
| 1 | 0 | 0.200 |
| 0 | 1 | 0.600 |
| 1 | 1 | 0.400 |


| $E$ | $C$ | $D$ | $P(E \mid C, D)$ |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0.200 |
| 1 | 0 | 0 | 0.800 |
| 0 | 1 | 0 | 0.600 |
| 1 | 1 | 0 | 0.400 |
| 0 | 0 | 1 | 0.800 |
| 1 | 0 | 1 | 0.200 |
| 0 | 1 | 1 | 0.800 |
| 1 | 1 | 1 | 0.200 |

## What are:

A:
B:
C:

D:
E:
What is the weight for the sample above?

In this question, we will perform likelihood weighting to estimate $P(C=1 \mid B=1, E=1)$.

Below are a set of weighted samples obtained by running likelihood weighting for the Bayes' net from the previous question. Use them to estimate $P(C=1 \mid B=1, E=1)$. Input -1 in the box below if the estimation cannot be made.


## Estimation:

Enter your answer here

(a) You are given the following samples:

$$
\begin{array}{llllllll}
+a & +b & -c & -d & +a & -b & -c & +d \\
+a & -b & +c & -d & +a & +b & +c & -d \\
-a & +b & +c & -d & -a & +b & -c & +d \\
-a & -b & +c & -d & -a & -b & +c & -d
\end{array}
$$

(i) Assume that these samples came from performing Prior Sampling, and calculate the sample estimate of $P(+c)$.
(ii) Now we will estimate $P(+c \mid+a,-d)$. Above, clearly cross out the samples that would not be used when doing Rejection Sampling for this task, and write down the sample estimate of $P(+c \mid+a,-d)$ below.
(b) Using Likelihood Weighting Sampling to estimate $P(-a \mid+b,-d)$, the following samples were obtained. Fill in the weight of each sample in the corresponding row.

$$
\left.\begin{array}{llll}
\text { Sample } & & & \text { Weight } \\
-a & +b & +c & -d
\end{array}\right) \square
$$

(c) From the weighted samples in the previous question, estimate $P(-a \mid+b,-d)$.
(e) Recall that during Gibbs Sampling, samples are generated through an iterative process.

Assume that the only evidence that is available is $A=+a$. Clearly fill in the circle(s) of the sequence(s) below that could have been generated by Gibbs Sampling.

Sequence 1

| $1:$ | $+a$ | $-b$ | $-c$ | $+d$ |
| :--- | :--- | :--- | :--- | :--- |
| $2:$ | $+a$ | $-b$ | $-c$ | $+d$ |
| $3:$ | $+a$ | $-b$ | $+c$ | $+d$ |

Sequence 3

| $1:$ | $+a$ | $-b$ | $-c$ | $+d$ |
| :---: | :---: | :---: | :---: | :---: |
| $2:$ | $+a$ | $-b$ | $-c$ | $-d$ |
| $3:$ | $+a$ | $+b$ | $-c$ | $-d$ |

Sequence 2

| $1:$ | $+a$ | $-b$ | $-c$ | $+d$ |
| :---: | :---: | :---: | :---: | :---: |
| $2:$ | $+a$ | $-b$ | $-c$ | $-d$ |
| $3:$ | $-a$ | $-b$ | $-c$ | $+d$ |

Sequence 4

$$
\begin{array}{l|llll}
\hline 1: & +a & -b & -c & +d \\
2: & +a & -b & -c & -d \\
3: & +a & +b & -c & +d
\end{array}
$$

