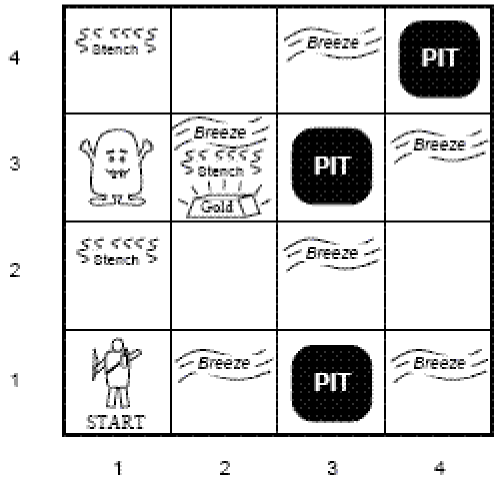
LOGICAL AGENTS

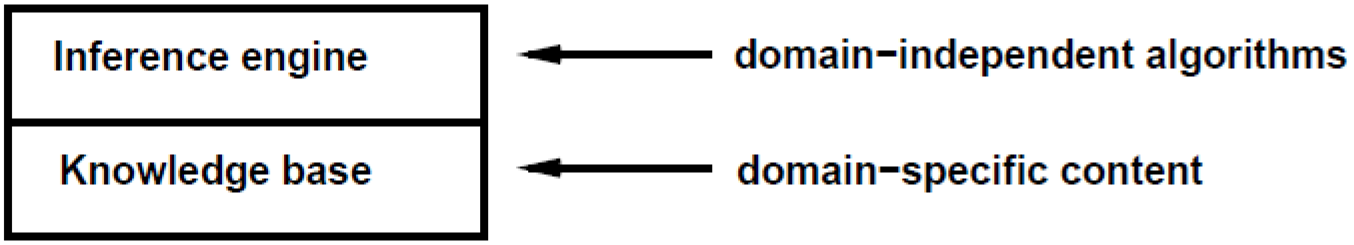


Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution

Knowledge Bases

- Knowledge base = set of sentences in a formal language
- Declarative approach to building an agent (or other system):
  - Tell it what it needs to know
- Then it can Ask itself what to do - answers should follow from the KB
- Agents can be viewed at the knowledge level
  - i.e., what they know, regardless of how implemented
- Or at the implementation level
  - i.e., data structures in KB and algorithms that manipulate them

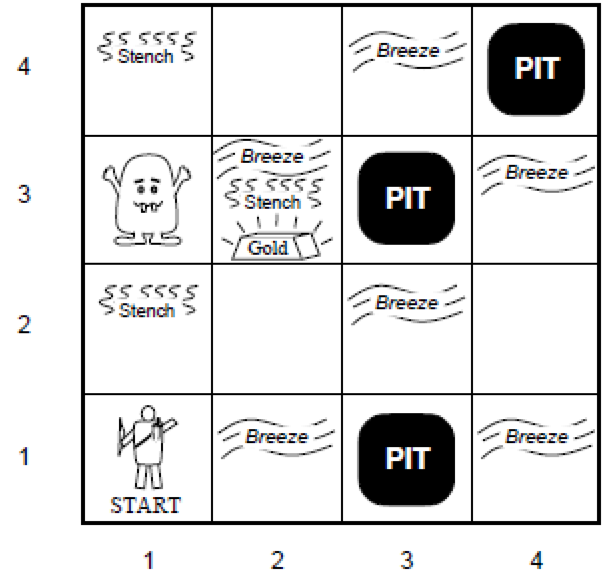


A simple Knowledge-Based Agent

- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world
  - Deduce appropriate actions

Wumpus World PEAS Description

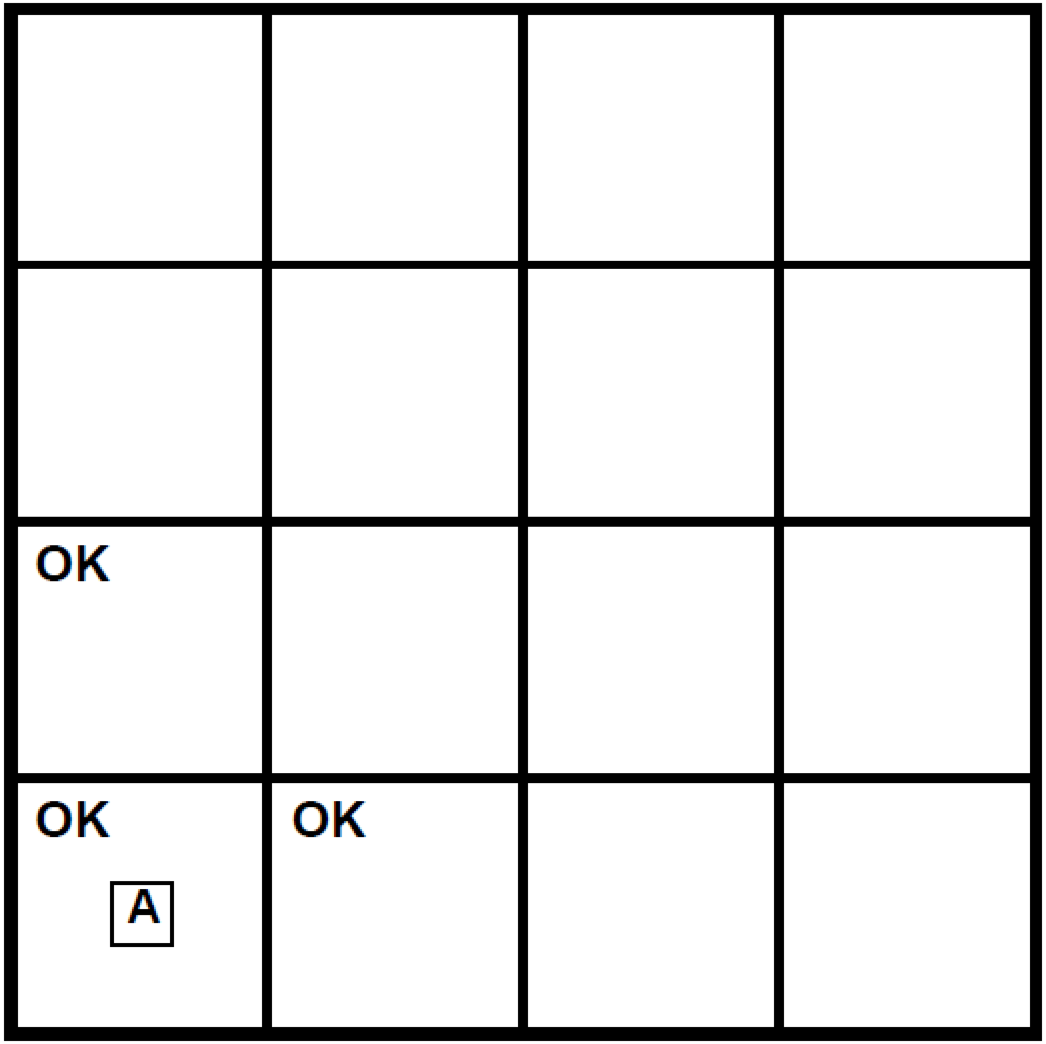
- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter if gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square
- Actuators Left turn, Right turn,
  - Forward, Grab, Release, Shoot
- Sensors
  - Breeze, Glitter, Smell



Wumpus World Characterization

- Observable??
  - No - only local perception
- Deterministic??
  - Yes - outcomes exactly specified
- Episodic??
  - No - sequential at the level of actions
- Static??
  - Yes - Wumpus and Pits do not move
- Discrete??
  - Yes
- Single-agent??
  - Yes - Wumpus is essentially a natural feature

Exploring a Wumpus World



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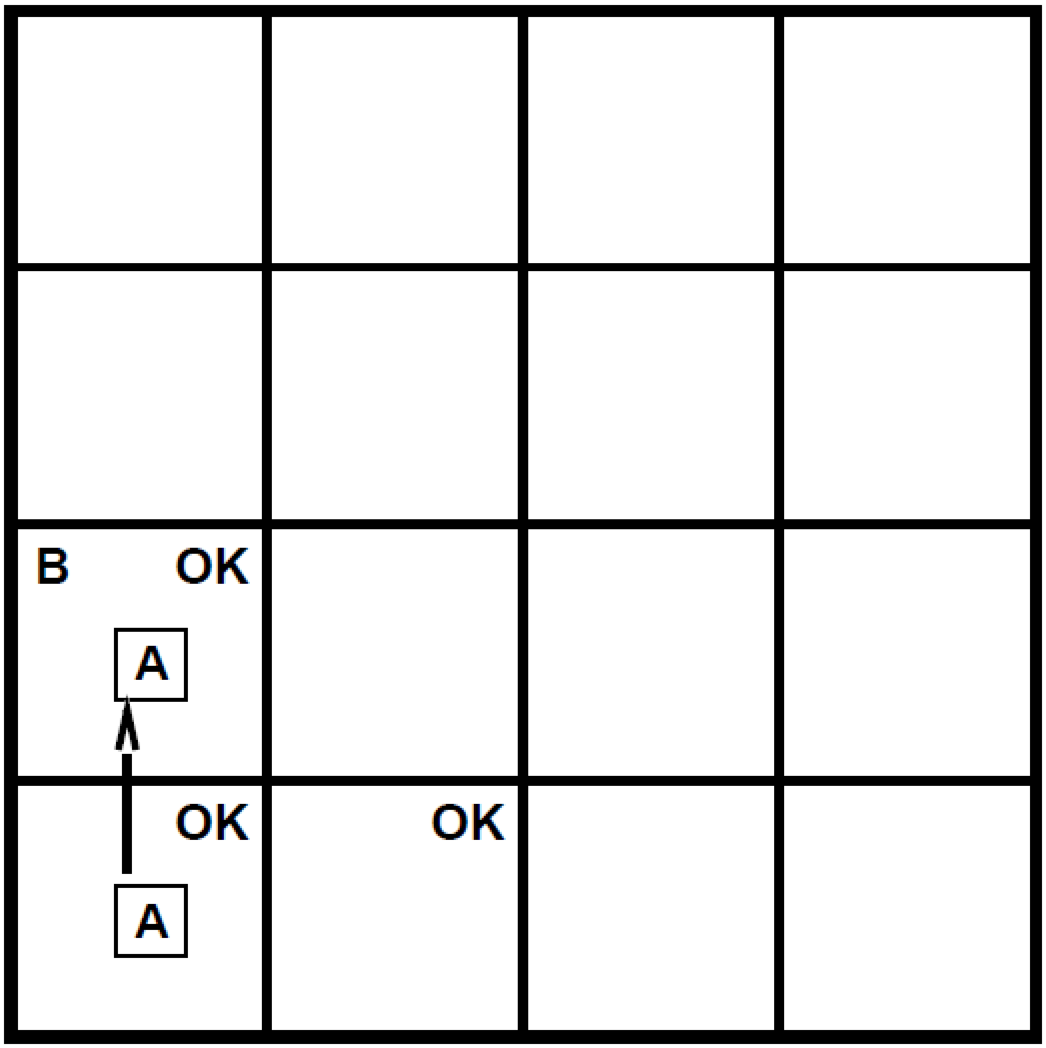

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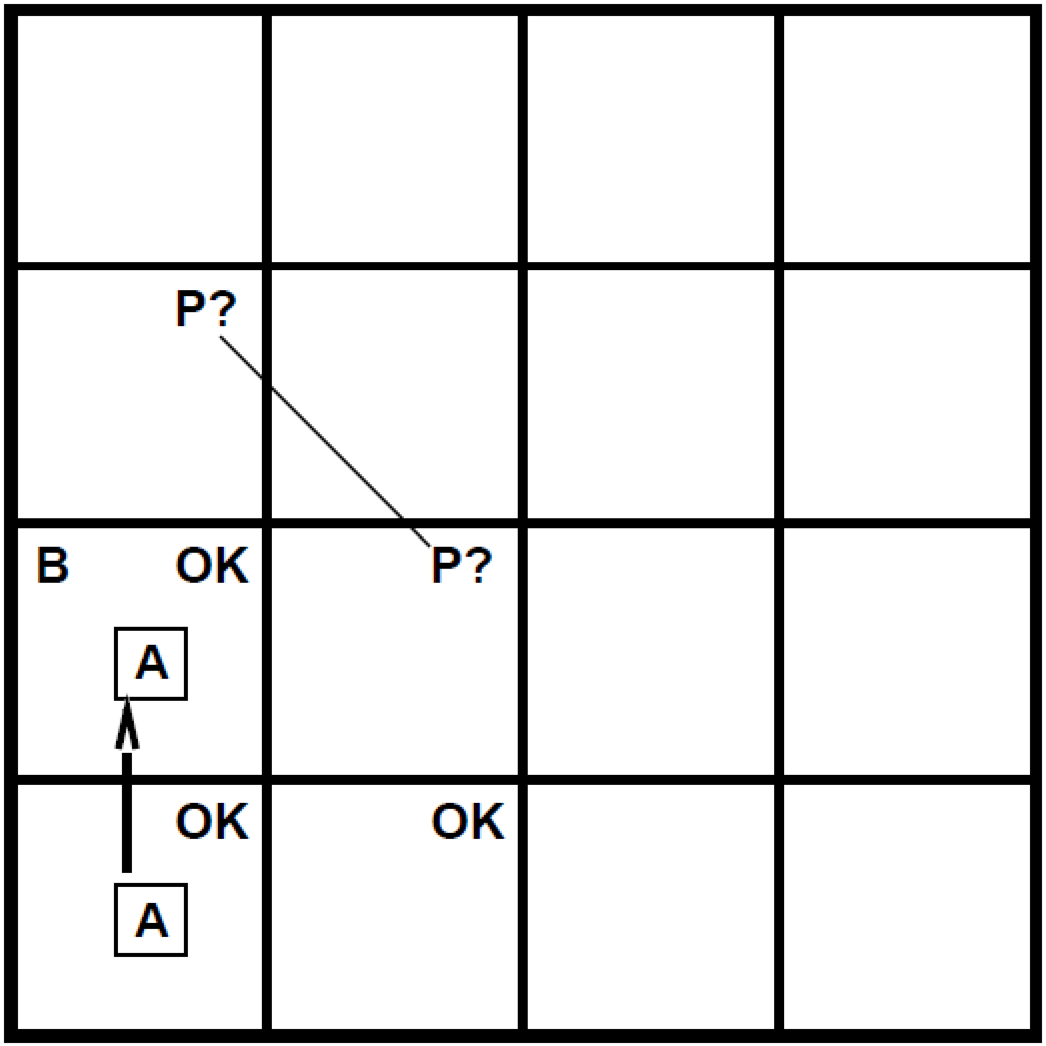

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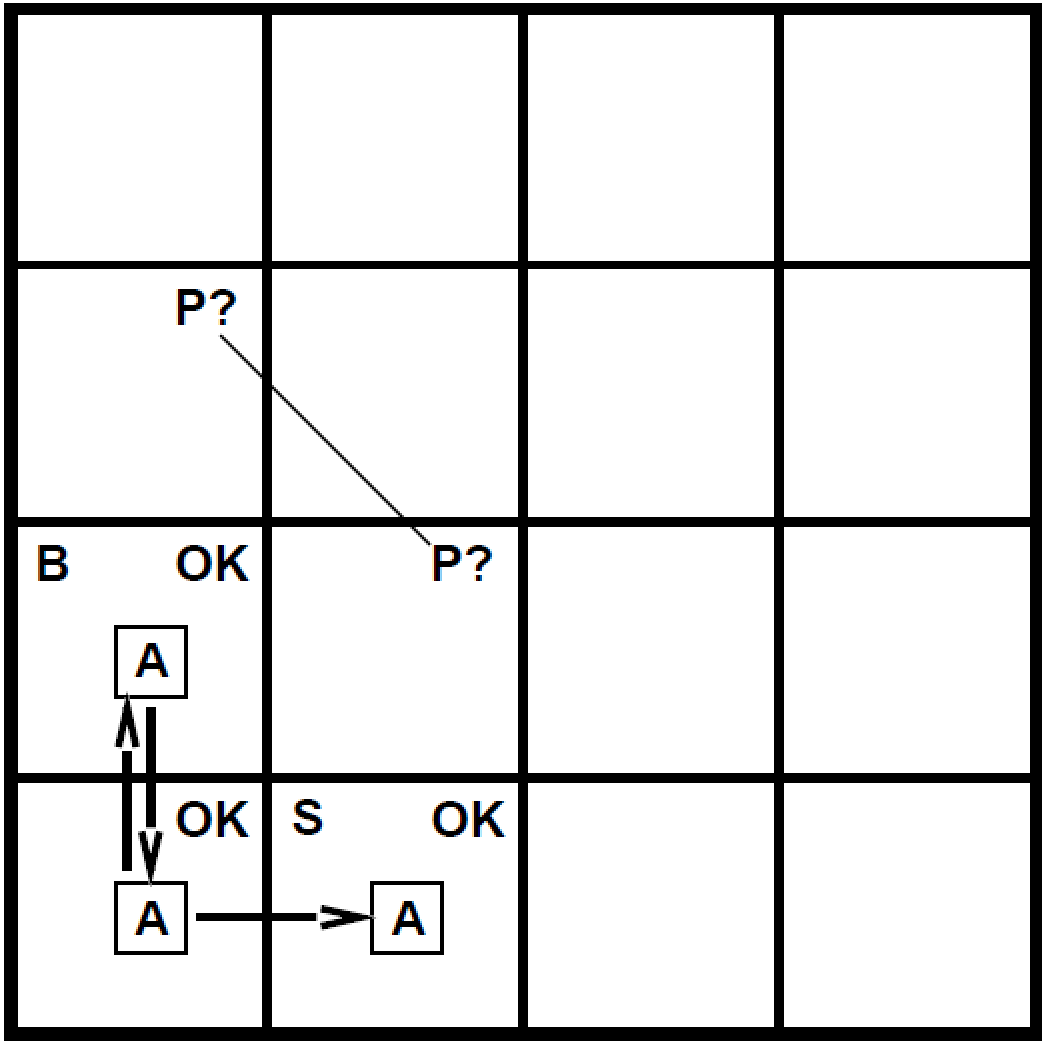

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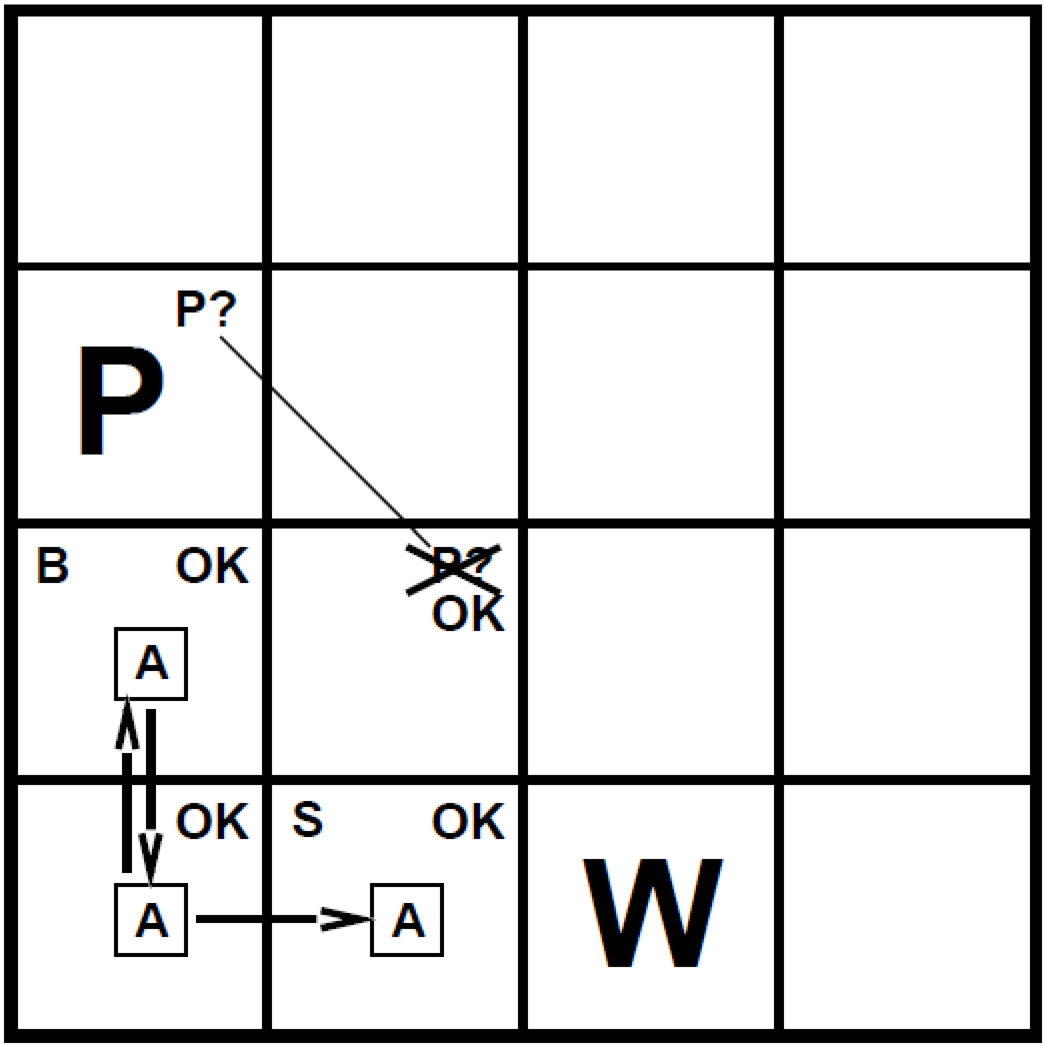

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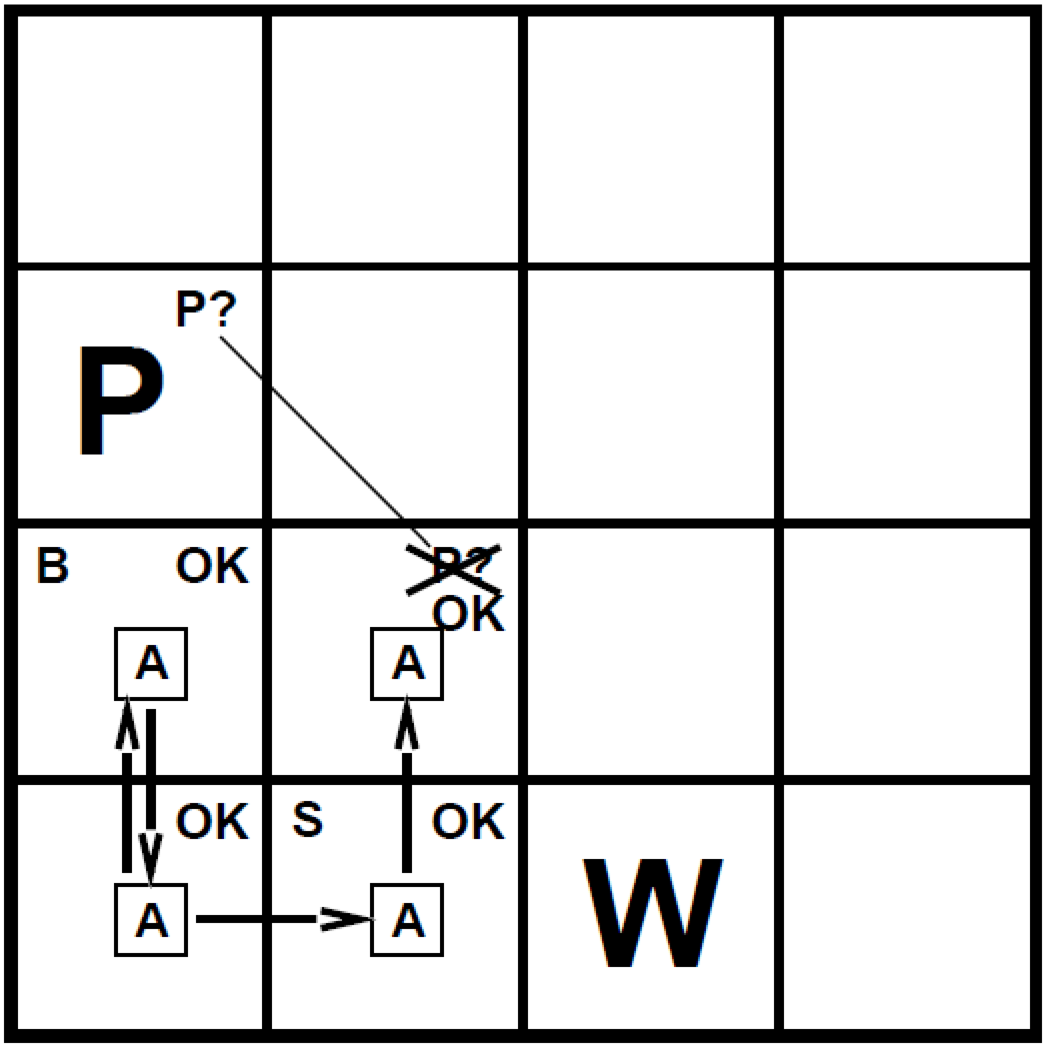

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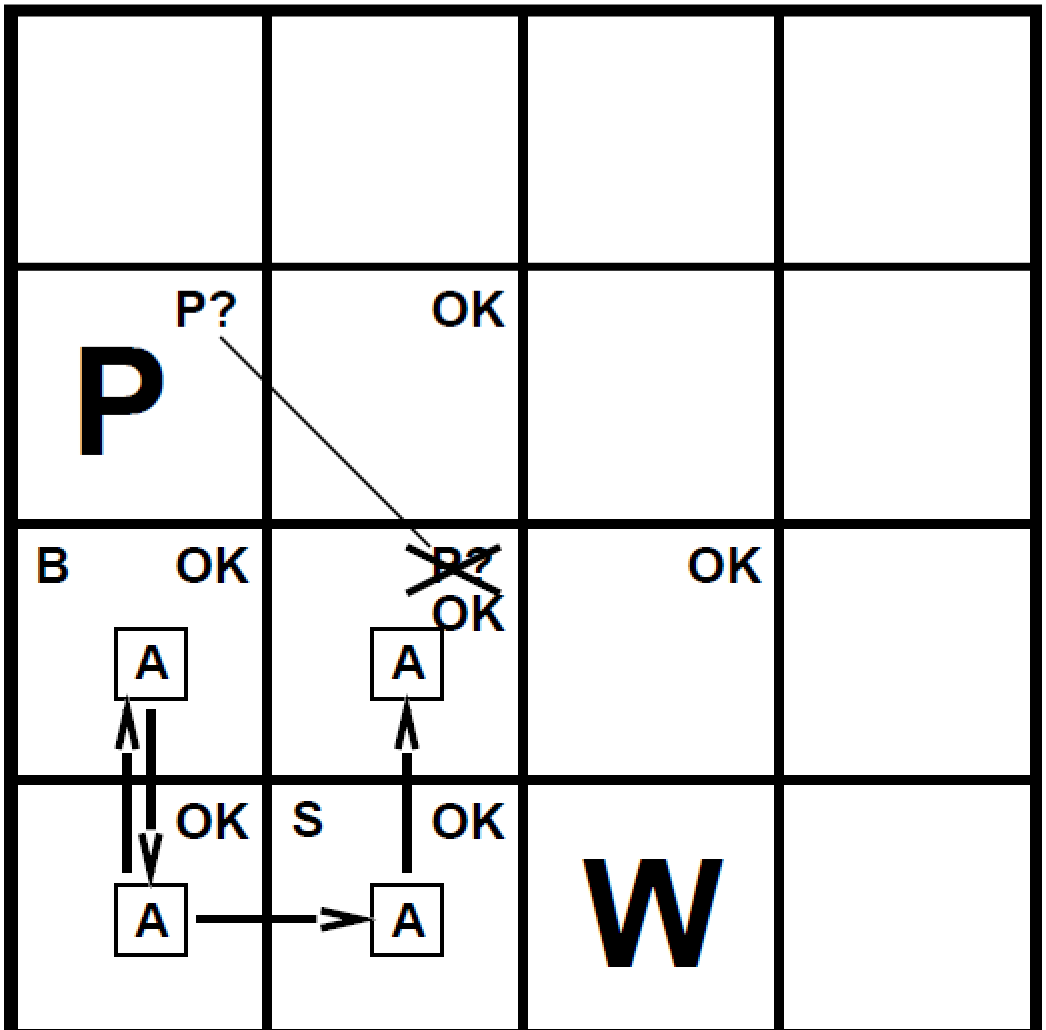
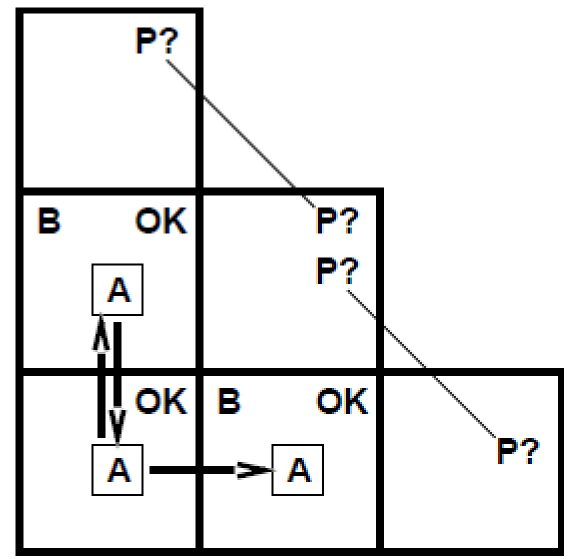
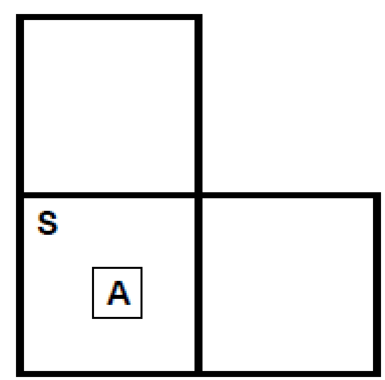


Other Tight Spots

- Breeze in (1,2) and (2,1)
  - => no safe actions
- Assuming pits uniformly distributed,
- (2,2) has pit w/ prob 0.86, vs. 0.31



- Smell in (1,1)
  - => cannot move
- Can use a strategy of coercion:
  - Shoot straight ahead
  - Wumpus was there => dead => safe
  - Wumpus wasn't there => safe



Logic in General

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the “meaning” of sentences;
  - i.e., define truth of a sentence in a world
- E.g., the language of arithmetic
  - x + 2 >= y is a sentence; x2 + y > is not a sentence
  - x + 2 >= y is true iff the number x + 2 is no less than the number y
  - x + 2 >= y is true in a world where x=7, y =1
  - x + 2 >= y is false in a world where x=0, y =6

Entailment

- Entailment means that one thing follows from another:

$$KB \models \alpha$$

- Knowledge base KB entails sentence α if and only if α is true in all worlds where KB is true
- E.g., x + y = 4 entails 4 = x + y
- Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Models

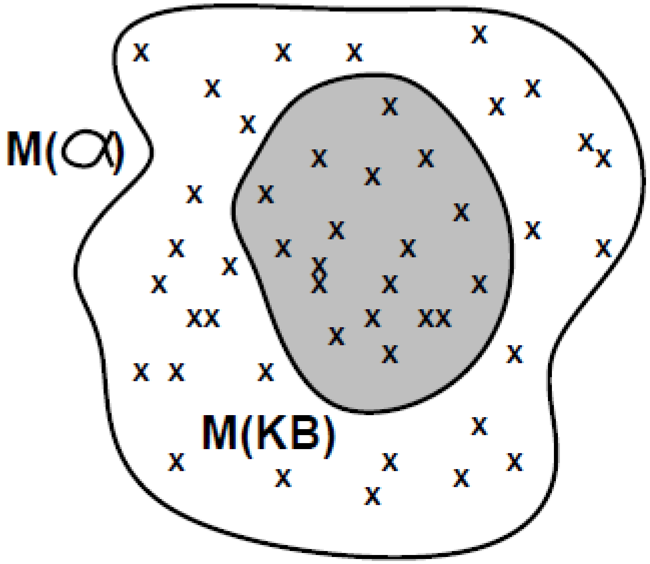
- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
- We say m is a model of a sentence α if α is true in m
- M(α) is the set of all models of α
- Then

$$KB \models \alpha$$

- if and only if

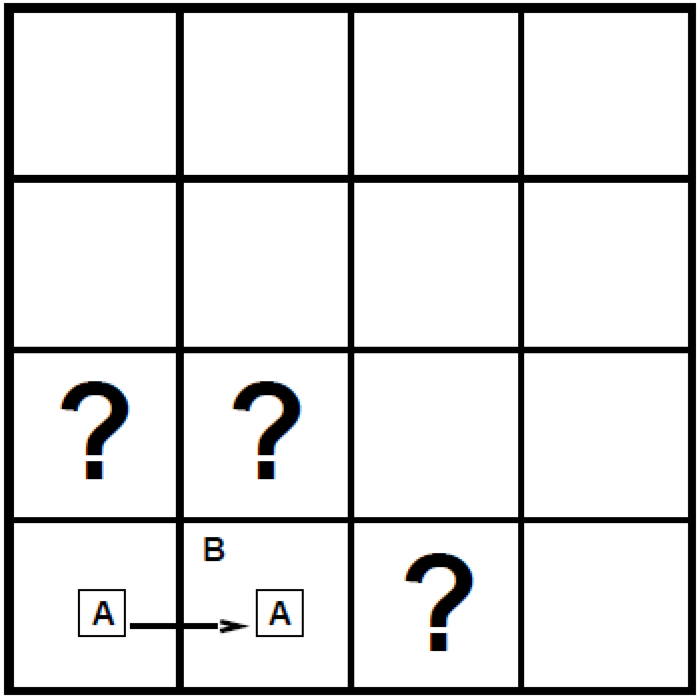
$$M(KB) \subseteq M(\alpha)$$

- E.g. KB = Giants won and Reds won
  - α = Giants won

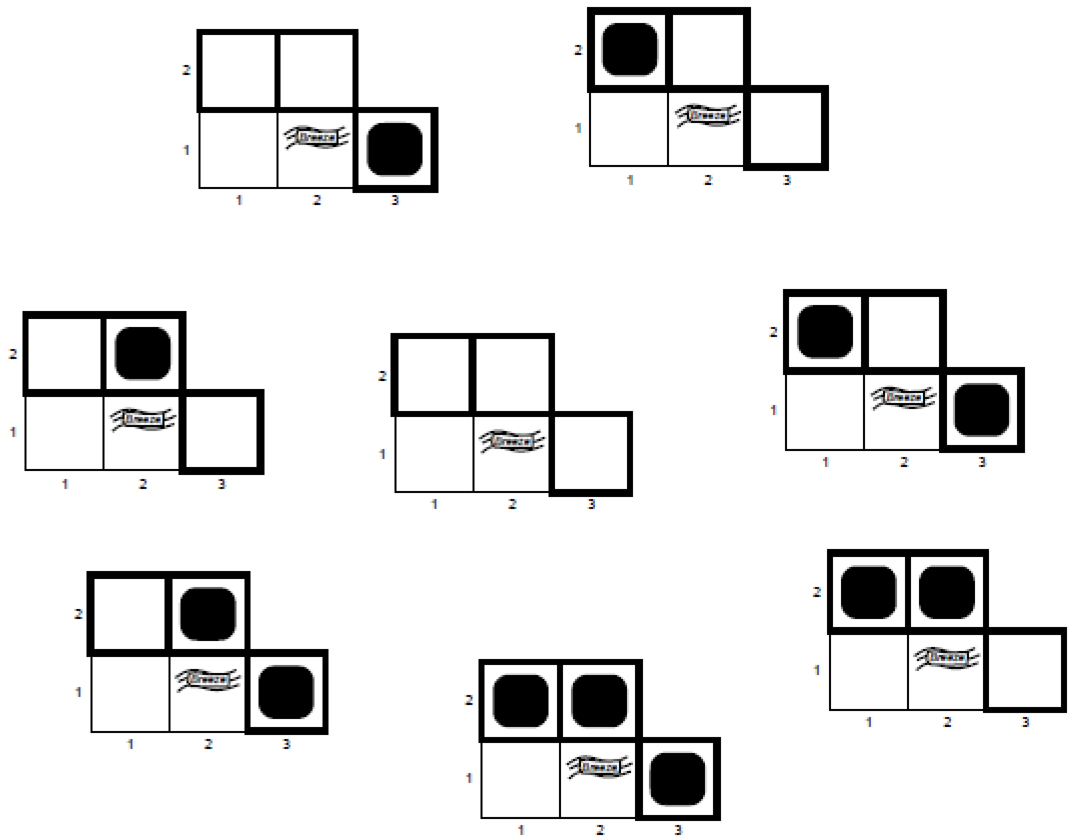


Entailment in the Wumpus World

- Situation after detecting nothing in [1,1], moving right, breeze in [2,1]
- Consider possible models for ?s
- assuming only pits, 3 Boolean choices => 8 possible models

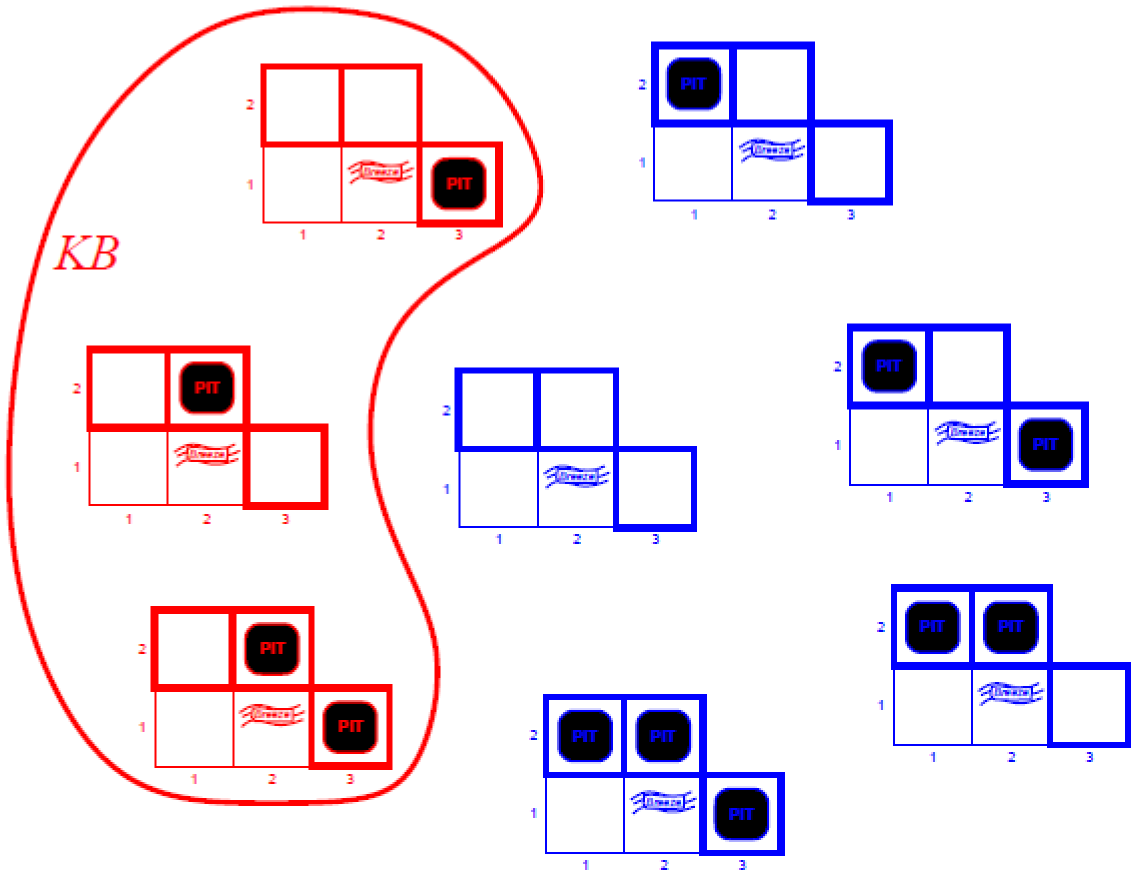


Wumpus Models



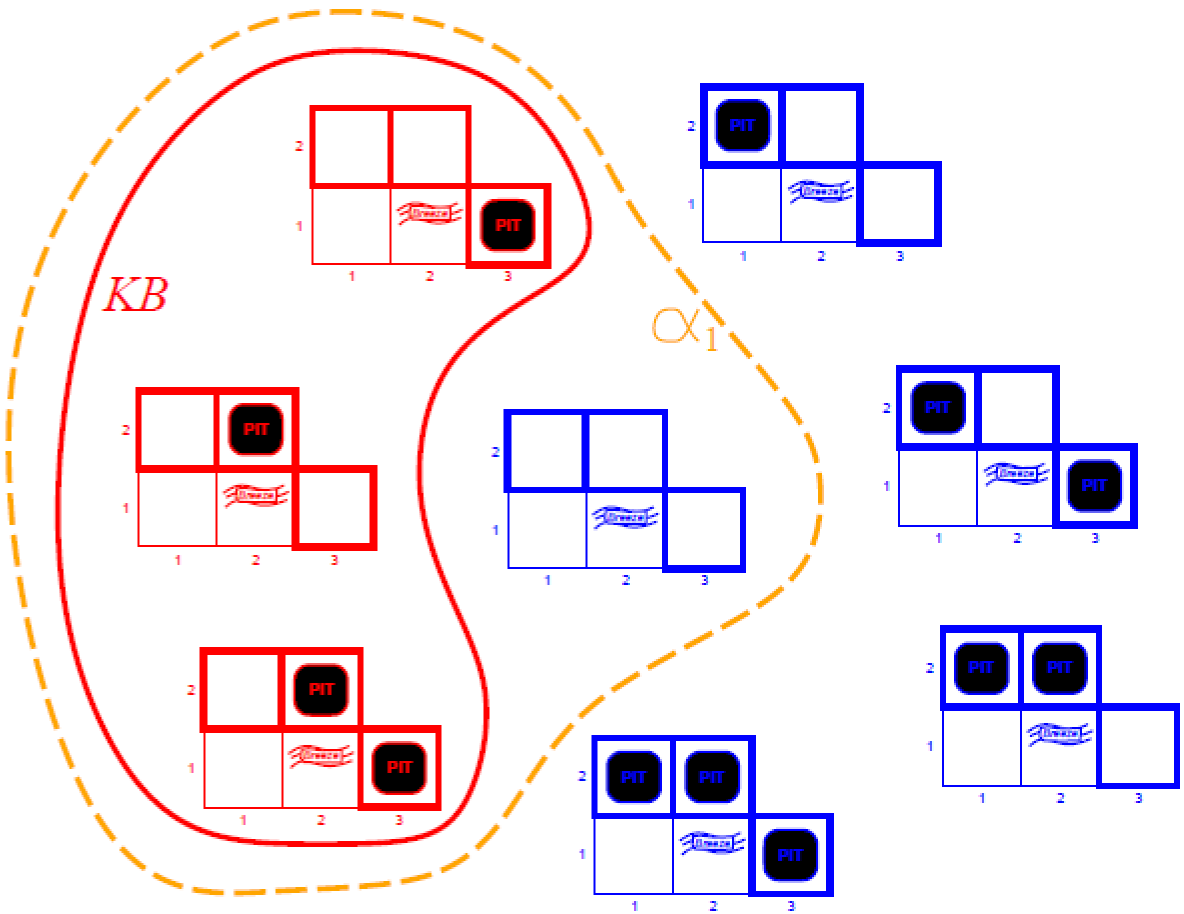
Wumpus Models

KB = wumpus-world rules + observations



Wumpus Models

- KB = wumpus-world rules + observations
- α1 = “[1,2] is safe”, KB |= α1, proved by model checking



Wumpus Models

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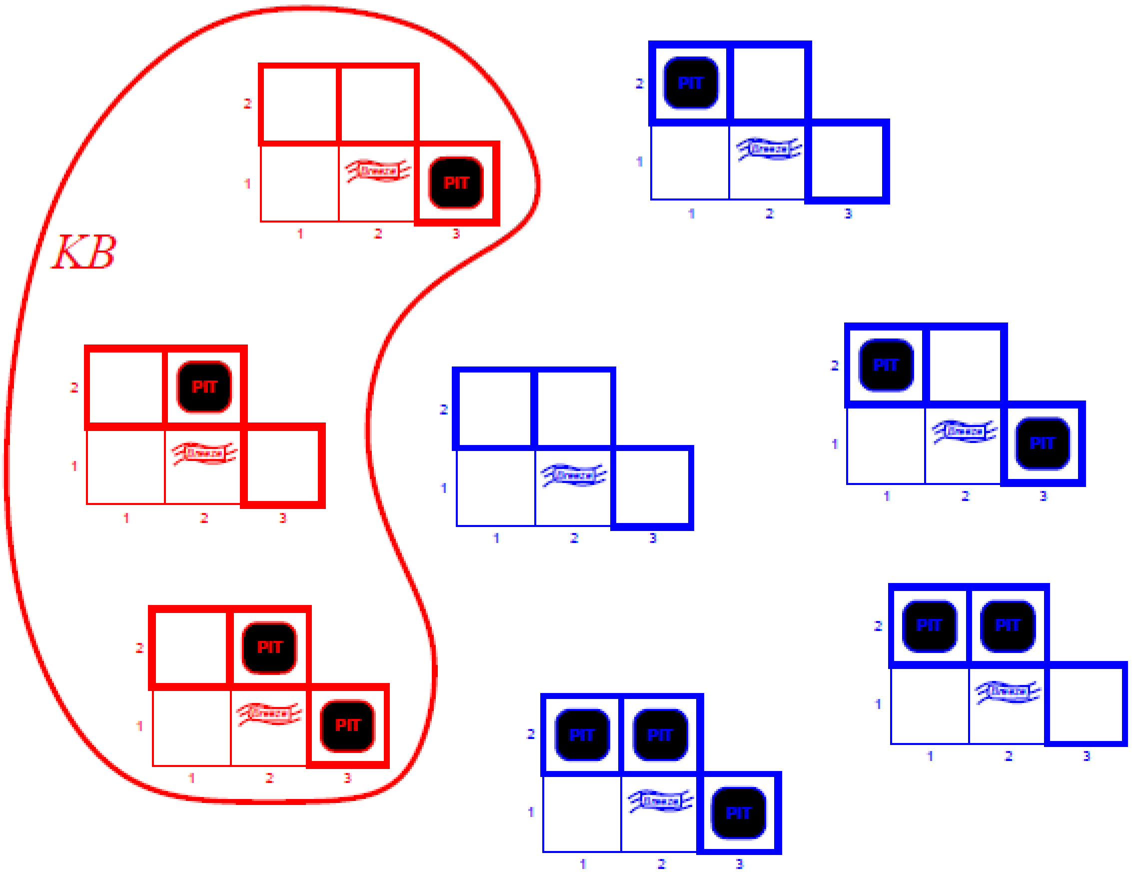
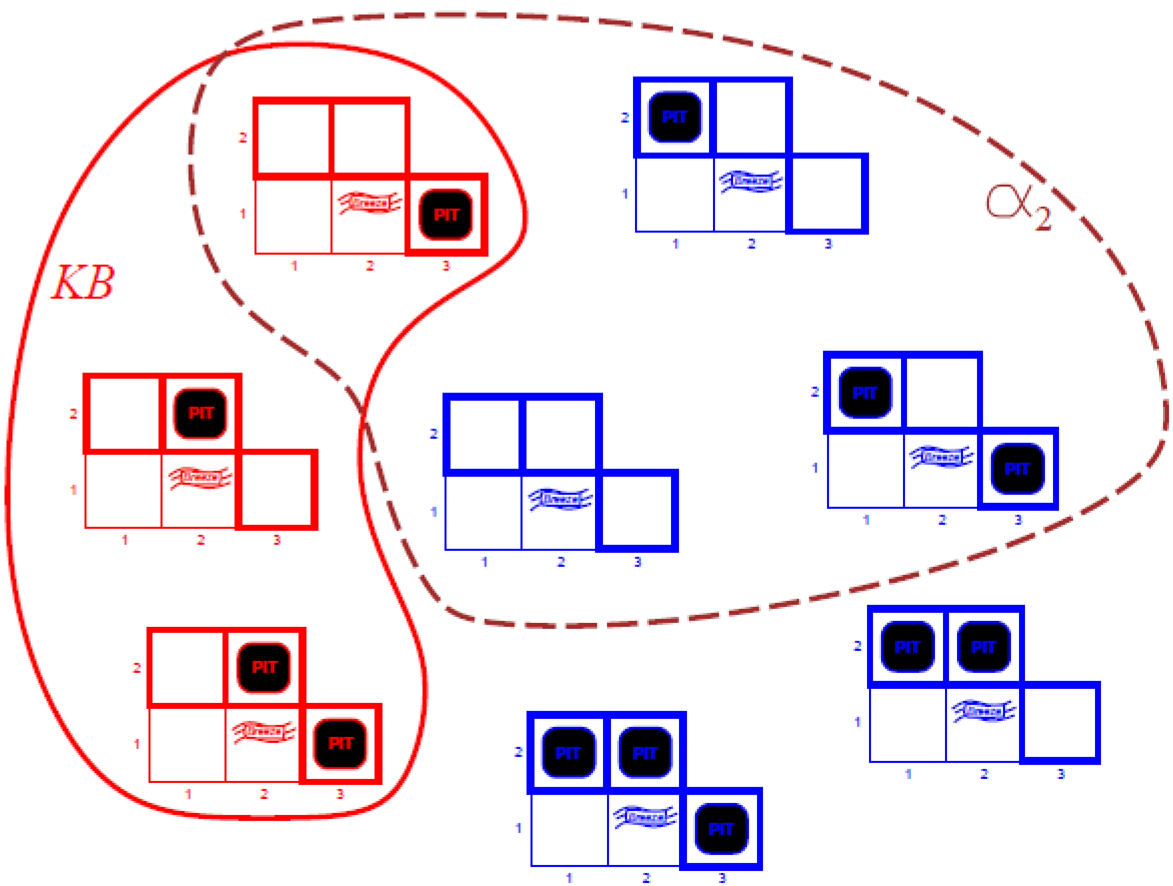


Wumpus models

- KB = wumpus-world rules + observations
- α2 = “[2,2] is safe”, KB |≠ α2



Inference

$$KB \vdash_i \alpha$$

- means sentence α can be derived from KB by procedure i

- Consequences of KB are a haystack; α is a needle.
- Entailment = needle in haystack; inference = finding it
- Soundness: i is sound if
  - whenever $KB \vdash_i \alpha$
  - it is also true that $KB \models \alpha$
- Completeness: i is complete if
  - whenever $KB \models \alpha$
  - it is also true that $KB \vdash_i \alpha$
- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose answer follows from what is known by the KB.

Propositional Logic: Syntax

- Propositional logic is the simplest logic - illustrates basic ideas
- The proposition symbols P_1, P_2 etc. are sentences
- If S is a sentence, $\neg S$ is a sentence (negation)
- If S1 and S2 are sentences, $S1 \wedge S2$ is a sentence (conjunction)
- If S1 and S2 are sentences, $S1 \vee S2$ is a sentence (disjunction)
- If S1 and S2 are sentences, $S1 \Rightarrow S2$ is a sentence (implication)
- If S1 and S2 are sentences, $S1 \Leftrightarrow S2$ is a sentence (biconditional)

Propositional Logic: Semantics

- Each model species true/false for each proposition symbol
- E.g. $\begin{matrix} P_{1,2} & P_{2,2} & P_{3,1} \\ true & true & false \end{matrix}$
- (With these symbols, 8 possible models, can be enumerated automatically.)
- Rules for evaluating truth with respect to a model m:

$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true **and**	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true **or**	S_2	is true
$S_1 \Rightarrow S_2$	is true iff	S_1	is false **or**	S_2	is true
i.e.,	is false iff	S_1	is true **and**	S_2	is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true **and**	$S_2 \Rightarrow S_1$	is true

$P_{1,2} \wedge (\neg P_{2,2} \vee \neg P_{3,1}) = true \wedge (false \vee true) = true \wedge true = true$

- Simple recursive process evaluates an arbitrary sentence

Truth Tables for Connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Wumpus World Sentences

- Let $P_{i,j}$ be true if there is a pit in [i, j].
- Let $B_{i,j}$ be true if there is a breeze in [i, j].
  - $\neg P_{1,1}$
  - $\neg B_{1,1}$
  - $B_{2,1}$
- “Pits cause breezes in adjacent squares”
  - $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$
  - $B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$
- “A square is breezy if and only if there is an adjacent pit”

Truth Tables for Inference

- Enumerate rows (different assignments to symbols), if KB is true in row, check that α is too

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Inference by Enumeration

- Depth-first enumeration of all models is sound and complete
- $O(2^n)$ for n symbols; problem is co-NP-complete

```
function TT-ENTAILS?(KB, α) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
           α, the query, a sentence in propositional logic

   symbols ← a list of the proposition symbols in KB and α
   return TT-CHECK-ALL(KB, α, symbols, [])
```

```
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
   if EMPTY?(symbols) then
       if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
       else return true
   else do
       P ← FIRST(symbols); rest ← REST(symbols)
       return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
              TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

Logical Equivalence

- Two sentences are logically equivalent if true in same models:

$$\alpha \equiv \beta$$

- if and only if

$$\alpha \models \beta$$

- and

$$\beta \models \alpha$$

$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	De Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	De Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$

Validity and Satisfiability

- A sentence is valid if it is true in all models,
  - e.g., True, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$
- Validity is connected to inference via the Deduction Theorem:
  - $KB \vDash \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid
- A sentence is satisfiable if it is true in some model
  - e.g., $A \vee B$, C
- A sentence is unsatisfiable if it is true in no models
  - e.g., $A \wedge \neg A$
- Satisfiability is connected to inference via the following:
  - $KB \vDash \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable
  - i.e., prove by reductio ad absurdum

Forward and Backward Chaining

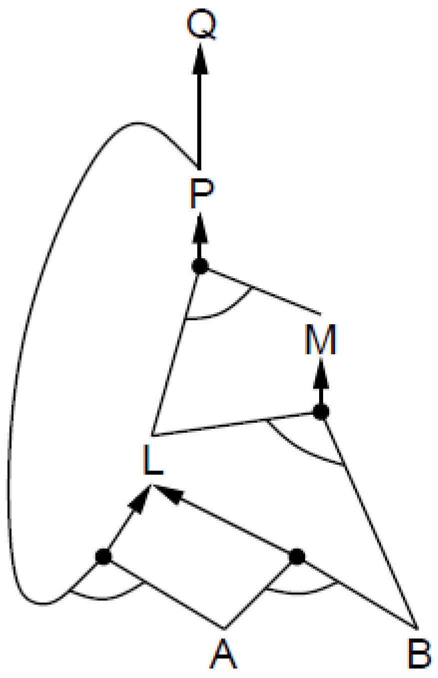
- Horn Form (restricted)
  - KB = conjunction of Horn clauses
  - Horn clause =
    - proposition symbol; or
    - (conjunction of symbols) ⇒ symbol
  - E.g., C ∧ (B ⇒ A) ∧ (C ∧ D) ⇒ B)
- Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

- Can be used with forward chaining or backward chaining.
- These algorithms are very natural and run in linear time

Forward Chaining

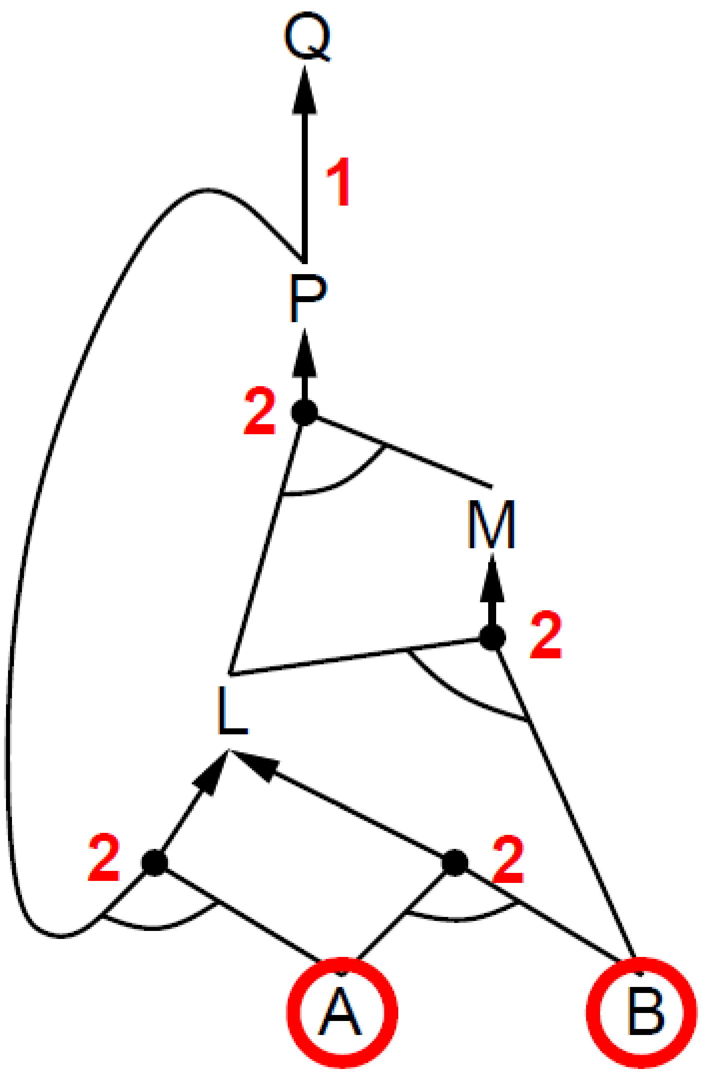
- Idea: fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found
  - $P \Rightarrow Q$
  - $L \wedge M \Rightarrow P$
  - $B \wedge L \Rightarrow M$
  - $A \wedge P \Rightarrow L$
  - $A \wedge B \Rightarrow L$
  - A
  - B



Forward Chaining Algorithm

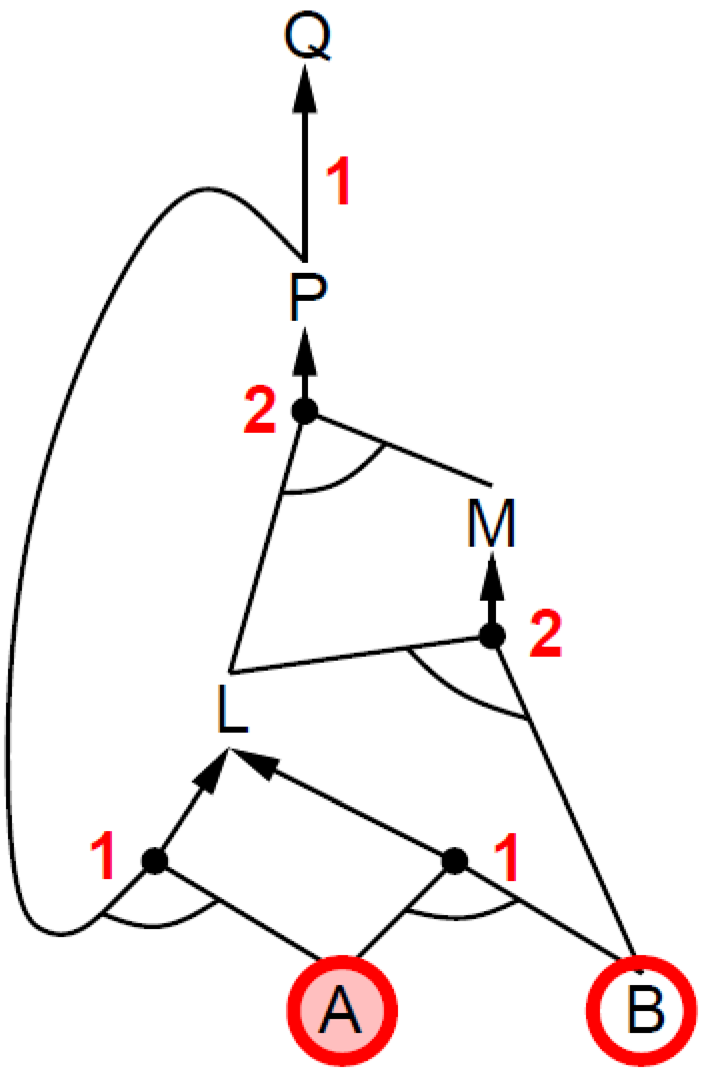
```
function PL-FC-ENTAILS?(KB, q) returns true or false
   inputs: KB, the knowledge base, a set of propositional Horn clauses
           q, the query, a proposition symbol
   local variables: count, a table, indexed by clause, initially the number of premises
                    inferred, a table, indexed by symbol, each entry initially false
                    agenda, a list of symbols, initially the symbols known in KB

   while agenda is not empty do
       p ← POP(agenda)
       unless inferred[p] do
           inferred[p] ← true
           for each Horn clause c in whose premise p appears do
               decrement count[c]
               if count[c] = 0 then do
                   if HEAD[c] = q then return true
                   PUSH(HEAD[c], agenda)
   return false
```

Forward Chaining Example



P ⇒ Q
L ∧ M ⇒ P
B ∧ L ⇒ M
A ∧ P ⇒ L
A ∧ B ⇒ L
A
B

Forward Chaining Example



P ⇒ Q
L ∧ M ⇒ P
B ∧ L ⇒ M
A ∧ P ⇒ L
A ∧ B ⇒ L
A
B

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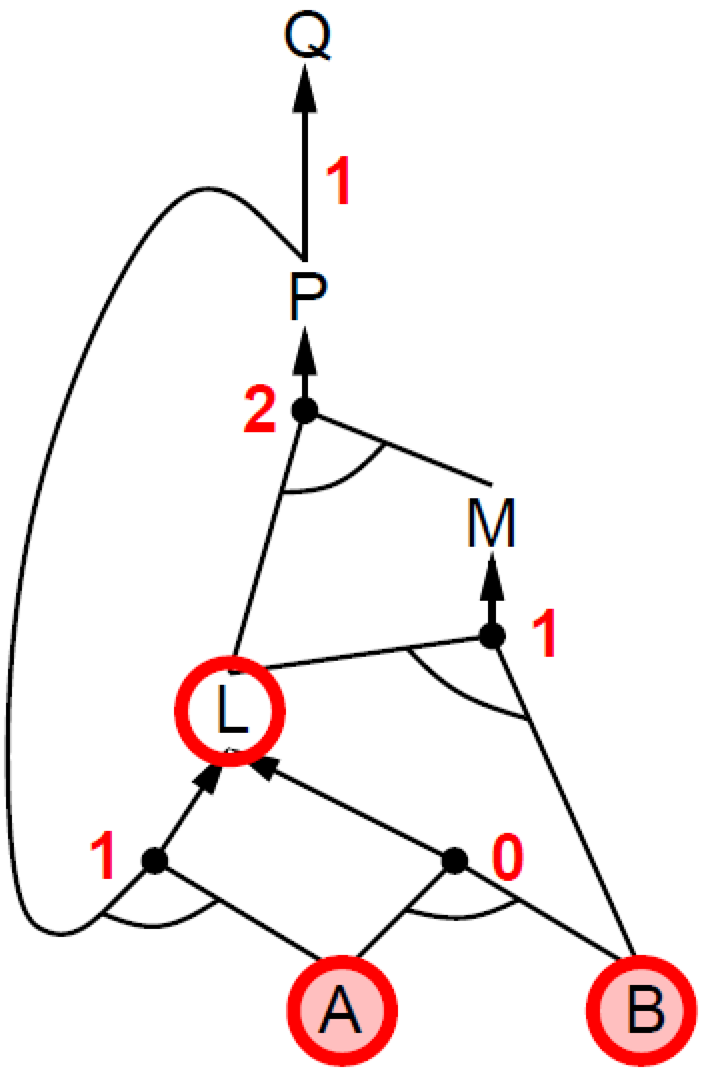

P ⇒ Q
L ∧ M ⇒ P
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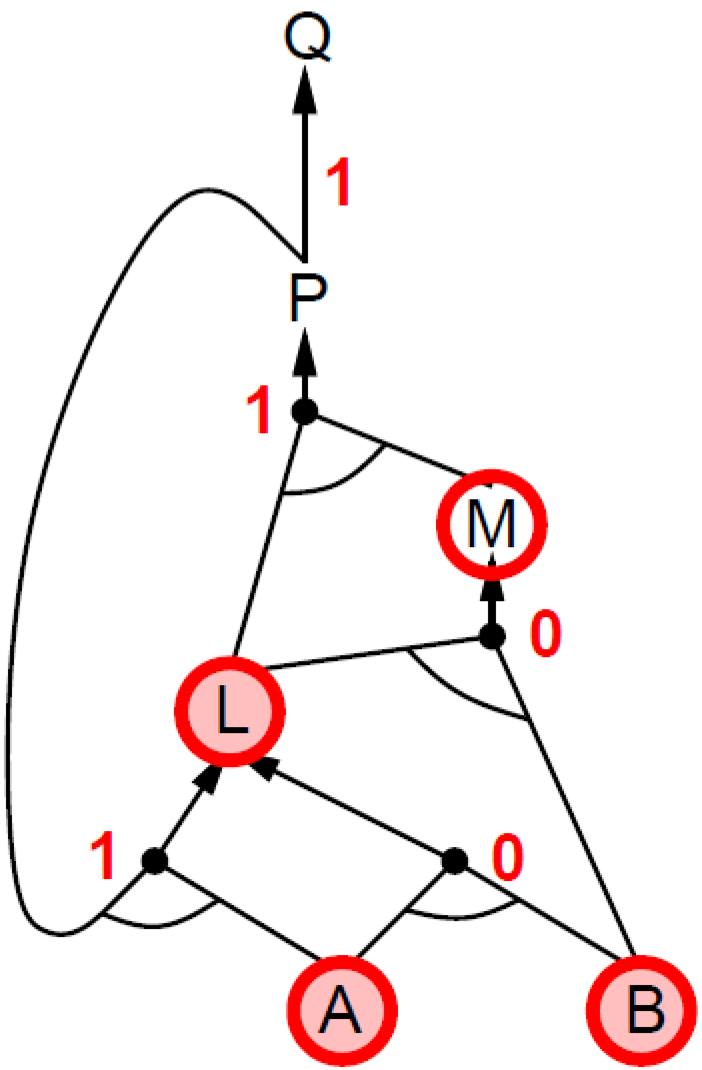
A
B

Forward Chaining Example



P ⇒ Q
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A ∧ P ⇒ L
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A
B

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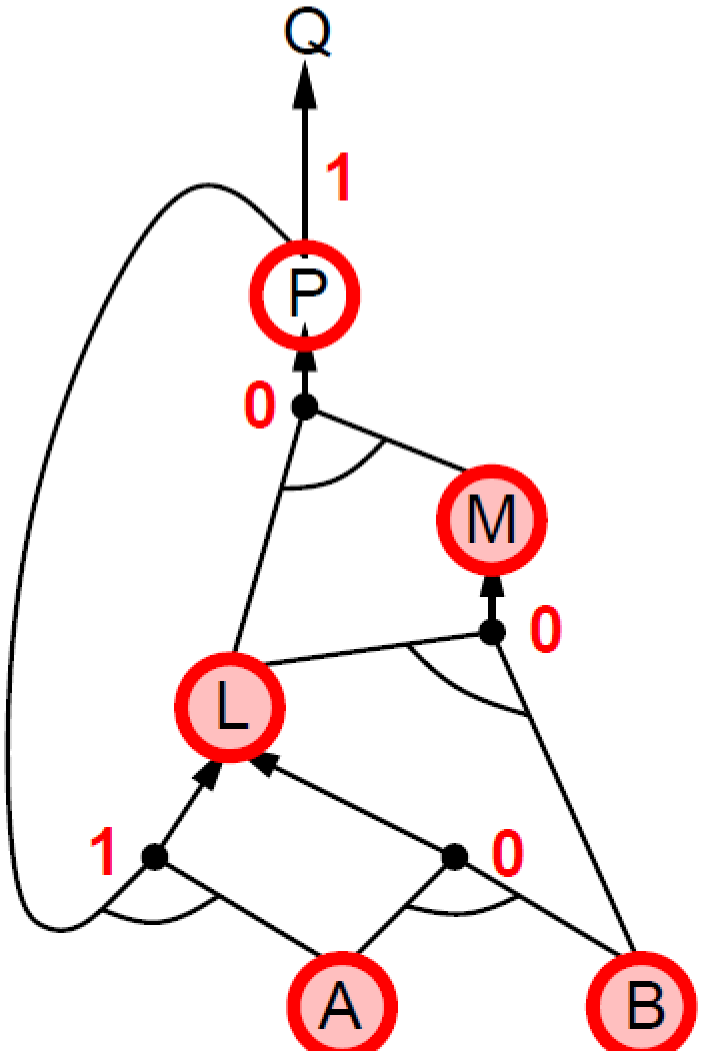

P ⇒ Q
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B ∧ L ⇒ M
A ∧ P ⇒ L
A ∧ B ⇒ L
A
B

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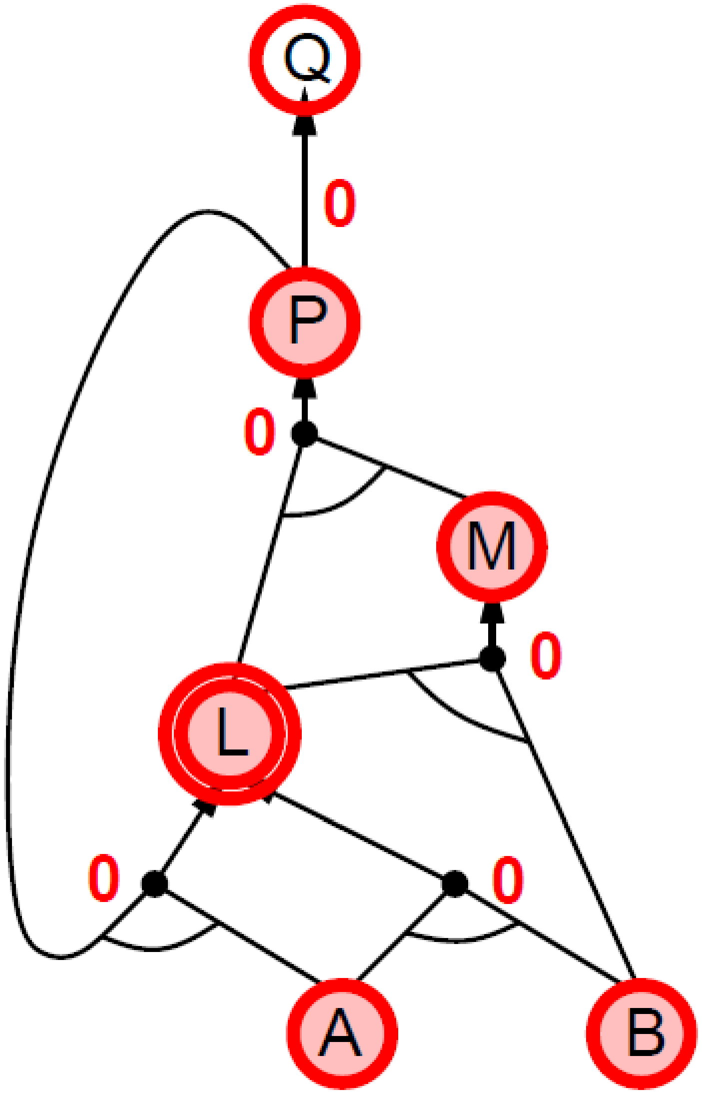

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B ∧ L ⇒ M
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A ∧ B ⇒ L
A
B

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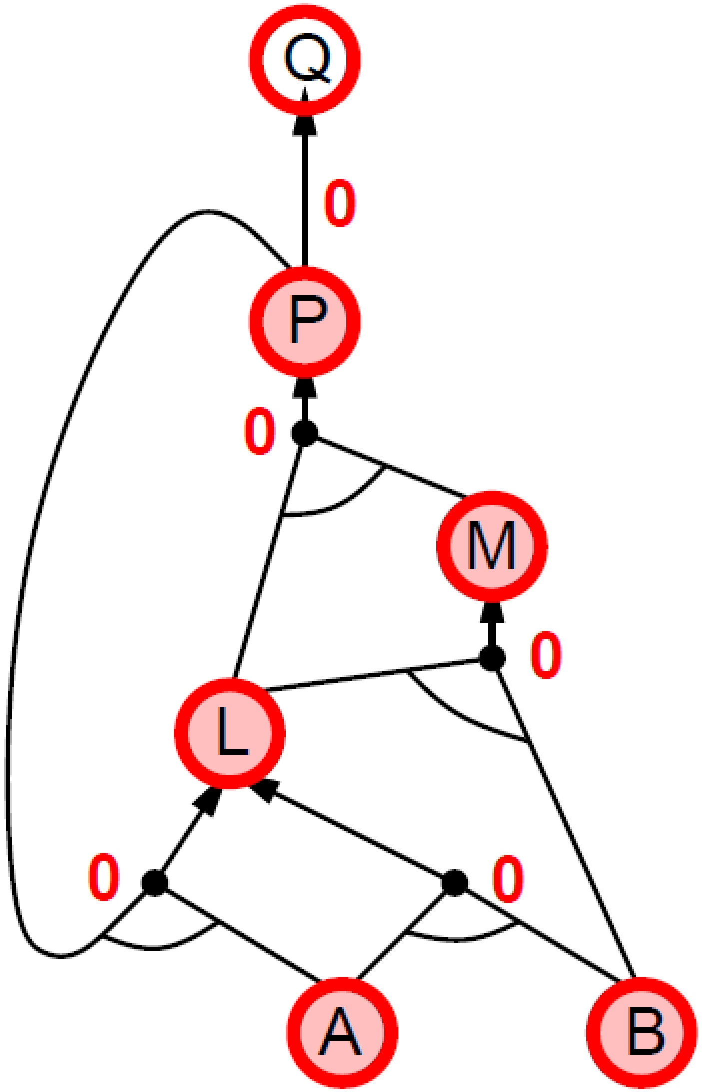

P ⇒ Q
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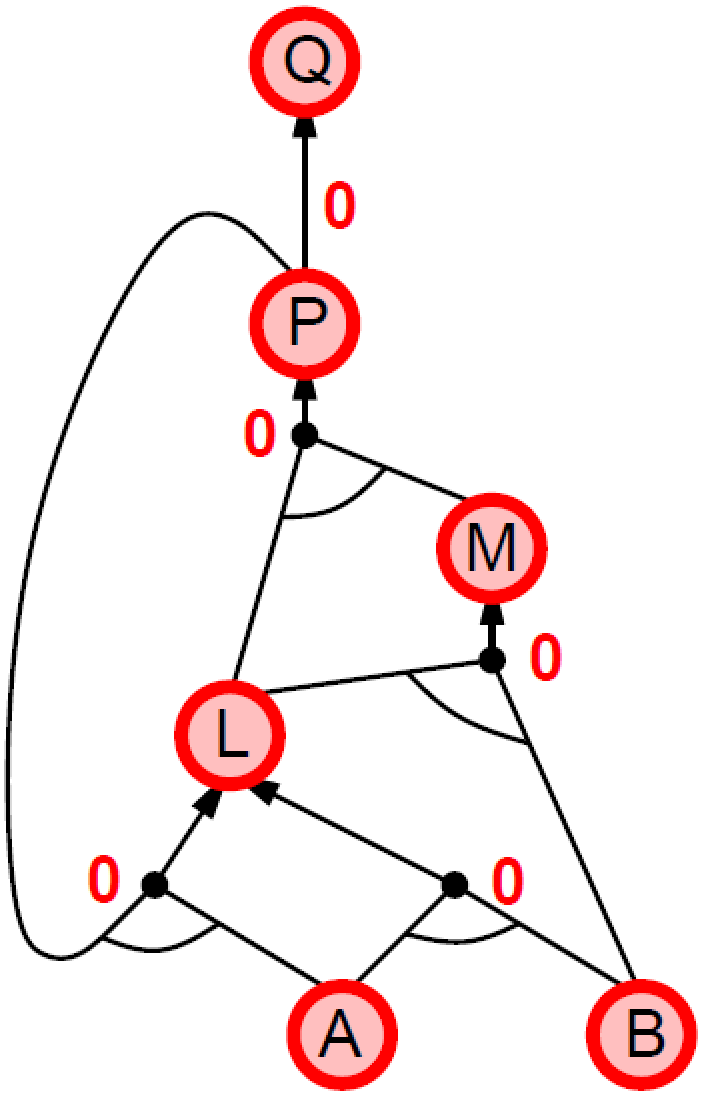
B ∧ L ⇒ M
A ∧ P ⇒ L
A ∧ B ⇒ L
A
B

Forward Chaining Example



$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

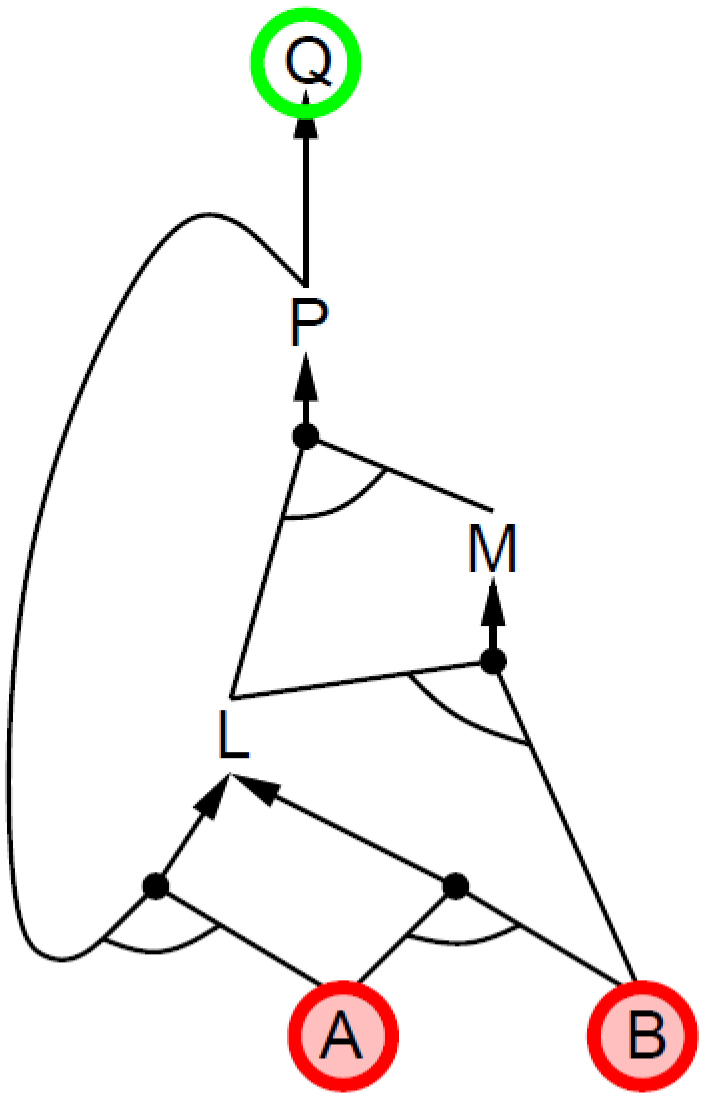
Proof of Completeness

- Forward chaining (FC) derives every atomic sentence that is entailed by KB
- 1. FC reaches a fixed point where no new atomic sentences are derived
- 2. Consider the final state as a model m, assigning true/false to symbols
- 3. Every clause in the original KB is true in m
  - Proof: Suppose a clause $a_1 \land \ldots \land a_k \Rightarrow b$ is false in m
  - Then $a_1 \land \ldots \land a_k$ is true in m and b is false in m
  - Therefore the algorithm has not reached a fixed point!
- 4. Hence m is a model of KB
- 5. If KB $\models$ q, q is true in every model of KB, including m

Backward Chaining

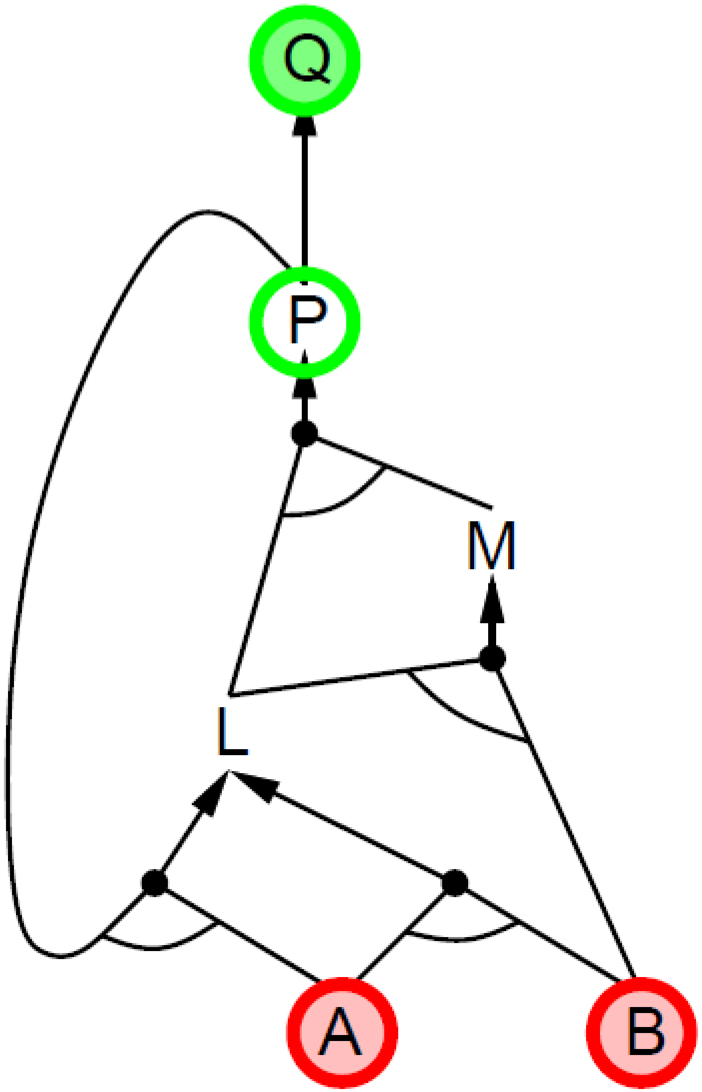
- Idea: work backwards from the query q:
  - To prove q by backward chaining,
    - Check if q is known already, or
    - Prove by backward chaining (BC) all premises of some rule concluding q
- Avoid loops: check if new subgoal is already on the goal stack
- Avoid repeated work: check if new subgoal
  - 1) has already been proved true, or
  - 2) has already failed

Backward Chaining Example



P ⇒ Q
L ∧ M ⇒ P
B ∧ L ⇒ M
A ∧ P ⇒ L
A ∧ B ⇒ L
A
B

Backward Chaining Example



P ⇒ Q
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B ∧ L ⇒ M
A ∧ P ⇒ L
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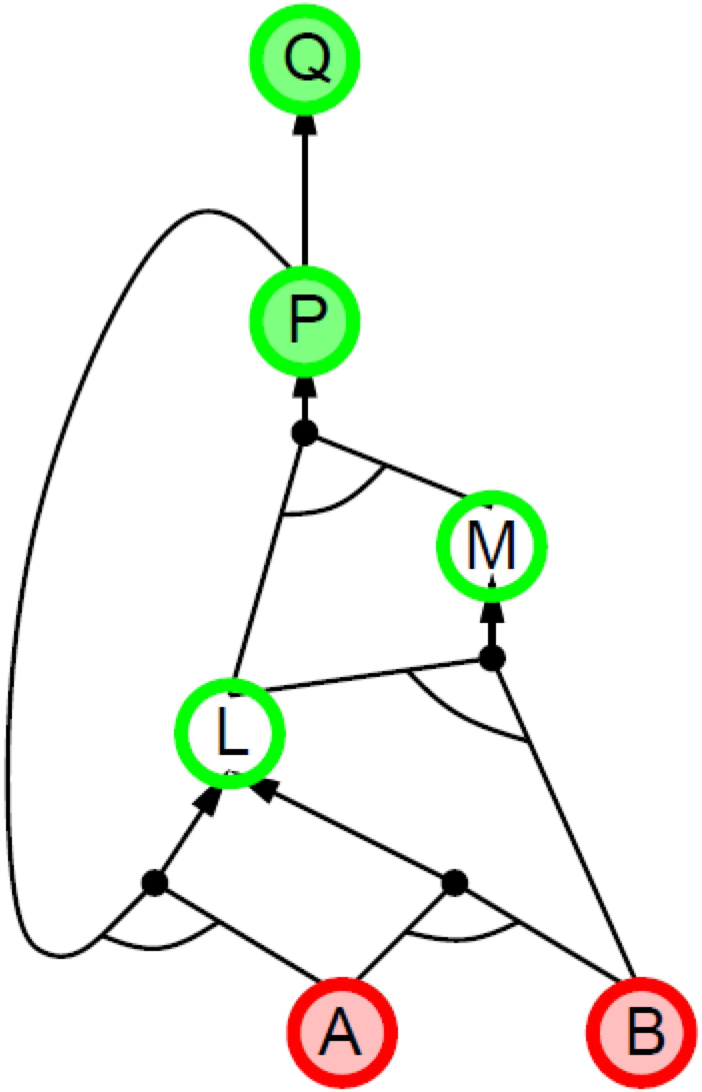
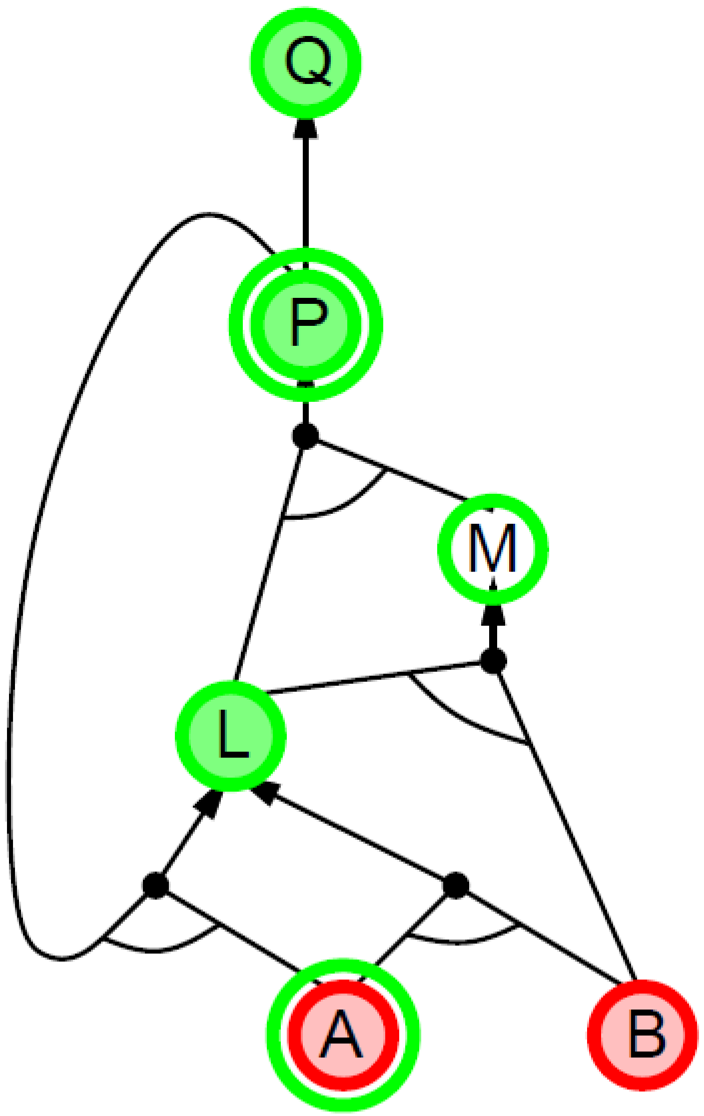
B

Backward Chaining Example



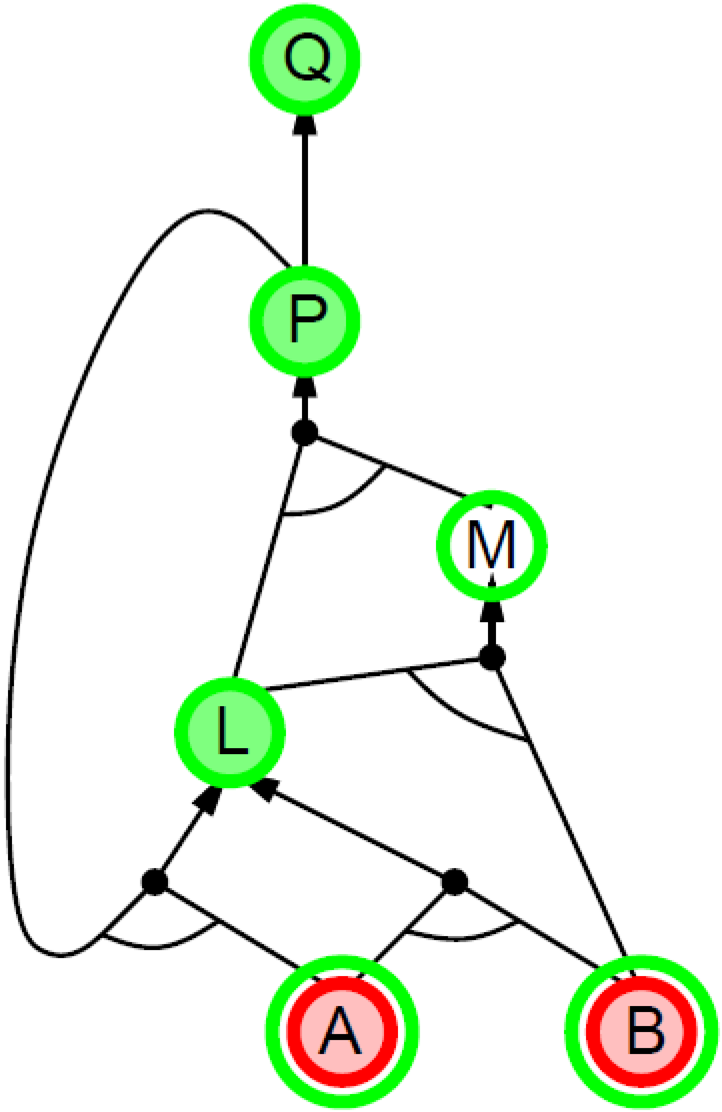
$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Backward Chaining Example



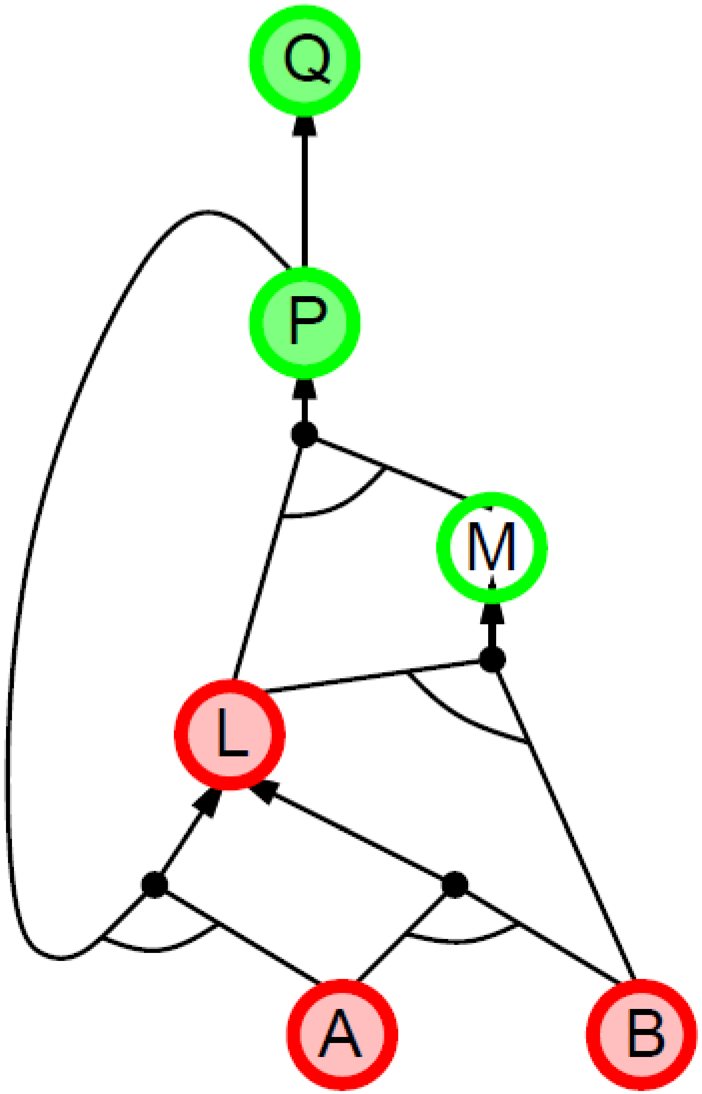
$P \Rightarrow Q$
$L \land M \Rightarrow P$
$B \land L \Rightarrow M$
$A \land P \Rightarrow L$
$A \land B \Rightarrow L$
A
B

Backward Chaining Example



P ⇒ Q
L ∧ M ⇒ P
B ∧ L ⇒ M
A ∧ P ⇒ L
A ∧ B ⇒ L
A
B

Backward Chaining Example



P ⇒ Q
L ∧ M ⇒ P
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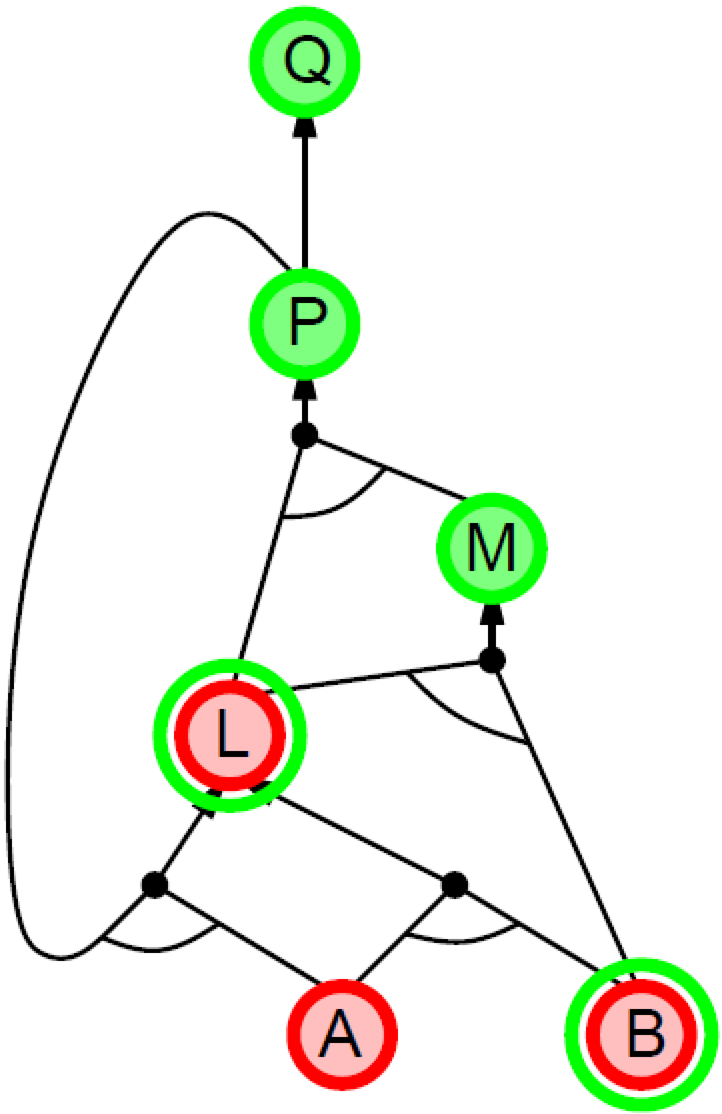
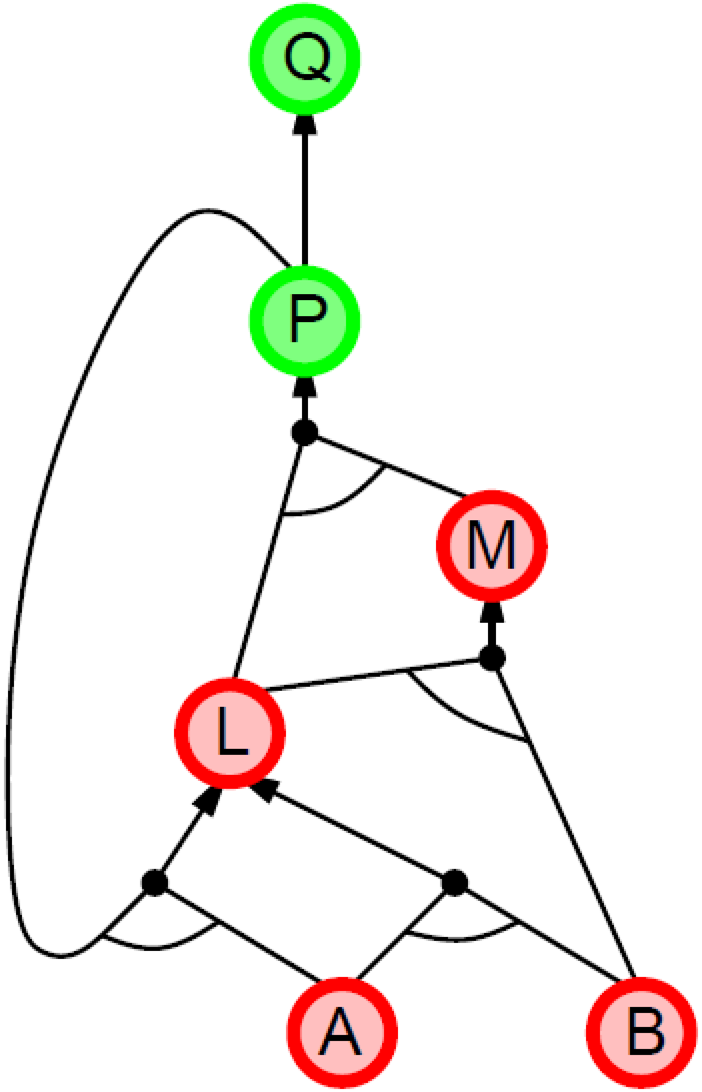
B

Backward Chaining Example



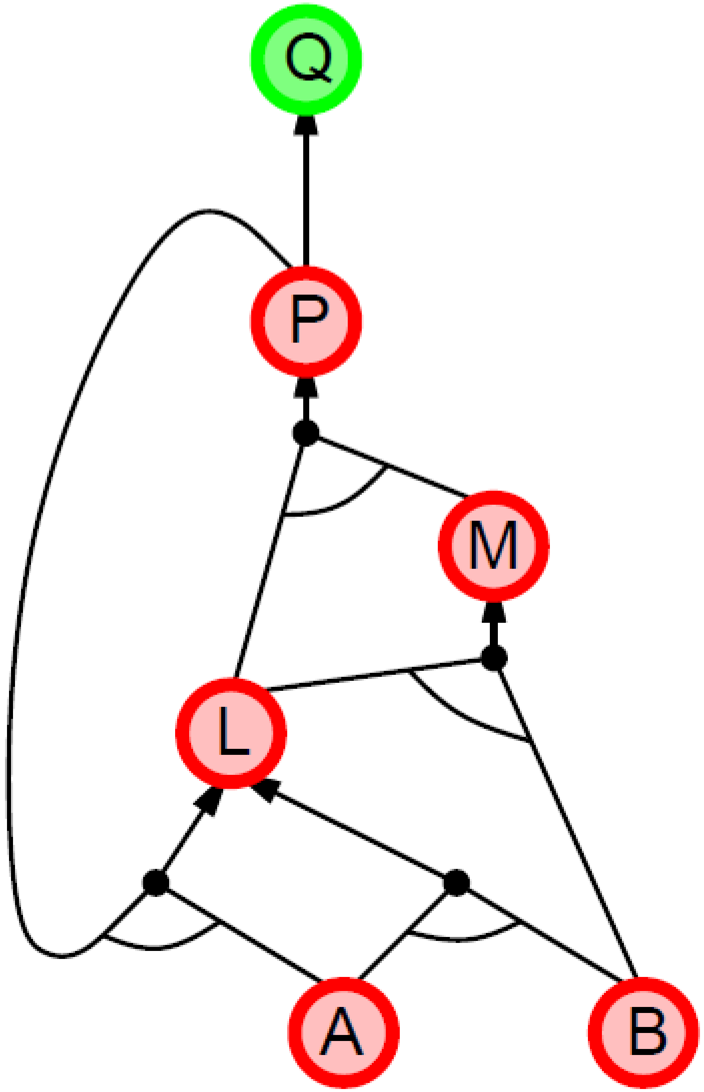
$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Backward Chaining Example



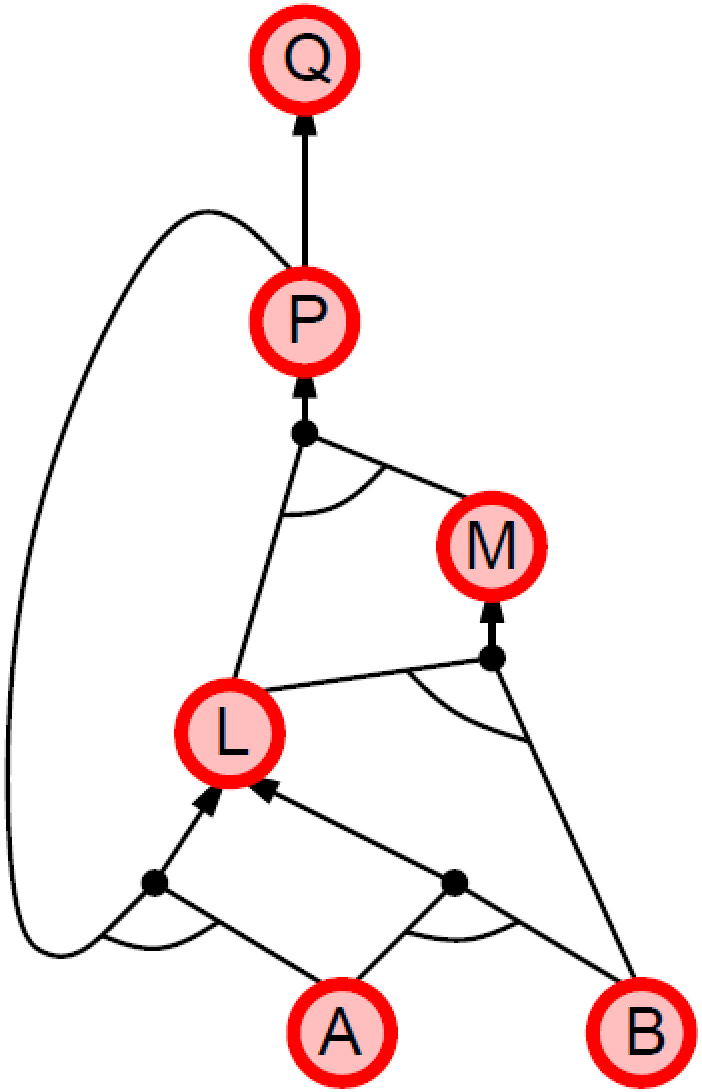
$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Backward Chaining Example



P ⇒ Q
L ∧ M ⇒ P
B ∧ L ⇒ M
A ∧ P ⇒ L
A ∧ B ⇒ L
A
B

Backward Chaining Example



$P \Rightarrow Q$
$L \land M \Rightarrow P$
$B \land L \Rightarrow M$
$A \land P \Rightarrow L$
$A \land B \Rightarrow L$
A
B

Forward vs. Backward Chaining

- FC is data-driven, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be much less than linear in size of KB

Resolution

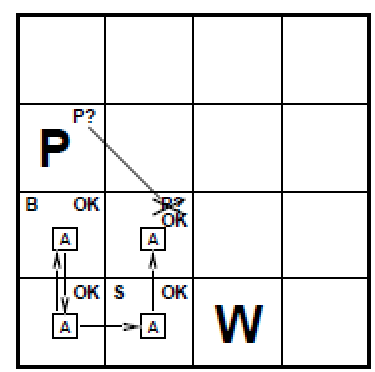
- Conjunctive Normal Form (CNF - universal)
  - Conjunction of disjunctions of literals
  - Disjunctions of literals means clauses
  - E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$

    Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \lor \cdots \lor \ell_k, \qquad m_1 \lor \cdots \lor m_n}{\ell_1 \lor \cdots \lor \ell_{i-1} \lor \ell_{i+1} \lor \cdots \lor \ell_k \lor m_1 \lor \cdots \lor m_{j-1} \lor m_{j+1} \lor \cdots \lor m_n}$$

- where l_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \lor P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



- Resolution is sound and complete for propositional logic

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

- Proof by contradiction, i.e., show KB ∧ ¬α : unsatisfiable

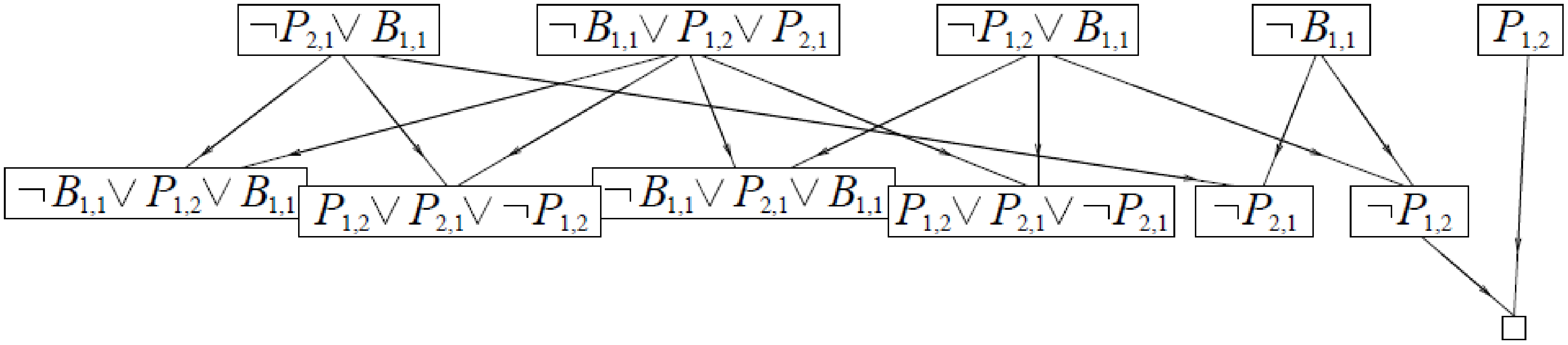
Resolution Algorithm

```
function PL-RESOLUTION(KB, α) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
           α, the query, a sentence in propositional logic

   clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
   new ← { }
   loop do
      for each C_i, C_j in clauses do
         resolvents ← PL-RESOLVE(C_i, C_j)
         if resolvents contains the empty clause then return true
         new ← new ∪ resolvents
      if new ⊆ clauses then return false
      clauses ← clauses ∪ new
```

Resolution Example

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \quad \alpha = \neg P_{1,2}$$



Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - Syntax: formal structure of sentences
  - Semantics: truth of sentences with respect to models
  - Entailment: necessary truth of one sentence given another
  - Inference: deriving sentences from other sentences
  - Soundness: derivations produce only entailed sentences
  - Completeness: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Forward, backward chaining are linear-time, complete for Horn clauses
- Resolution is complete for propositional logic
- Propositional logic lacks expressive power

Summary

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution