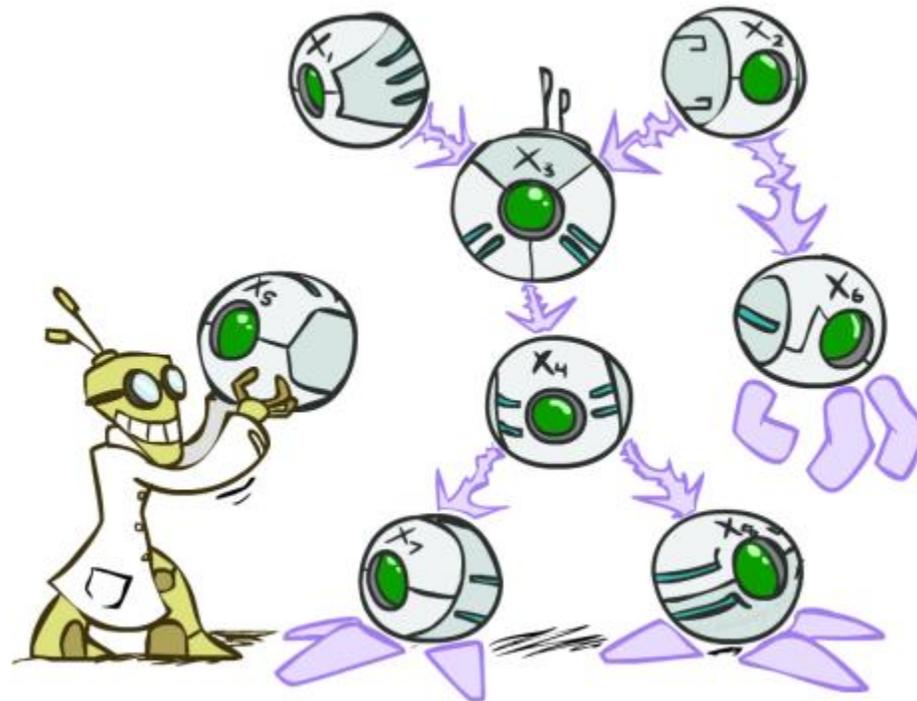


# CSCI 446: Artificial Intelligence

## Bayes' Nets



Instructors: Michele Van Dyne

# Today

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- Review
  - Independence
  - Conditional Independence
- Bayes Nets
  - Big Picture
  - Semantics

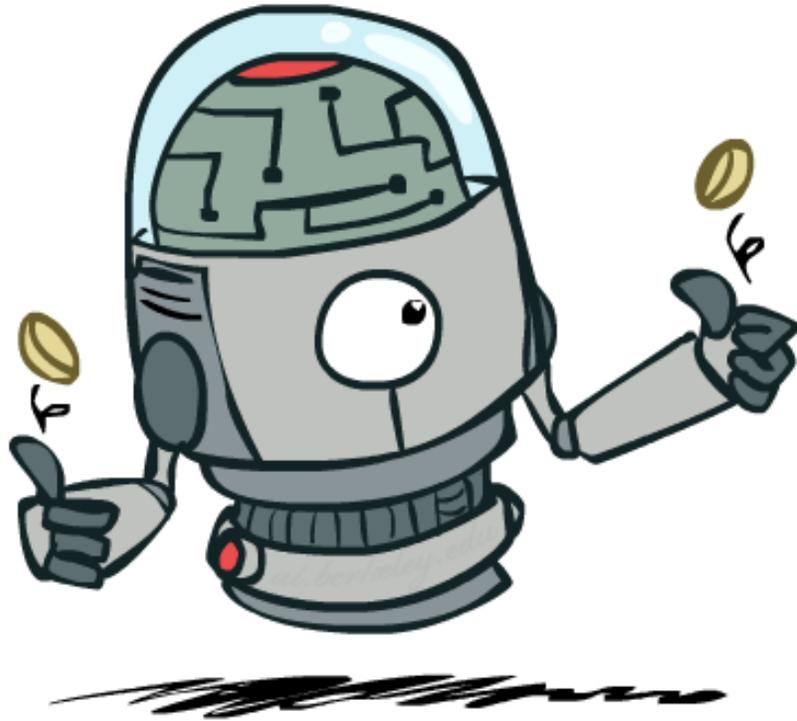
# Probabilistic Models

- Models describe how (a portion of) the world works
- **Models are always simplifications**
  - May not account for every variable
  - May not account for all interactions between variables
  - “All models are wrong; but some are useful.”  
– George E. P. Box
- What do we do with probabilistic models?
  - We (or our agents) need to reason about unknown variables, given evidence
  - Example: explanation (diagnostic reasoning)
  - Example: prediction (causal reasoning)
  - Example: value of information



# Independence

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# Independence

- Two variables are *independent* if:

$$\forall x, y : P(x, y) = P(x)P(y)$$

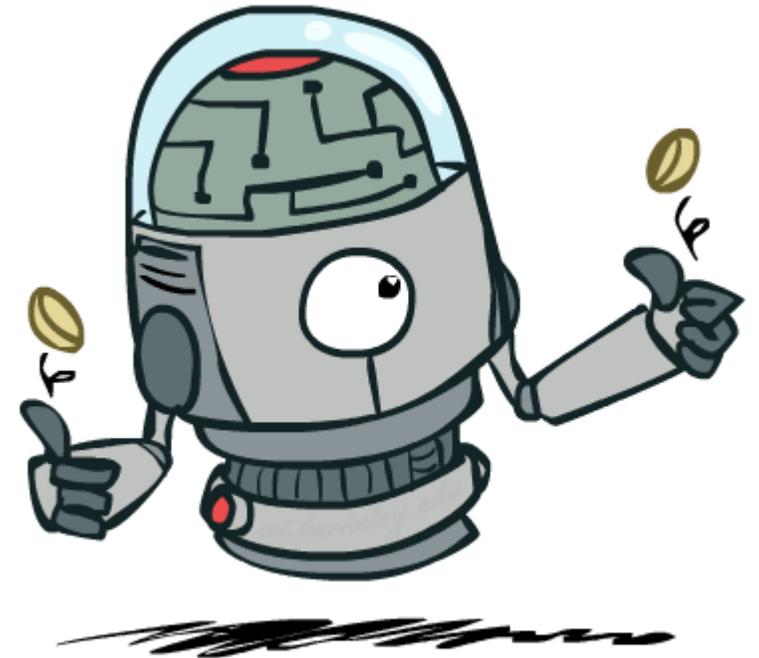
- This says that their joint distribution *factors* into a product two simpler distributions
- Another form:

$$\forall x, y : P(x|y) = P(x)$$

- We write:  $X \perp\!\!\!\perp Y$

- Independence is a simplifying *modeling assumption*

- Empirical* joint distributions: at best “close” to independent
- What could we assume for {Weather, Traffic, Cavity, Toothache}?



# Example: Independence?

$P_1(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$P(T)$

T	P
hot	0.5
cold	0.5

$P_2(T, W)$

T	W	P
hot	sun	0.3
hot	rain	0.2
cold	sun	0.3
cold	rain	0.2

$P(W)$

W	P
sun	0.6
rain	0.4

# Example: Independence

- N fair, independent coin flips:

$P(X_1)$

H	0.5
T	0.5

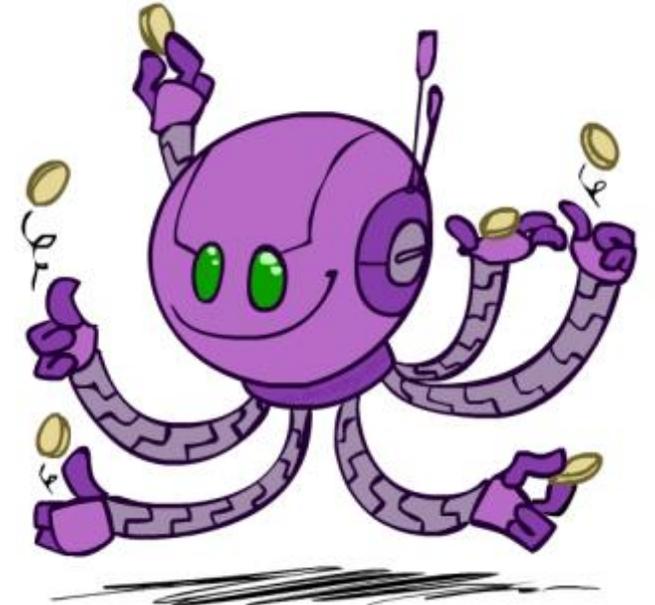
$P(X_2)$

H	0.5
T	0.5

...

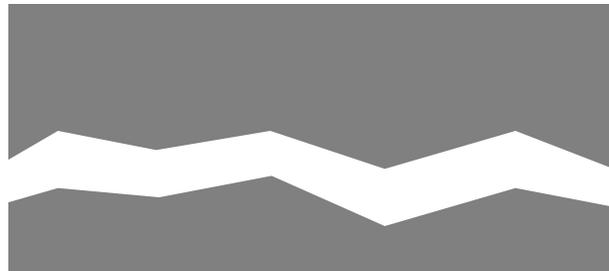
$P(X_n)$

H	0.5
T	0.5



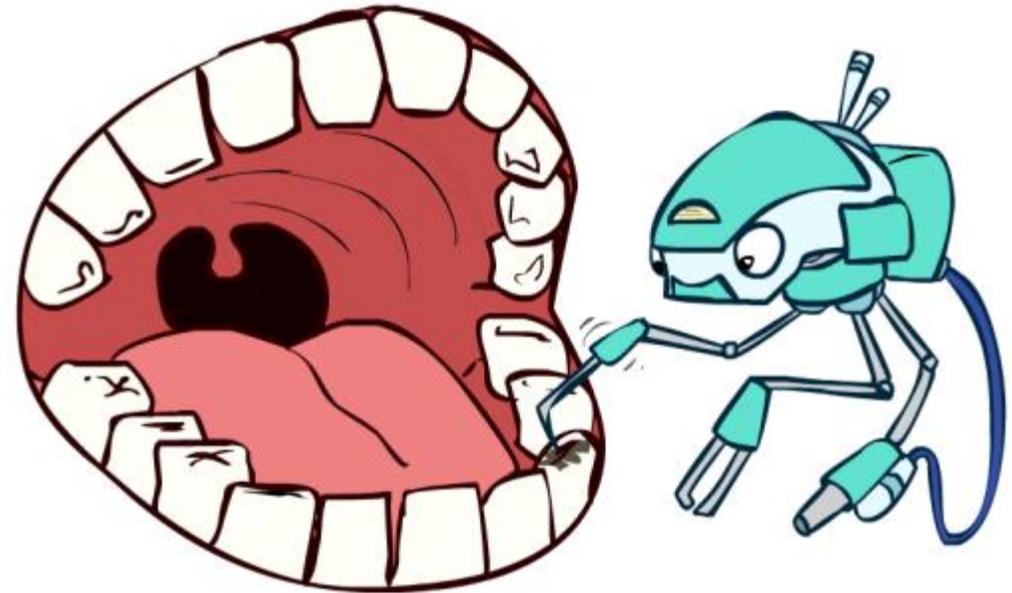
$P(X_1, X_2, \dots, X_n)$

$2^n$



# Conditional Independence

- $P(\text{Toothache}, \text{Cavity}, \text{Catch})$
- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:
  - $P(+\text{catch} \mid +\text{toothache}, +\text{cavity}) = P(+\text{catch} \mid +\text{cavity})$
- The same independence holds if I don't have a cavity:
  - $P(+\text{catch} \mid +\text{toothache}, -\text{cavity}) = P(+\text{catch} \mid -\text{cavity})$
- Catch is *conditionally independent* of Toothache given Cavity:
  - $P(\text{Catch} \mid \text{Toothache}, \text{Cavity}) = P(\text{Catch} \mid \text{Cavity})$
- Equivalent statements:
  - $P(\text{Toothache} \mid \text{Catch}, \text{Cavity}) = P(\text{Toothache} \mid \text{Cavity})$
  - $P(\text{Toothache}, \text{Catch} \mid \text{Cavity}) = P(\text{Toothache} \mid \text{Cavity}) P(\text{Catch} \mid \text{Cavity})$
  - One can be derived from the other easily



# Conditional Independence

- Unconditional (absolute) independence very rare (why?)
- *Conditional independence* is our most basic and robust form of knowledge about uncertain environments.
- X is conditionally independent of Y given Z

$$X \perp\!\!\!\perp Y | Z$$

if and only if:

$$\forall x, y, z : P(x, y | z) = P(x | z)P(y | z)$$

or, equivalently, if and only if

$$\forall x, y, z : P(x | z, y) = P(x | z)$$

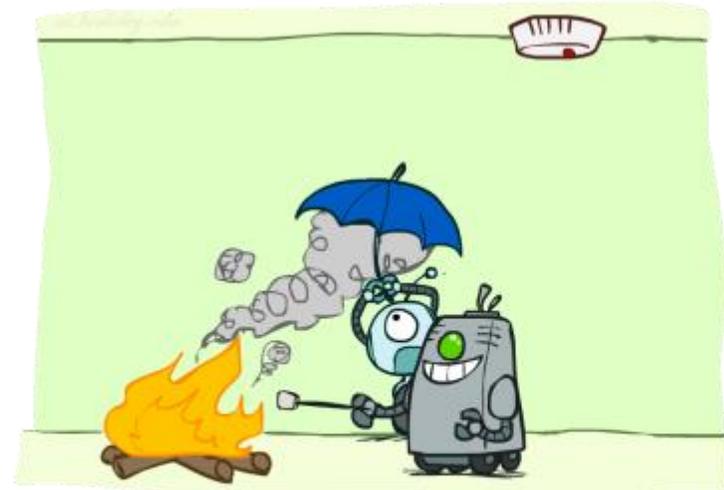
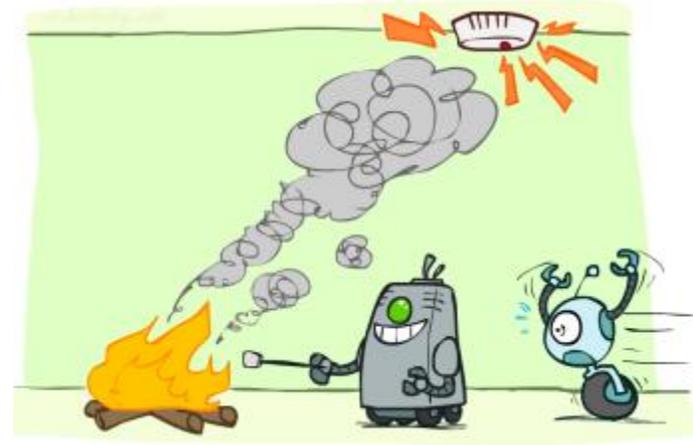
# Conditional Independence

- What about this domain:
  - Traffic
  - Umbrella
  - Raining



# Conditional Independence

- What about this domain:
  - Fire
  - Smoke
  - Alarm



# Conditional Independence and the Chain Rule

- Chain rule:  $P(X_1, X_2, \dots, X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2) \dots$

- Trivial decomposition:

$$P(\text{Traffic, Rain, Umbrella}) = P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain, Traffic})$$

- With assumption of conditional independence:

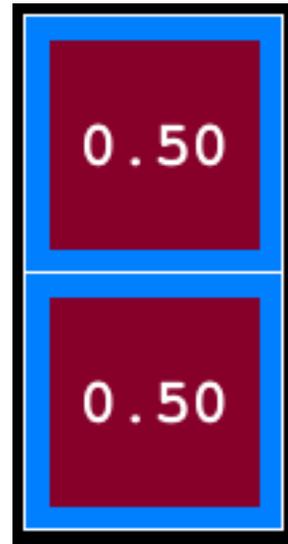
$$P(\text{Traffic, Rain, Umbrella}) = P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain})$$

- Bayes' nets / graphical models help us express conditional independence assumptions



# Ghostbusters Chain Rule

- Each sensor depends only on where the ghost is
- That means, the two sensors are conditionally independent, given the ghost position
- T: Top square is red  
B: Bottom square is red  
G: Ghost is in the top



- Givens:  
 $P(+g) = 0.5$   
 $P(-g) = 0.5$   
 $P(+t \mid +g) = 0.8$   
 $P(+t \mid -g) = 0.4$   
 $P(+b \mid +g) = 0.4$   
 $P(+b \mid -g) = 0.8$

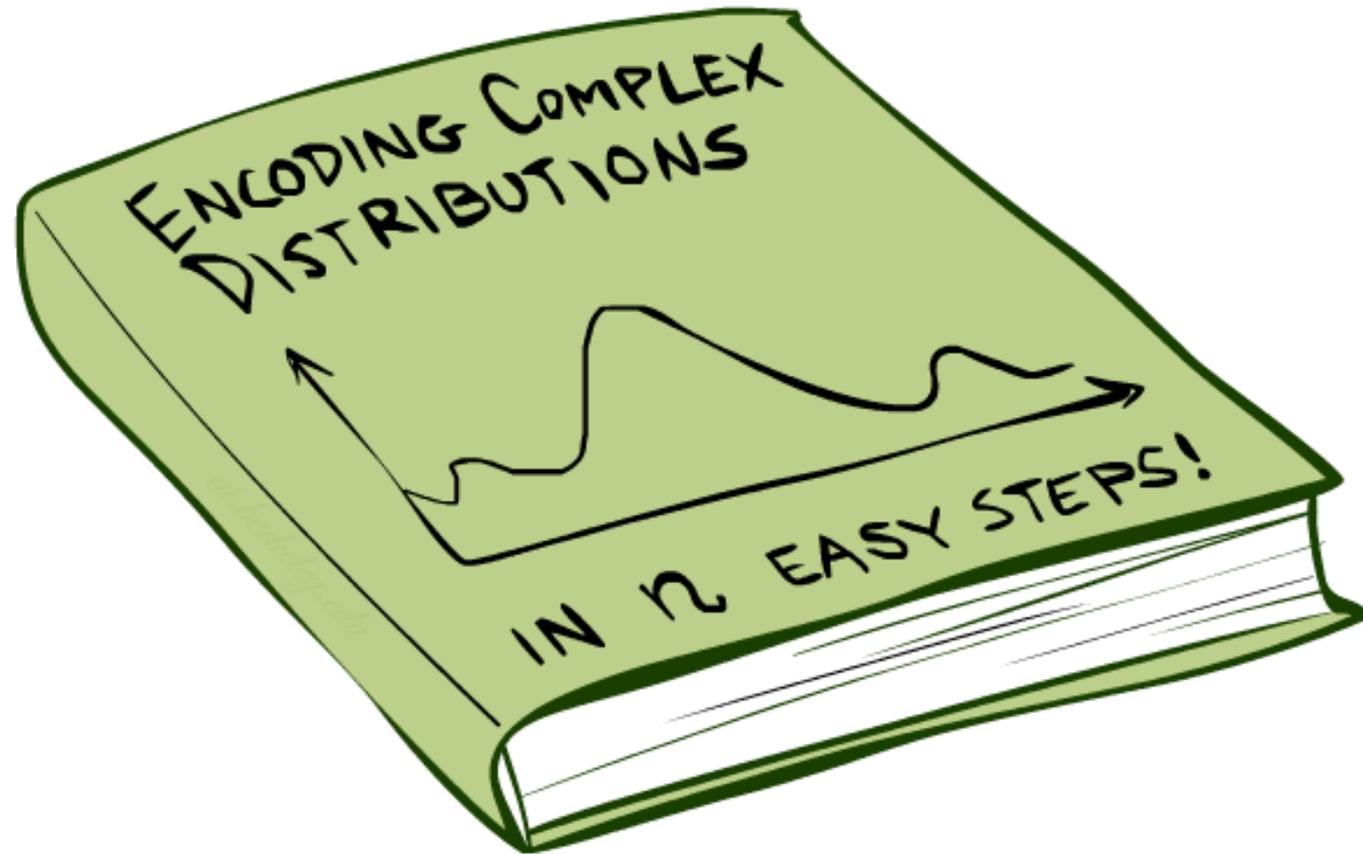
$$P(T,B,G) = P(G) P(T \mid G) P(B \mid G)$$

T	B	G	P(T,B,G)
+t	+b	+g	0.16
+t	+b	-g	0.16
+t	-b	+g	0.24
+t	-b	-g	0.04
-t	+b	+g	0.04
-t	+b	-g	0.24
-t	-b	+g	0.06
-t	-b	-g	0.06



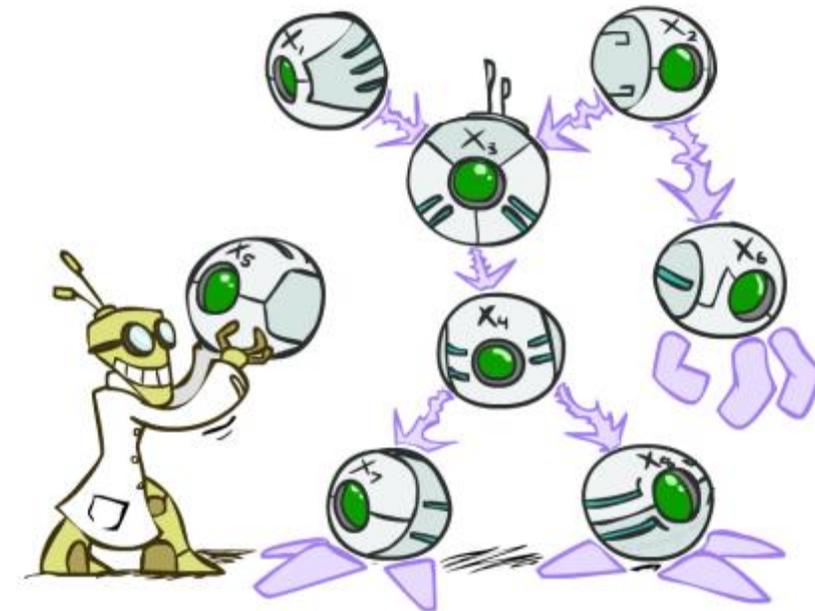
# Bayes' Nets: Big Picture

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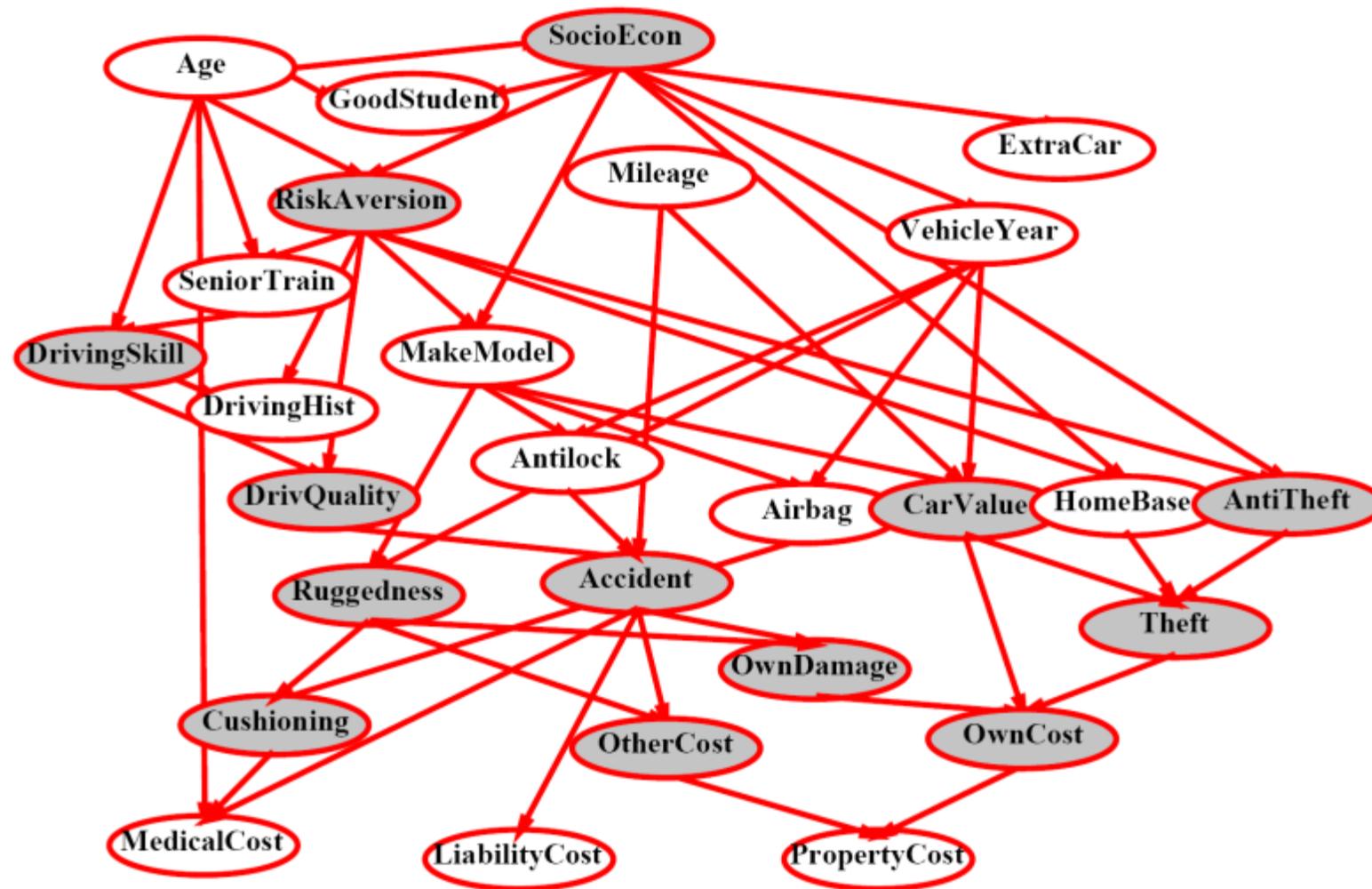


# Bayes' Nets: Big Picture

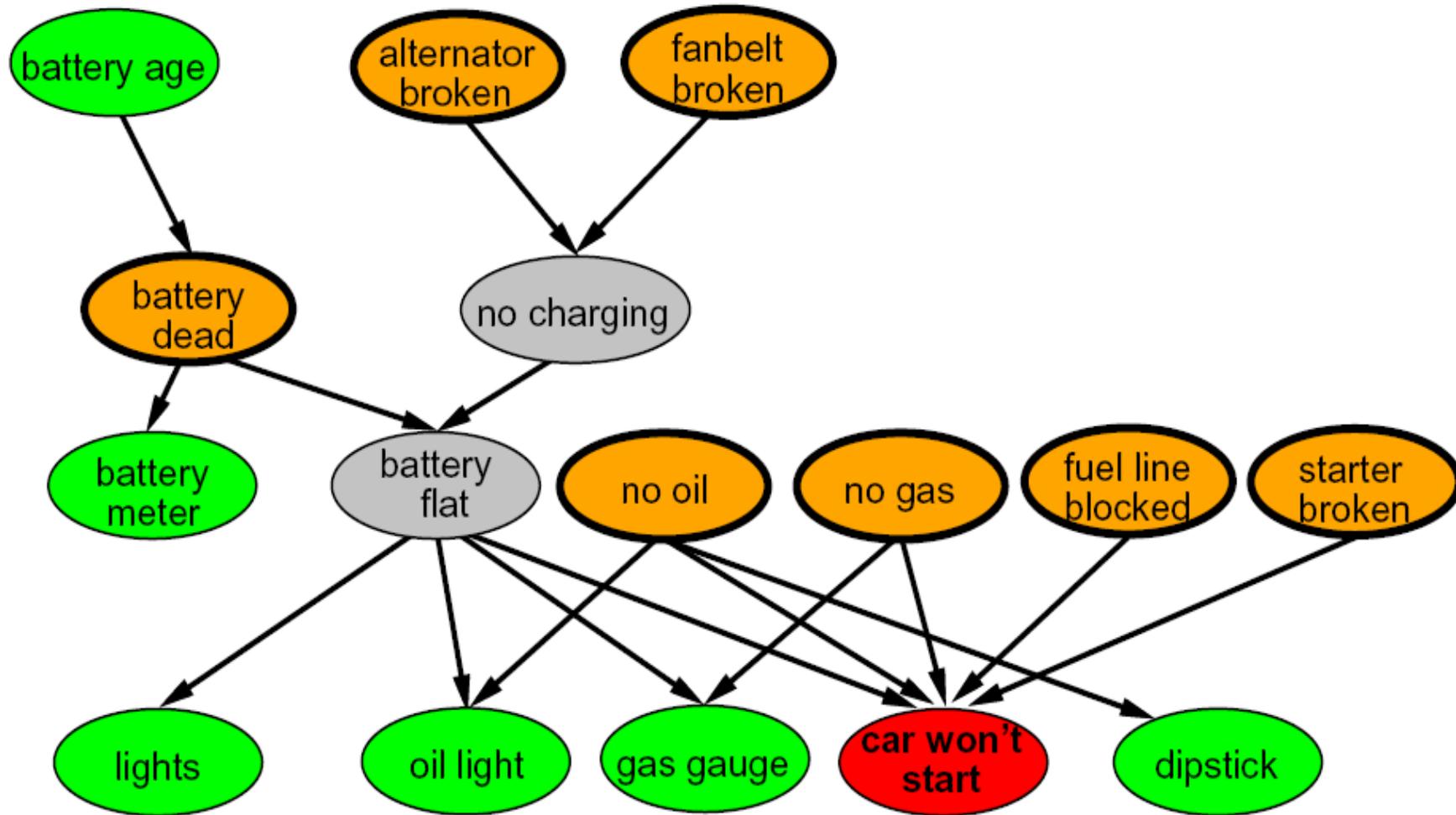
- Two problems with using full joint distribution tables as our probabilistic models:
  - Unless there are only a few variables, the joint is WAY too big to represent explicitly
  - Hard to learn (estimate) anything empirically about more than a few variables at a time
- **Bayes' nets:** a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
  - More properly called **graphical models**
  - We describe how variables locally interact
  - Local interactions chain together to give global, indirect interactions
  - For about 10 min, we'll be vague about how these interactions are specified



# Example Bayes' Net: Insurance

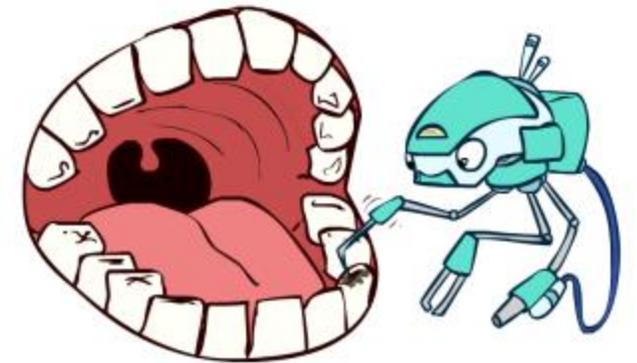
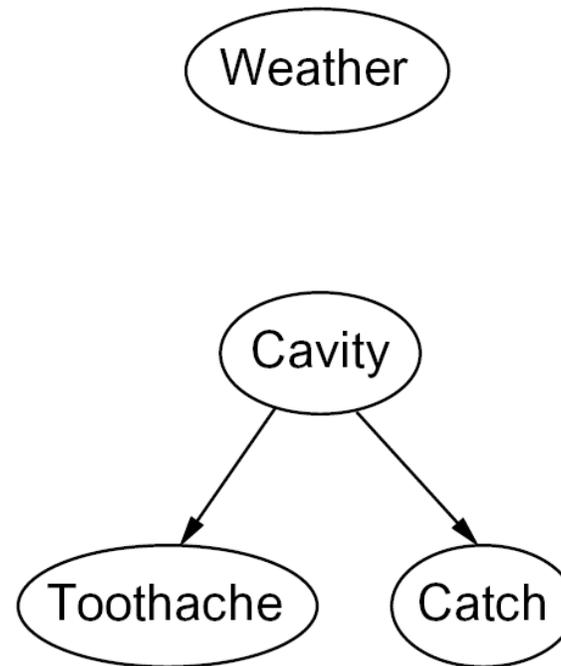


# Example Bayes' Net: Car



# Graphical Model Notation

- **Nodes: variables (with domains)**
  - Can be assigned (observed) or unassigned (unobserved)
- **Arcs: interactions**
  - Similar to CSP constraints
  - Indicate “direct influence” between variables
  - Formally: encode conditional independence (more later)
- For now: imagine that arrows mean direct causation (in general, they don't!)



# Example: Coin Flips

- N independent coin flips



- No interactions between variables: **absolute independence**

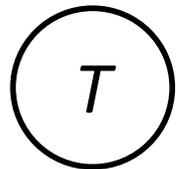
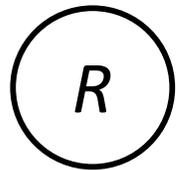
# Example: Traffic

- Variables:

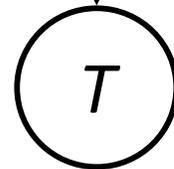
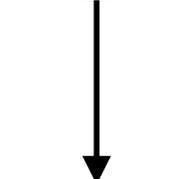
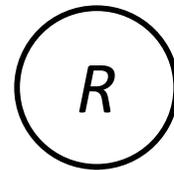
- R: It rains
- T: There is traffic



- Model 1: independence



- Model 2: rain causes traffic



- Why is an agent using model 2 better?

# Example: Traffic II

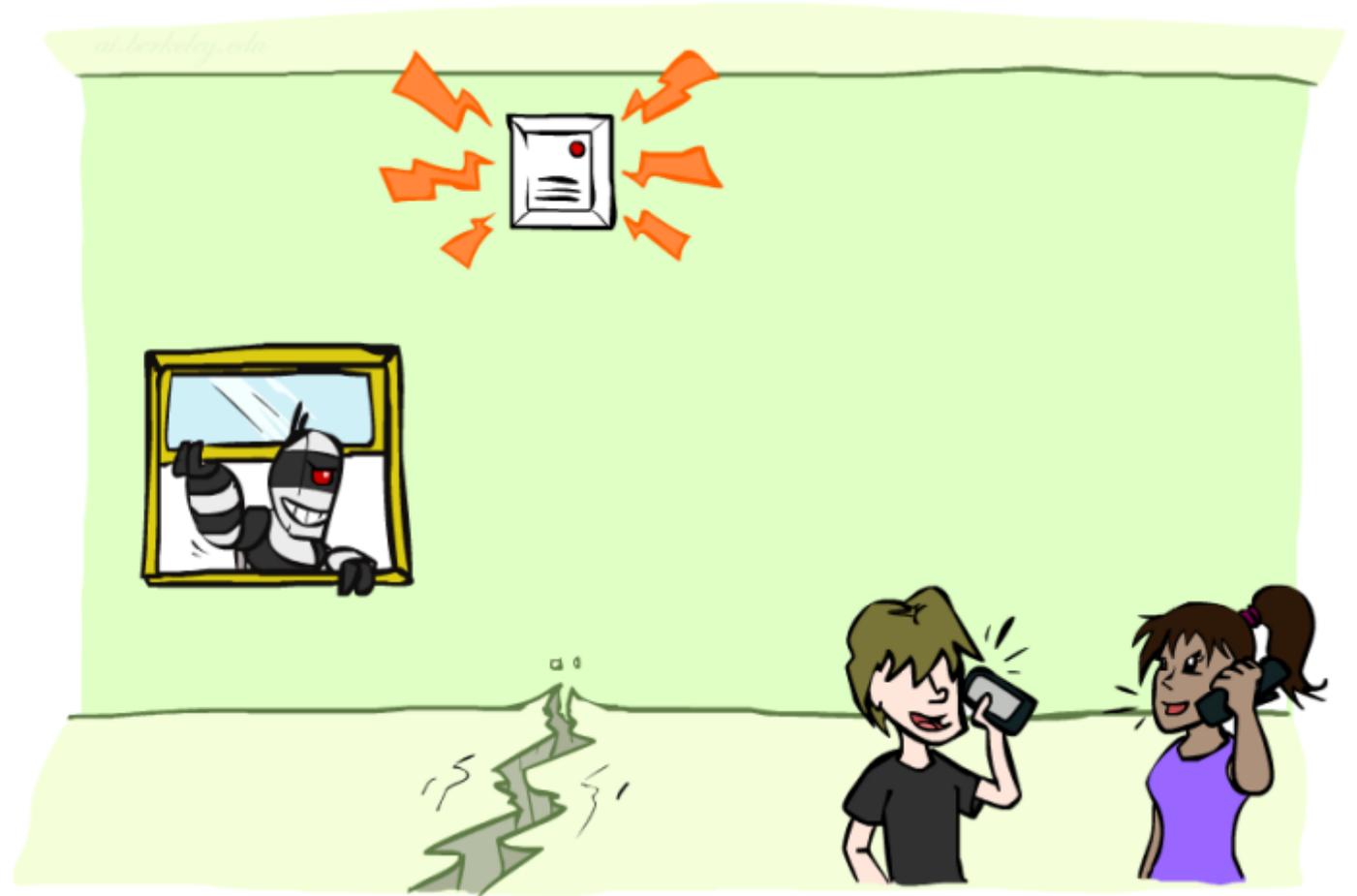
- Let's build a causal graphical model!
- Variables
  - T: Traffic
  - R: It rains
  - L: Low pressure
  - D: Roof drips
  - B: Ballgame
  - C: Cavity



# Example: Alarm Network

- Variables

- B: Burglary
- A: Alarm goes off
- M: Mary calls
- J: John calls
- E: Earthquake!



# Bayes' Net Semantics

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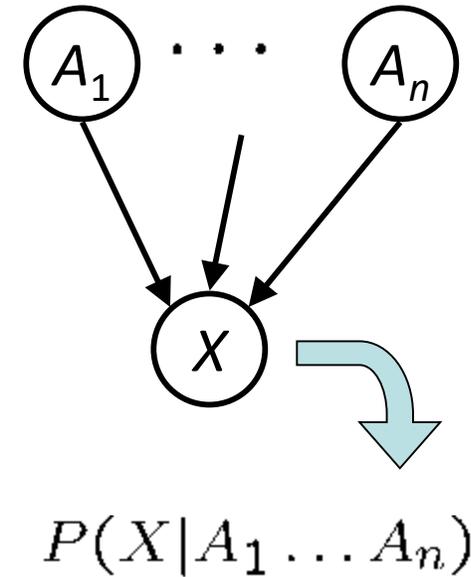
# Bayes' Net Semantics



- A set of nodes, one per variable  $X$
- A directed, acyclic graph
- A conditional distribution for each node
  - A collection of distributions over  $X$ , one for each combination of parents' values

$$P(X|a_1 \dots a_n)$$

- CPT: conditional probability table
- Description of a noisy “causal” process



*A Bayes net = Topology (graph) + Local Conditional Probabilities*

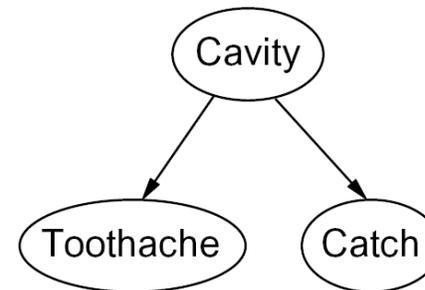
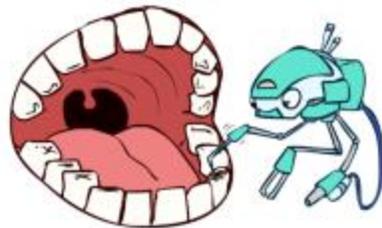
# Probabilities in BNs



- Bayes' nets **implicitly** encode joint distributions
  - As a product of local conditional distributions
  - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

- Example:



$$P(+cavity, +catch, -toothache)$$

# Probabilities in BNs



- Why are we guaranteed that setting

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

results in a proper joint distribution?

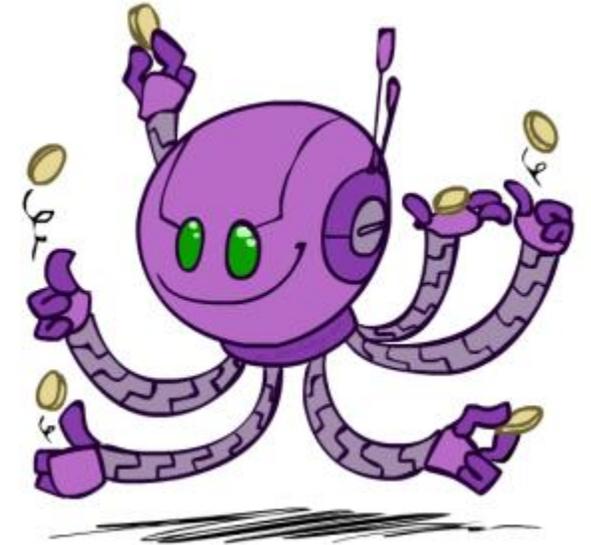
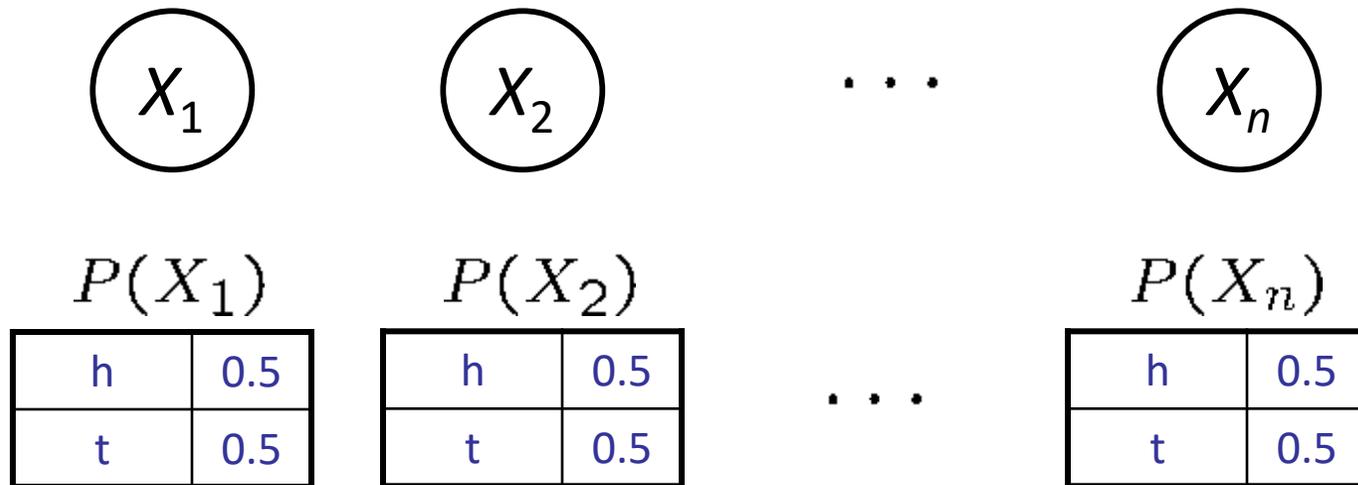
- Chain rule (valid for all distributions):  $P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | x_1 \dots x_{i-1})$

- Assume conditional independences:  $P(x_i | x_1, \dots, x_{i-1}) = P(x_i | \text{parents}(X_i))$

→ Consequence:  $P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$

- Not every BN can represent every joint distribution
  - The topology enforces certain conditional independencies

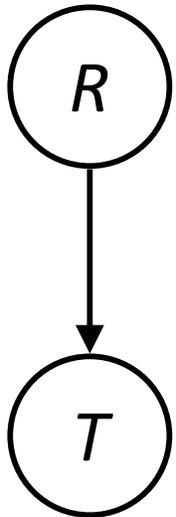
# Example: Coin Flips



$$P(h, h, t, h) =$$

*Only distributions whose variables are absolutely independent can be represented by a Bayes' net with no arcs.*

# Example: Traffic

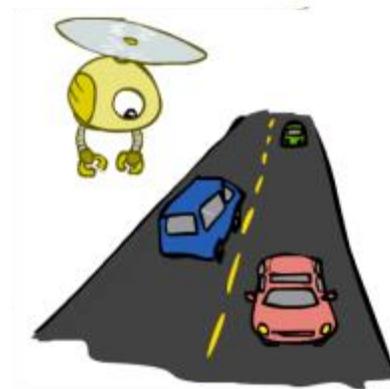

$$P(R)$$

+r	1/4
-r	3/4

$$P(+r, -t) =$$

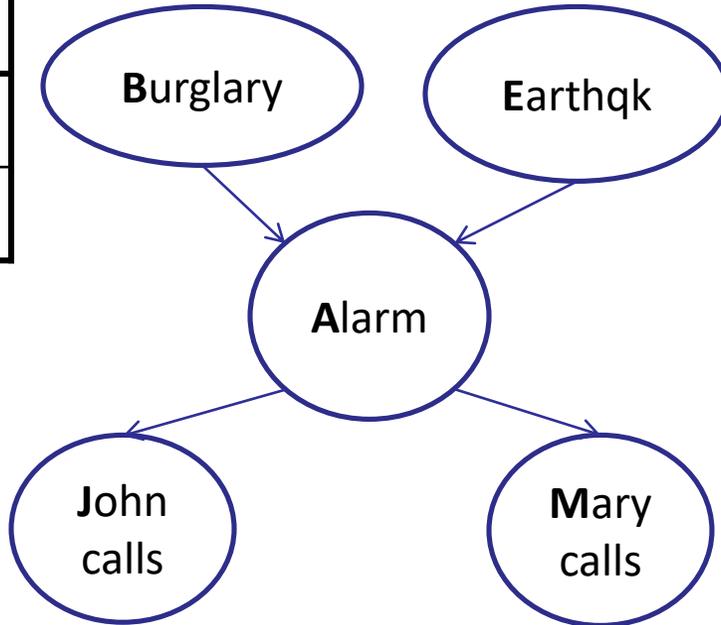
$$P(T|R)$$

+r	+t	3/4
+r	-t	1/4
-r	+t	1/2
-r	-t	1/2

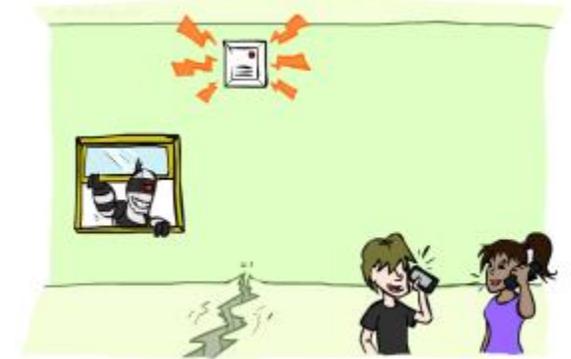


# Example: Alarm Network

B	P(B)
+b	0.001
-b	0.999



E	P(E)
+e	0.002
-e	0.998



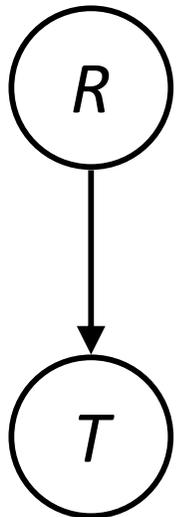
A	J	P(J A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

A	M	P(M A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

B	E	A	P(A B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999

# Example: Traffic

- Causal direction



$P(R)$

+r	1/4
-r	3/4

$P(T|R)$

+r	+t	3/4
	-t	1/4

-r	+t	1/2
	-t	1/2

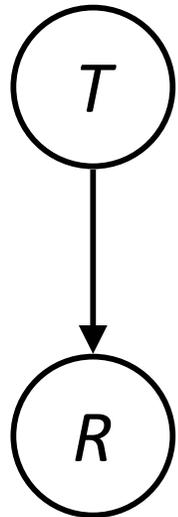
$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16



# Example: Reverse Traffic

- Reverse causality?



$P(T)$

+t	9/16
-t	7/16

$P(R|T)$

+t	+r	1/3
	-r	2/3

-t	+r	1/7
	-r	6/7

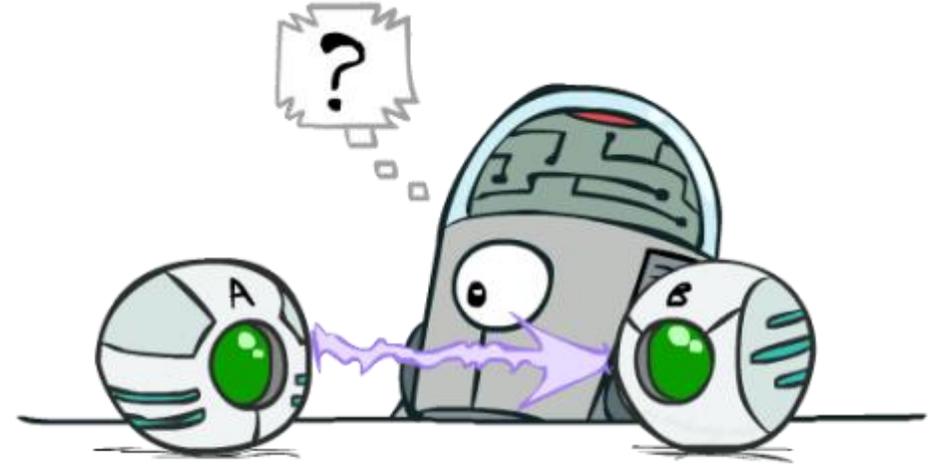


$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16

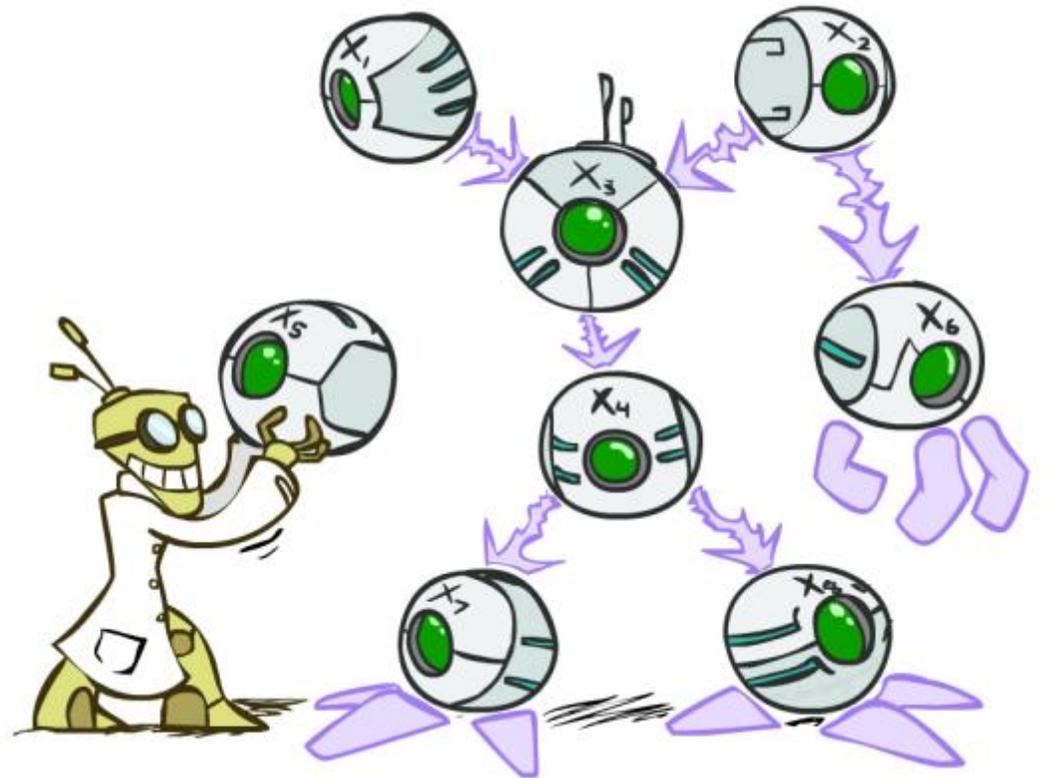
# Causality?

- When Bayes' nets reflect the true causal patterns:
  - Often simpler (nodes have fewer parents)
  - Often easier to think about
  - Often easier to elicit from experts
- BNs need not actually be causal
  - Sometimes no causal net exists over the domain (especially if variables are missing)
  - E.g. consider the variables *Traffic* and *Drips*
  - End up with arrows that reflect correlation, not causation
- What do the arrows really mean?
  - Topology may happen to encode causal structure
  - **Topology really encodes conditional independence**
$$P(x_i | x_1, \dots, x_{i-1}) = P(x_i | \text{parents}(X_i))$$



# Bayes' Nets

- So far: how a Bayes' net encodes a joint distribution
- Next: how to answer queries about that distribution
  - Today:
    - First assembled BNs using an intuitive notion of conditional independence as causality
    - Then saw that key property is conditional independence
  - Main goal: answer queries about conditional independence and influence
- After that: how to answer numerical queries (inference)



# Today

- Review
  - Independence
  - Conditional Independence
- Bayes Nets
  - Big Picture
  - Semantics

