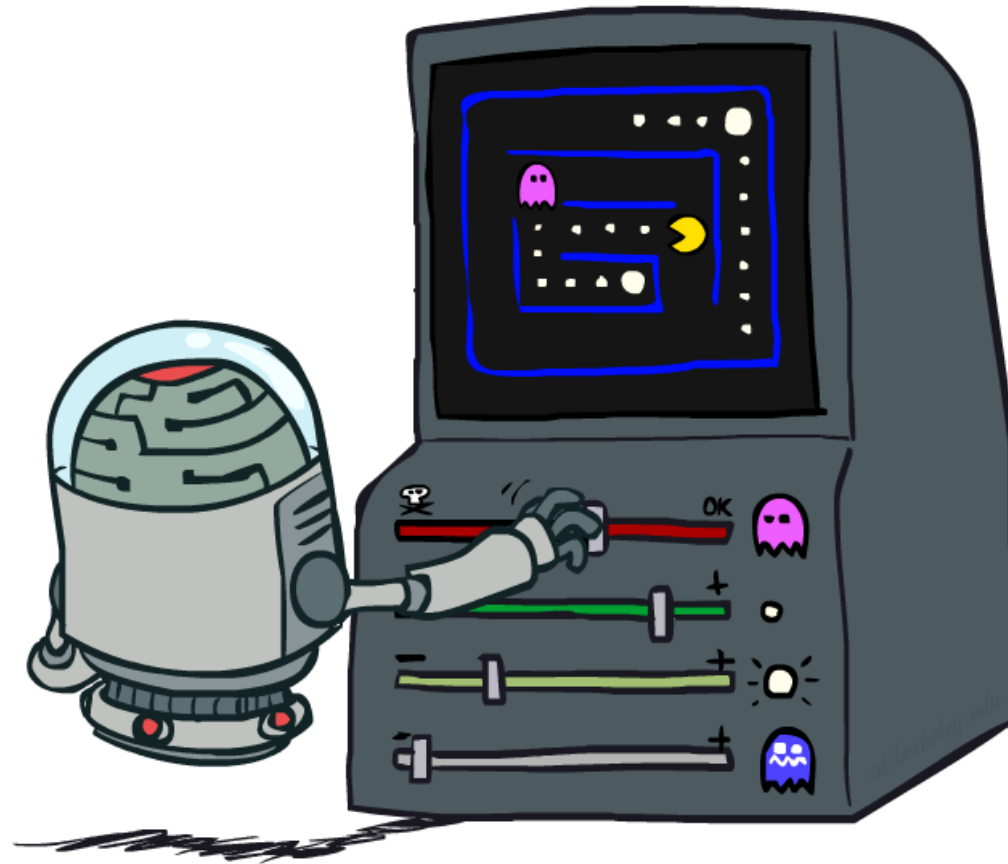# CSCI 446: Artificial Intelligence

Reinforcement Learning II

Instructor: Michele Van Dyne

Today

- Exploration vs Exploitation

- Approximate Q-Learning

- Q-Learning and Least Squares

  - Overfitting: Why Limiting Capacity Can Help

- Policy Search

Reinforcement Learning

- We still assume an MDP:
    - A set of states s ∈ S
    - A set of actions (per state) A
    - A model T(s,a,s')
    - A reward function R(s,a,s')
- Still looking for a policy π(s)

- New twist: don't know T or R, so must try out actions

- Big idea: Compute all averages over T using sample outcomes

The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V*, Q*, π*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

Unknown MDP: Model-Based

Goal	Technique
Compute V*, Q*, π*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

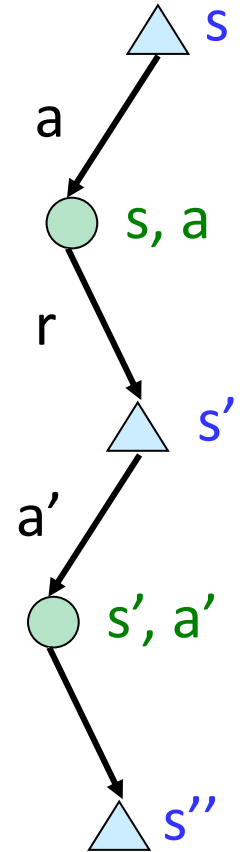| Goal	Technique
Compute V*, Q*, π*	Q-learning
Evaluate a fixed policy π	Value Learning

Model-Free Learning

- ## Model-free (temporal difference) learning
  - Experience world through episodes

$$(s, a, r, s', a', r', s'', a'', r'', s'''' \ldots)$$

  - Update estimates each transition (s, a, r, s')

  - Over time, updates will mimic Bellman updates

Q-Learning

- We'd like to do Q-value updates to each Q-state:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

  - But can't compute this update without knowing T, R

- Instead, compute average as we go
  - Receive a sample transition (s,a,r,s')
 - This sample suggests

    $$Q(s,a) \approx r + \gamma \max_{a'} Q(s',a')$$

 - But we want to average over results from (s,a) (Why?)
  - So keep a running average

    $$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[r + \gamma \max_{a'} Q(s',a') \right]$$

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!

- This is called off-policy learning

- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - ... but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)

[Demo: Q-learning – auto – cliff grid (L11D1)]
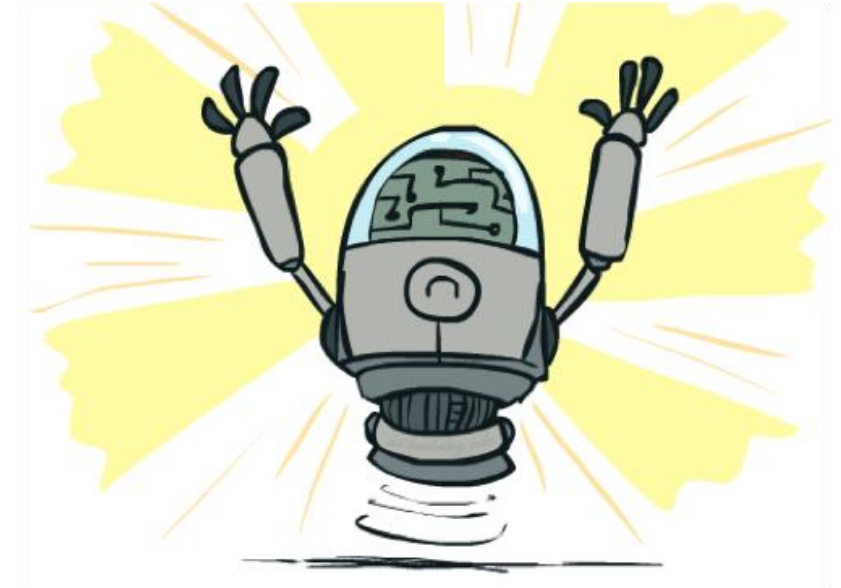
Exploration vs. Exploitation
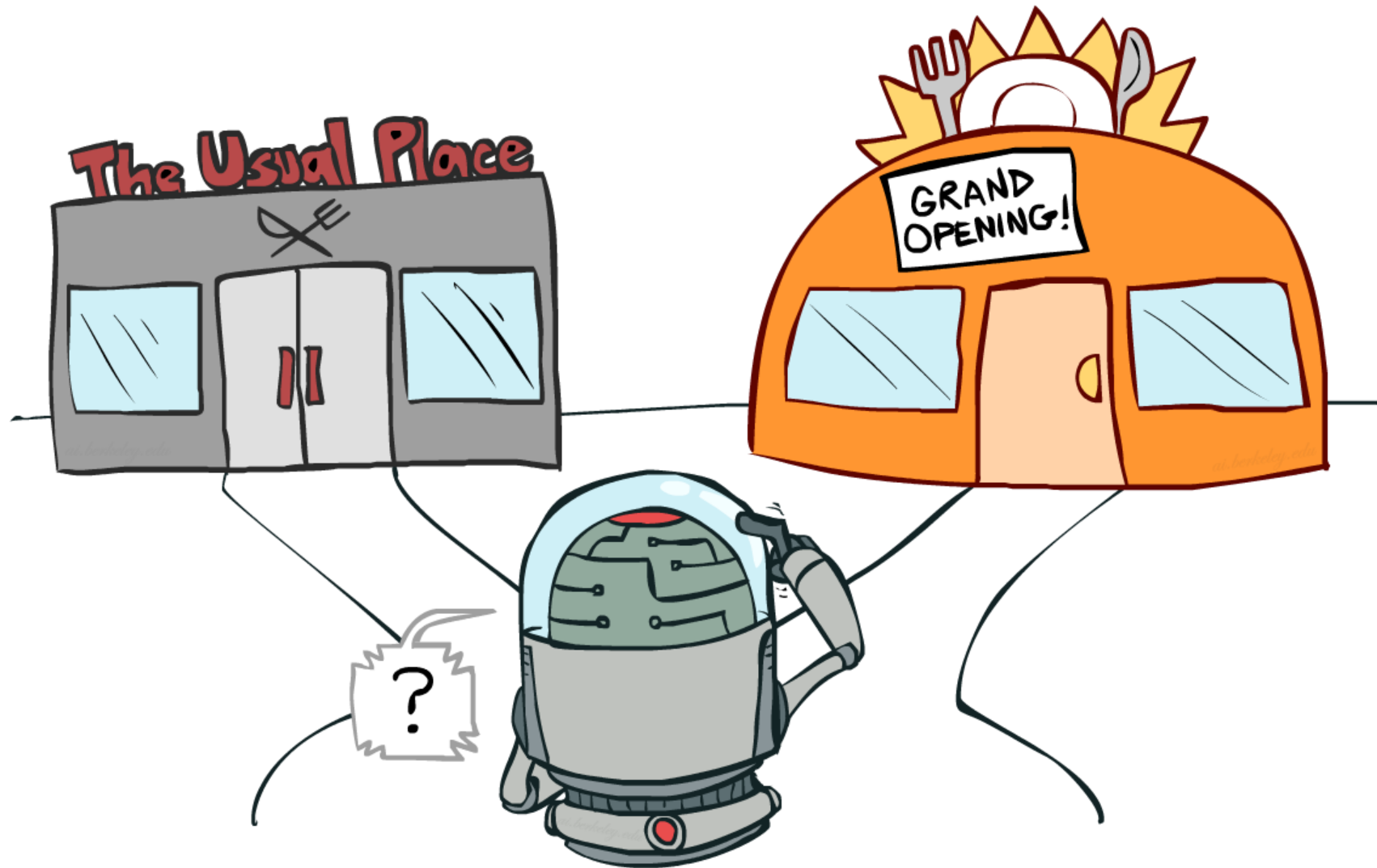
How to Explore?

- **Several schemes for forcing exploration**
  - Simplest: random actions (ε-greedy)
    - Every time step, flip a coin
    - With (small) probability ε, act randomly
    - With (large) probability $1-\varepsilon$, act on current policy

  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions

[Demo: Q-learning – manual exploration – bridge grid (L11D2)]
[Demo: Q-learning – epsilon-greedy -- crawler (L11D3)]

Exploration Functions

- ## When to explore?
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

- ## Exploration function
  - Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g. $f(u, n) = u + k/n$

 Regular Q-Update: $\quad Q(s, a) \leftarrow_\alpha R(s, a, s') + \gamma \max_{a'} Q(s', a')$

  Modified Q-Update: $Q(s, a) \leftarrow_\alpha R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$

  - Note: this propagates the "bonus" back to states that lead to unknown states as well!

[Demo: exploration – Q-learning – crawler – exploration function (L11D4)]

Regret

- Even if you learn the optimal policy, you still make mistakes along the way
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret
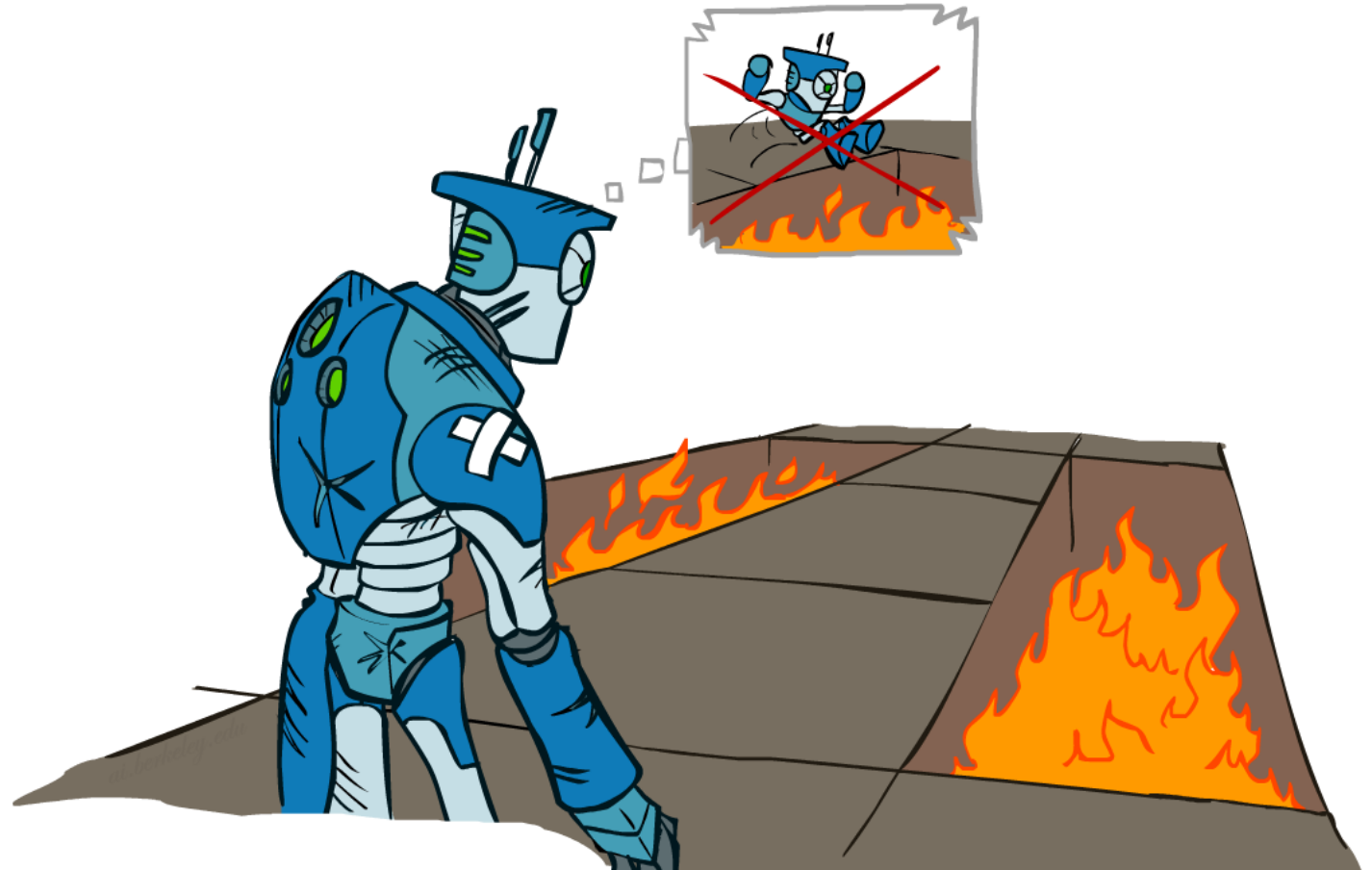
Approximate Q-Learning
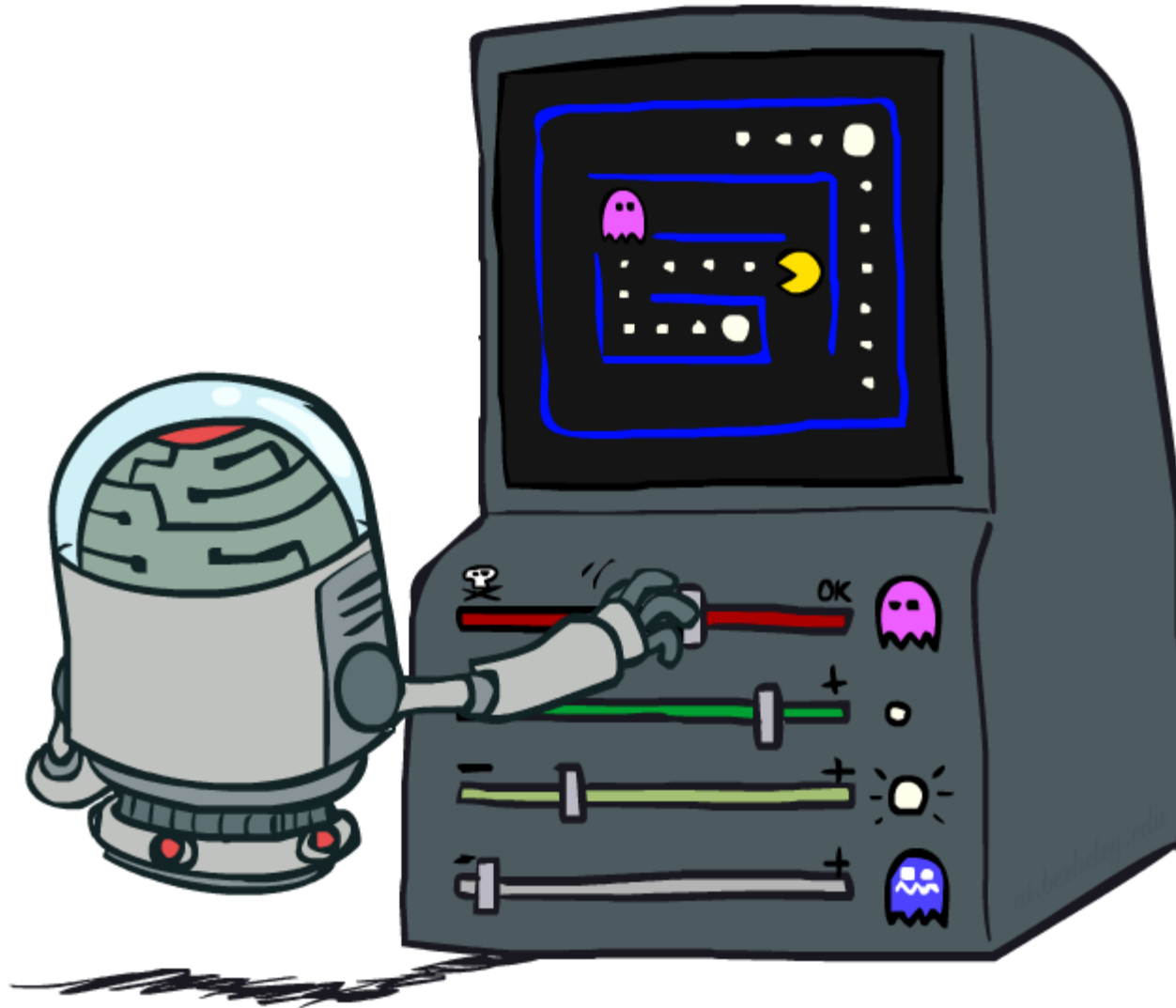
Generalizing Across States

- Basic Q-Learning keeps a table of all q-values

- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory

- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again
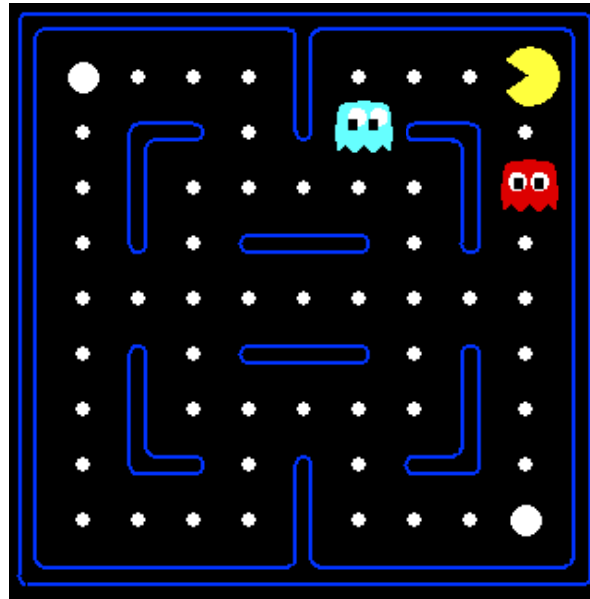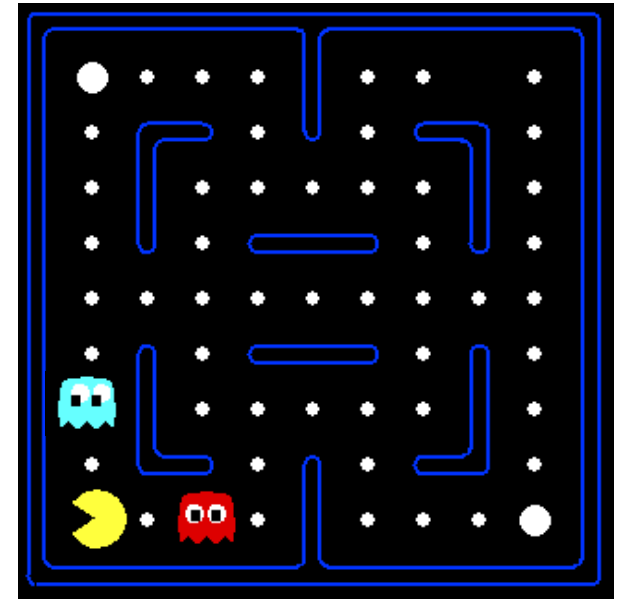
[demo – RL pacman]
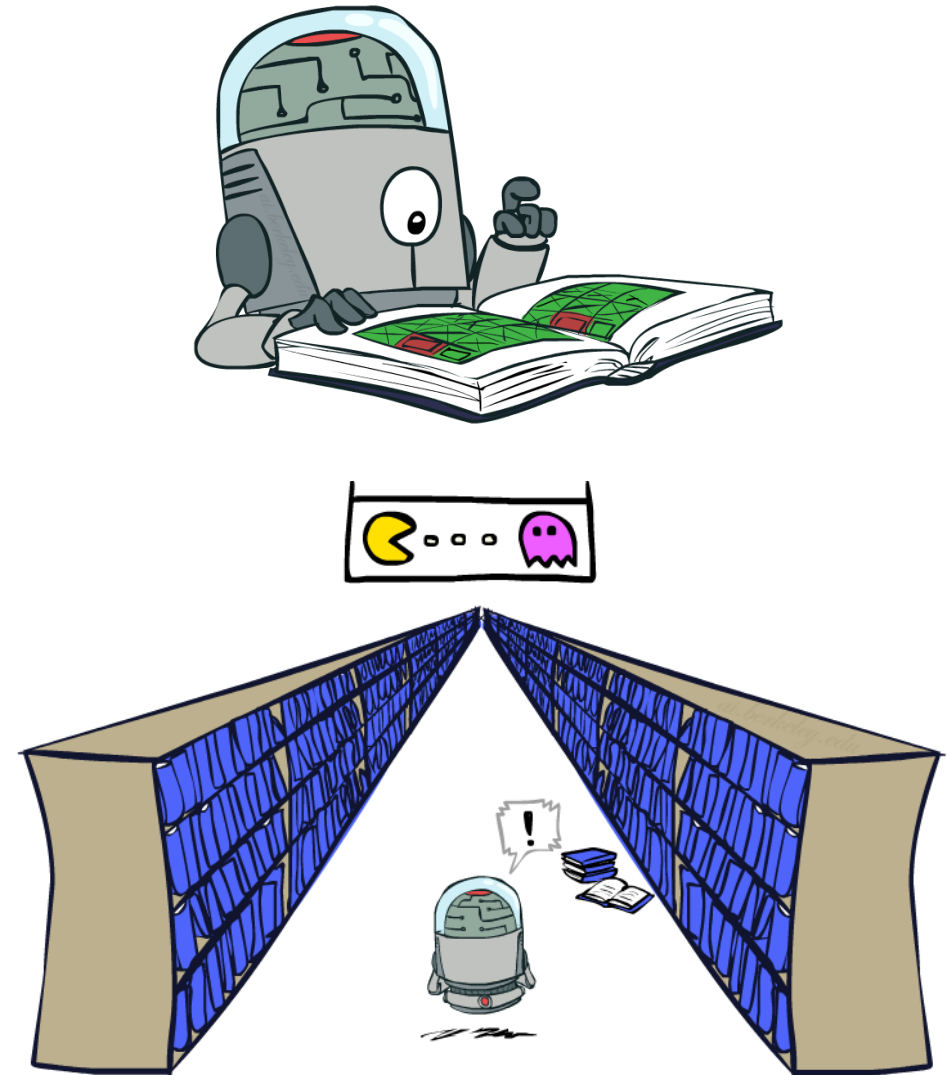
Example: Pacman

Let's say we discover through experience that this state is bad:
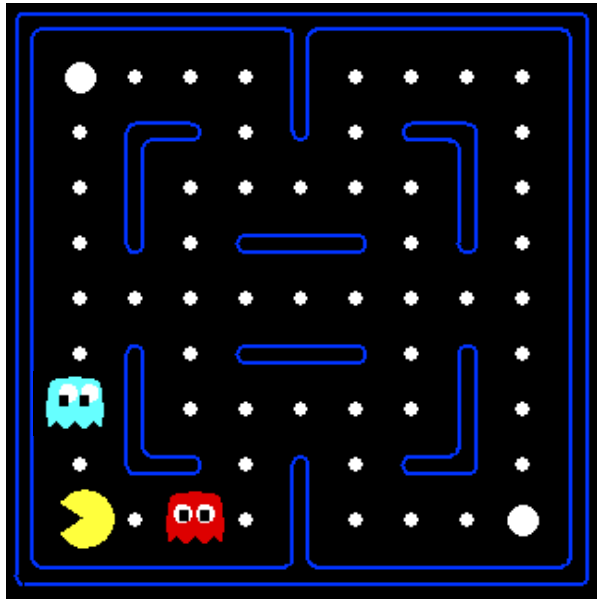

In naïve q-learning, we know nothing about this state:

Or even this one!

Feature-Based Representations

- **Solution: describe a state using a vector of features (properties)**
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - $1 / (\text{dist to dot})^2$
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)
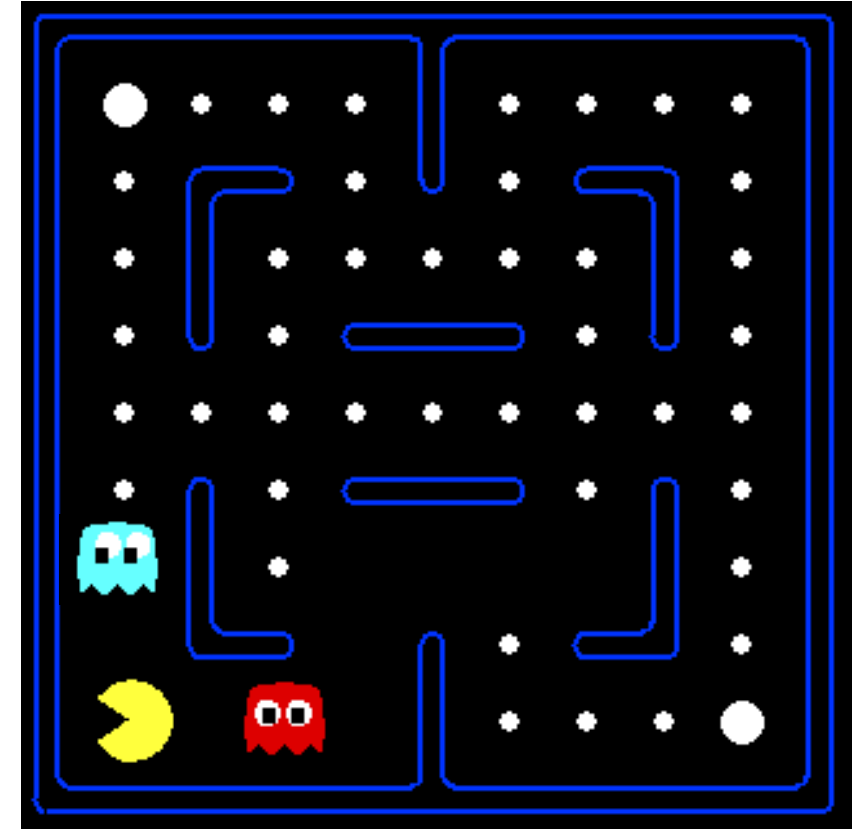
Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \ldots + w_n f_n(s, a)$$

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Q-learning with linear Q-functions:

  $$\text{transition } = (s,a,r,s')$$

  $$\text{difference} = \left[r + \gamma \max_{a'} Q(s',a') \right] - Q(s,a)$$

  $$Q(s,a) \leftarrow Q(s,a) + \alpha \, [\text{difference}] \qquad \text{Exact Q's}$$

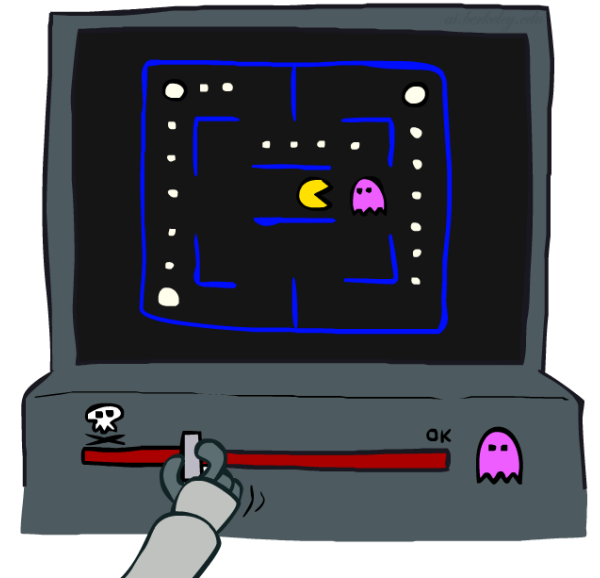 $$w_i \leftarrow w_i + \alpha \, [\text{difference}] \, f_i(s,a) \qquad \text{Approximate Q's}$$

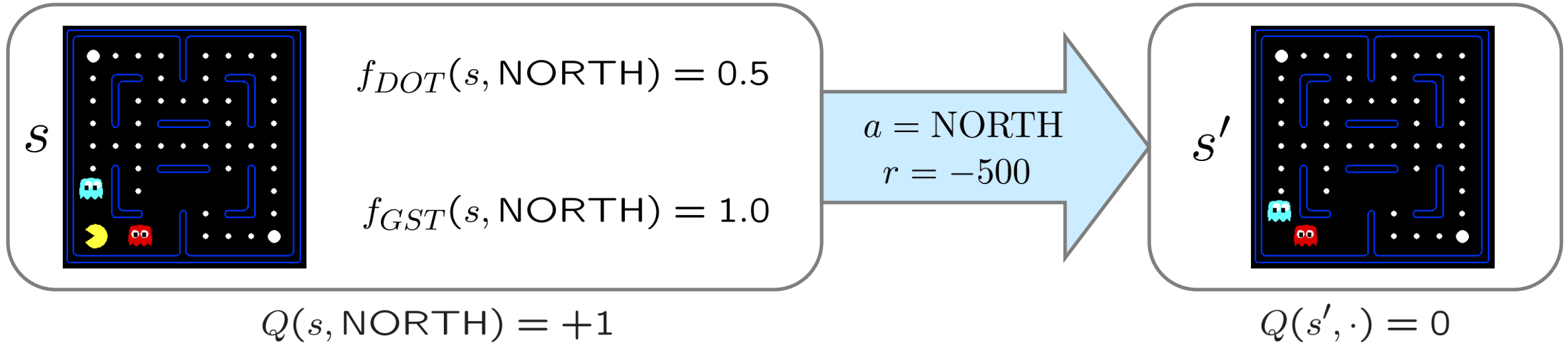- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features

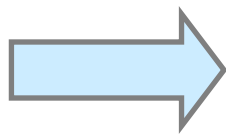- Formal justification: online least squares

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

$f_{DOT}(s, \mathsf{NORTH}) = 0.5$

$a = \mathsf{NORTH}$
$r = -500$

$f_{GST}(s, \mathsf{NORTH}) = 1.0$

s'

$Q(s, \mathsf{NORTH}) = +1$

$Q(s', \cdot) = 0$

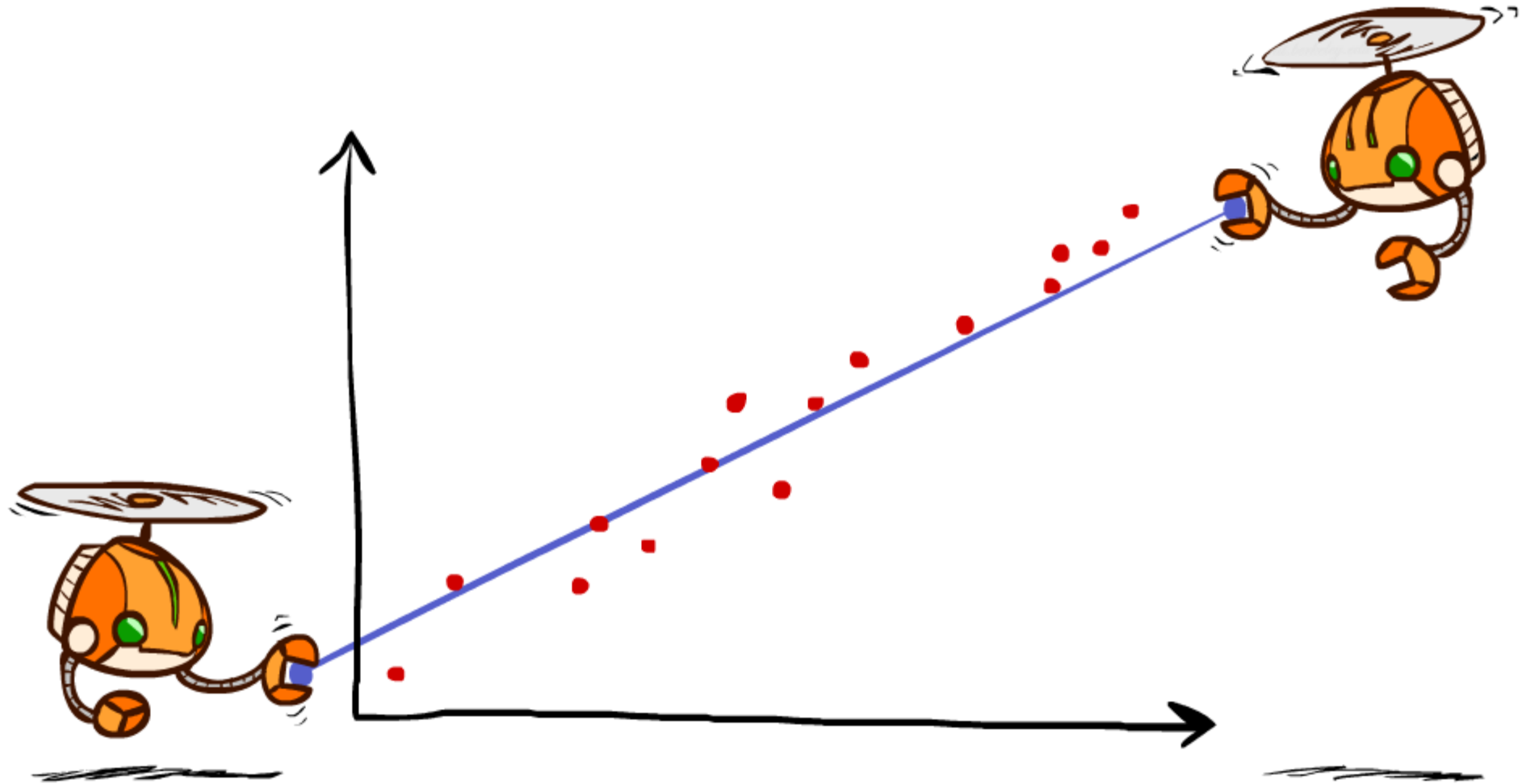$r + \gamma \max_{a'} Q(s', a') = -500 + 0$

$\text{difference} = -501$

$w_{DOT} \leftarrow 4.0 + \alpha[-501]\,0.5$
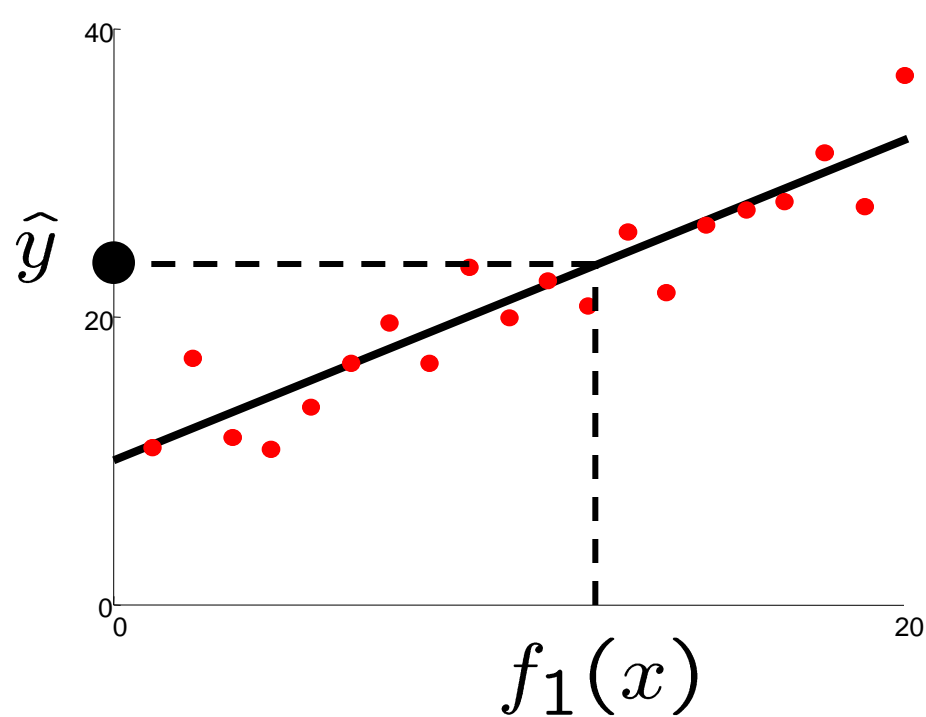$w_{GST} \leftarrow -1.0 + \alpha[-501]\,1.0$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

[Demo: approximate Q-learning pacman (L11D10)]
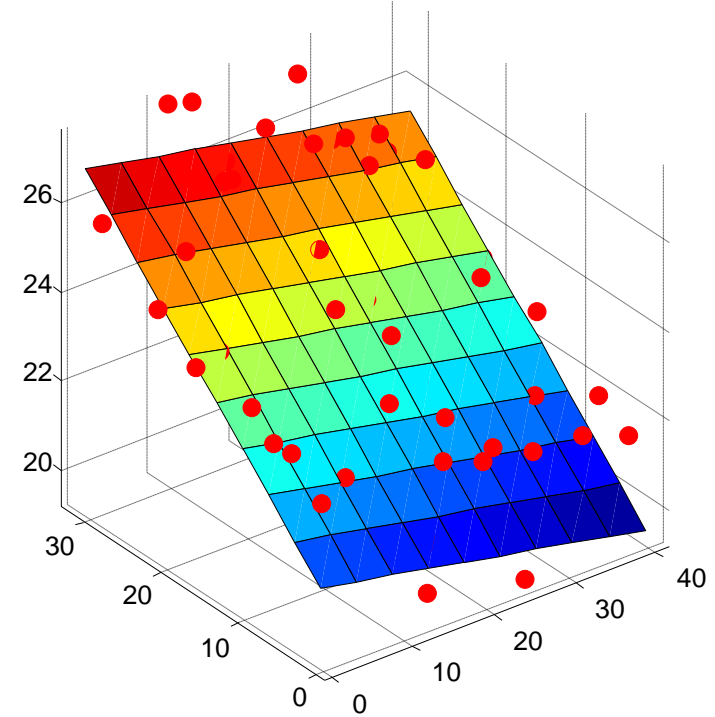
Q-Learning and Least Squares

Linear Approximation: Regression*
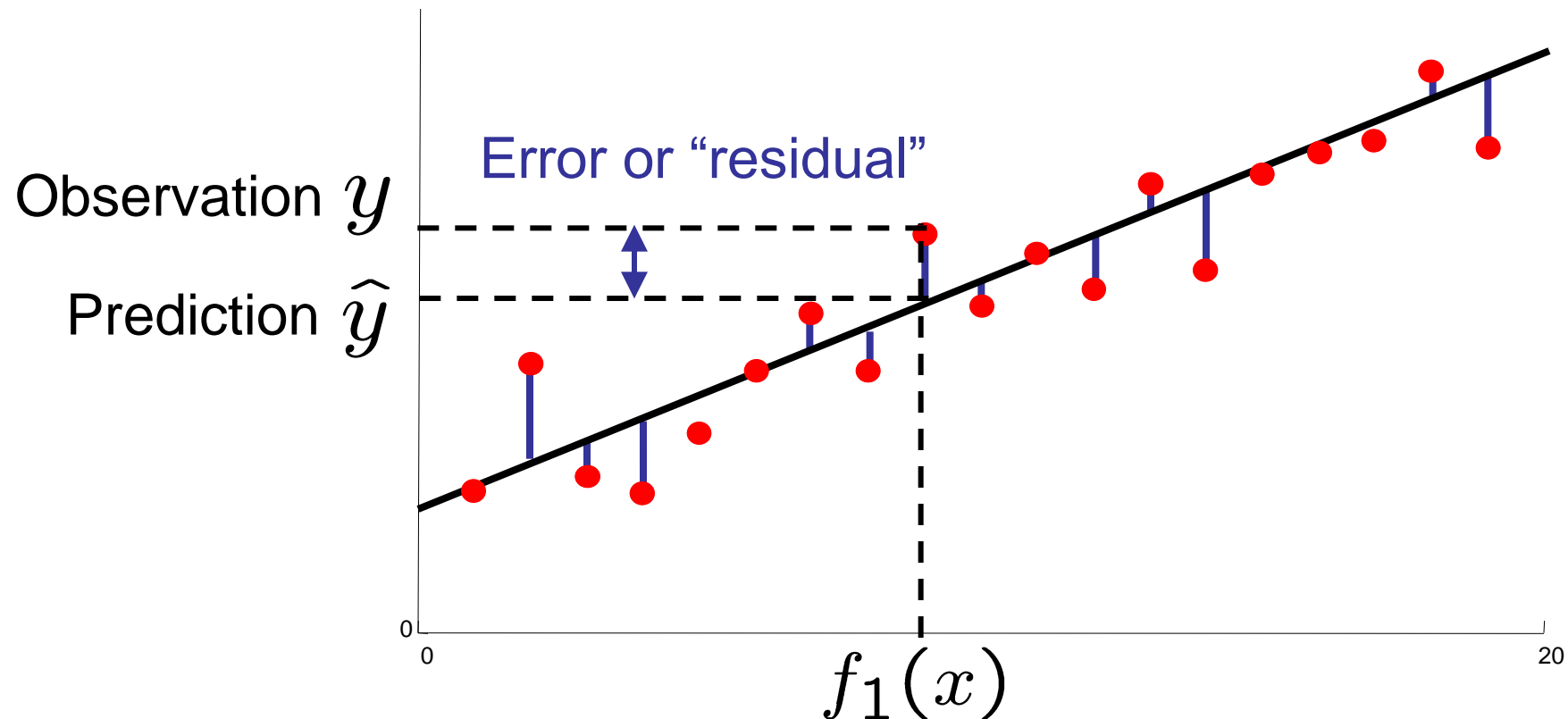

Prediction:
$$\hat{y} = w_0 + w_1 f_1(x)$$

Prediction:
$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares*
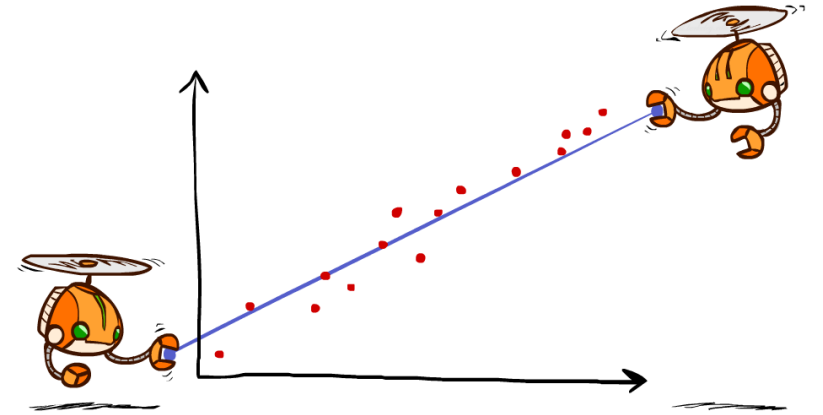
$$\text{total error} = \sum_i (y_i - \widehat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$

Error or "residual"

Observation y

Prediction $\widehat{y}$

$f_1(x)$

Minimizing Error*

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial \, \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)$$
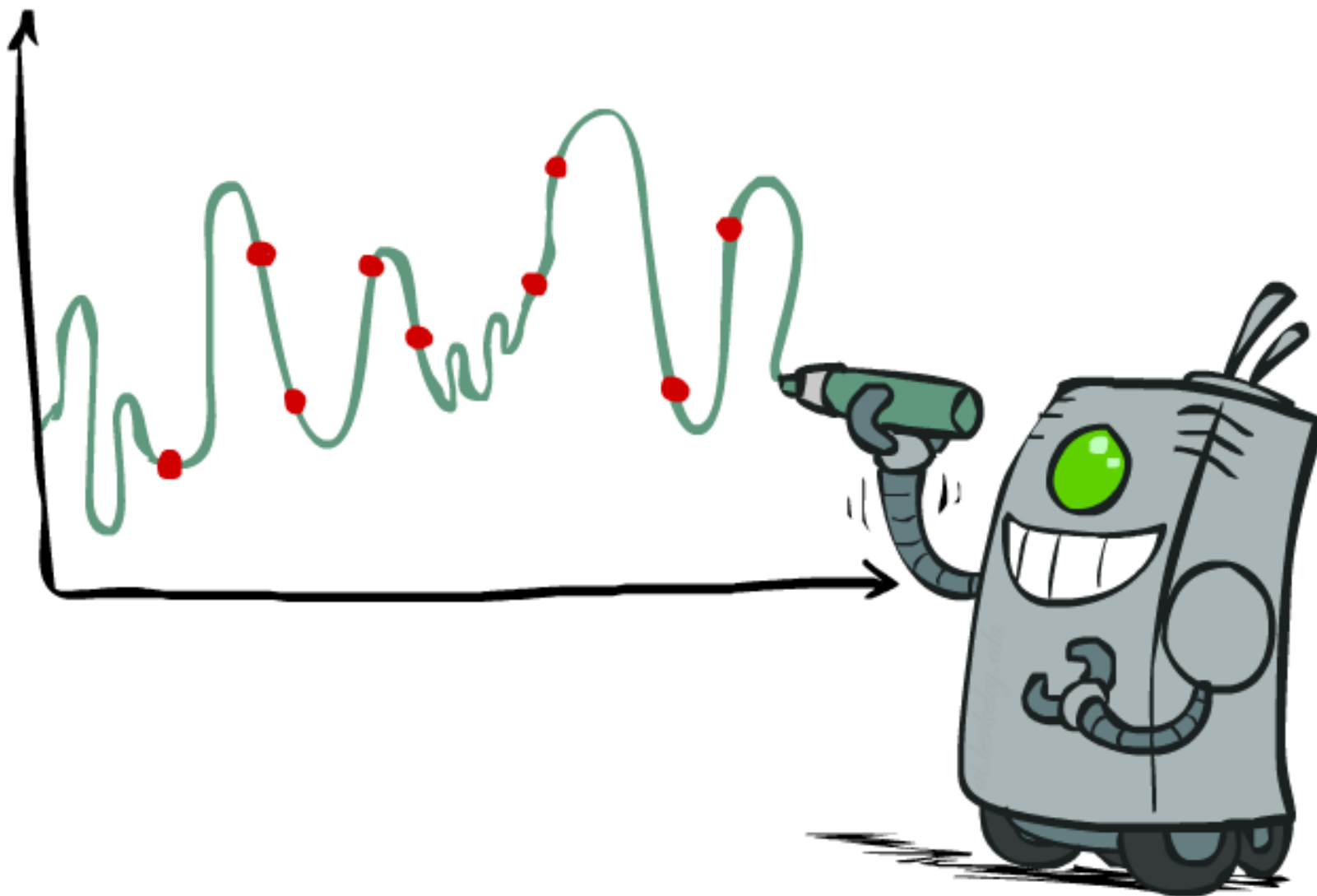
Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a)\right] f_m(s, a)$$

"target"   "prediction"
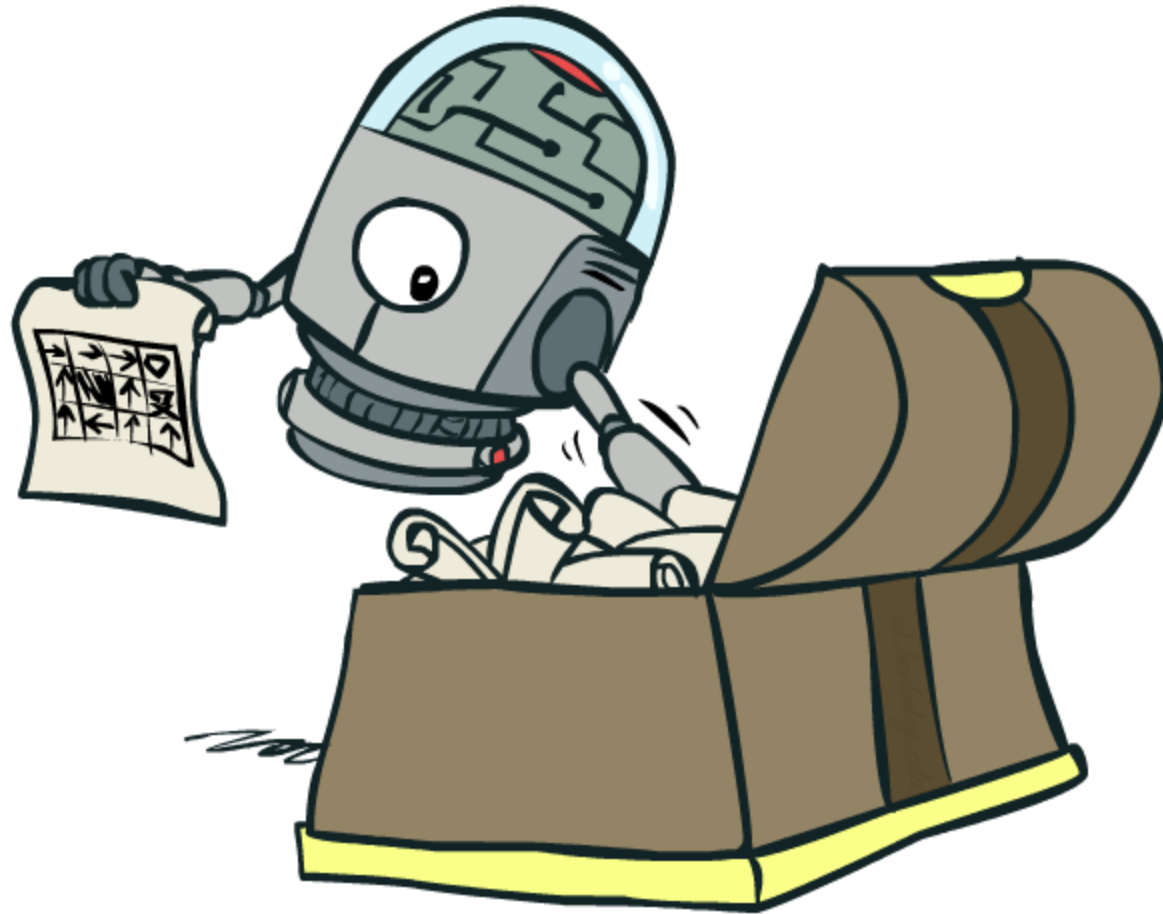
Overfitting: Why Limiting Capacity Can Help*

Policy Search

Policy Search

- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - Q-learning's priority: get Q-values close (modeling)
  - Action selection priority: get ordering of Q-values right (prediction)
  - We'll see this distinction between modeling and prediction again later in the course

- Solution: learn policies that maximize rewards, not the values that predict them

- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by hill climbing on feature weights

Policy Search

- Simplest policy search:
  - Start with an initial linear value function or Q-function
  - Nudge each feature weight up and down and see if your policy is better than before

- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

- Better methods exploit lookahead structure, sample wisely, change multiple parameters…

Policy Search

[Andrew Ng]   [Video: HELICOPTER]

Conclusion

- We're done with Part I: Search and Planning!

- We've seen how AI methods can solve problems in:
  - Search
  - Constraint Satisfaction Problems
  - Games
  - Markov Decision Problems
  - Reinforcement Learning

- Next up: Part II: Uncertainty and Learning!

Today

- Exploration vs Exploitation
- Approximate Q-Learning
- Q-Learning and Least Squares
  - Overfitting: Why Limiting Capacity Can Help
- Policy Search