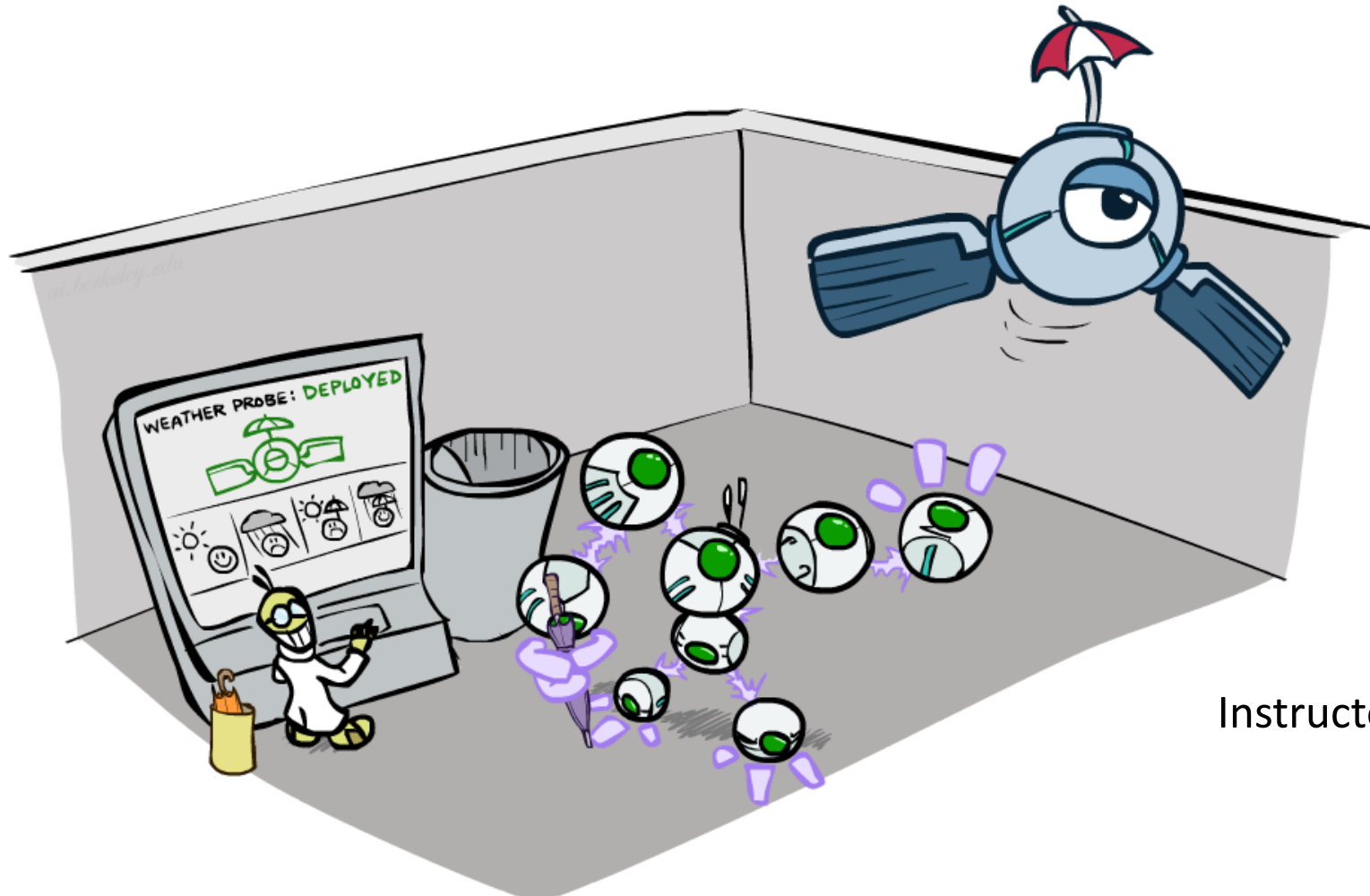


# CSCI 446: Artificial Intelligence

## Decision Networks and Value of Perfect Information



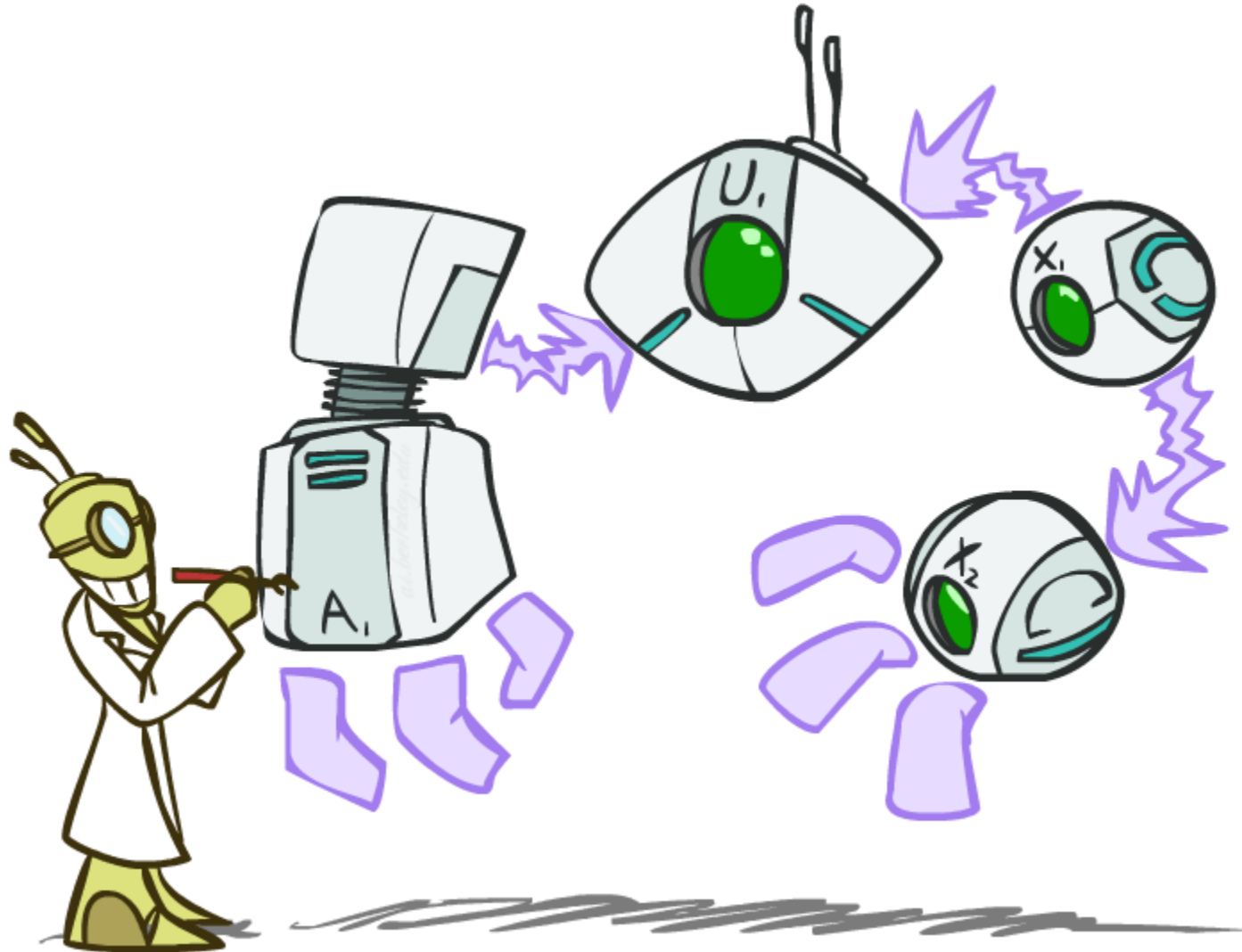
Instructor: Michele Van Dyne

# Outline

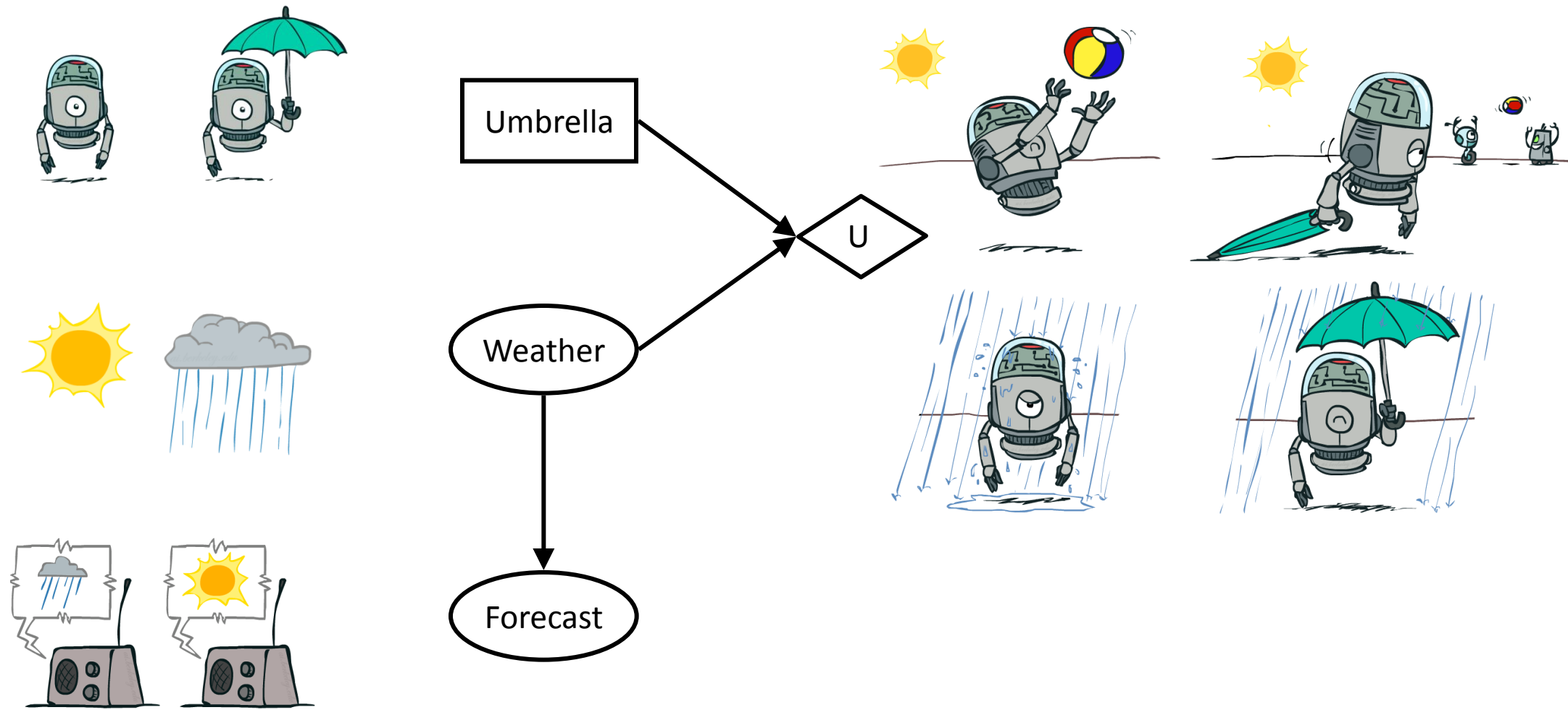
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- Decision Networks
- Value of Information
- POMDPs

# Decision Networks



# Decision Networks



# Decision Networks

- **MEU: choose the action which maximizes the expected utility given the evidence**

- Can directly operationalize this with decision networks

- Bayes nets with nodes for utility and actions
- Lets us calculate the expected utility for each action

- New node types:



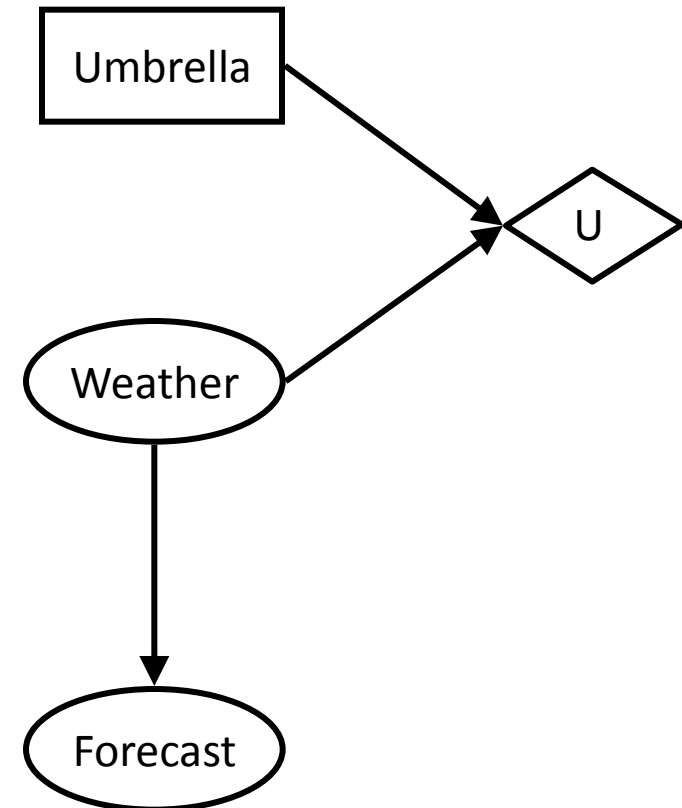
- Chance nodes (just like BNs)



- Actions (rectangles, cannot have parents, act as observed evidence)

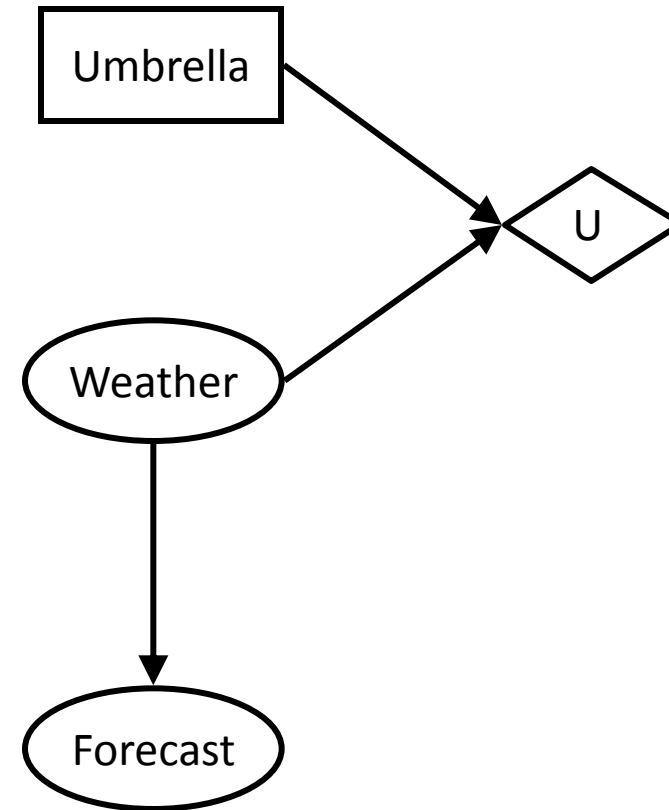


- Utility node (diamond, depends on action and chance nodes)



# Decision Networks

- Action selection
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action



# Decision Networks

Umbrella = leave

$$EU(\text{leave}) = \sum_w P(w)U(\text{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

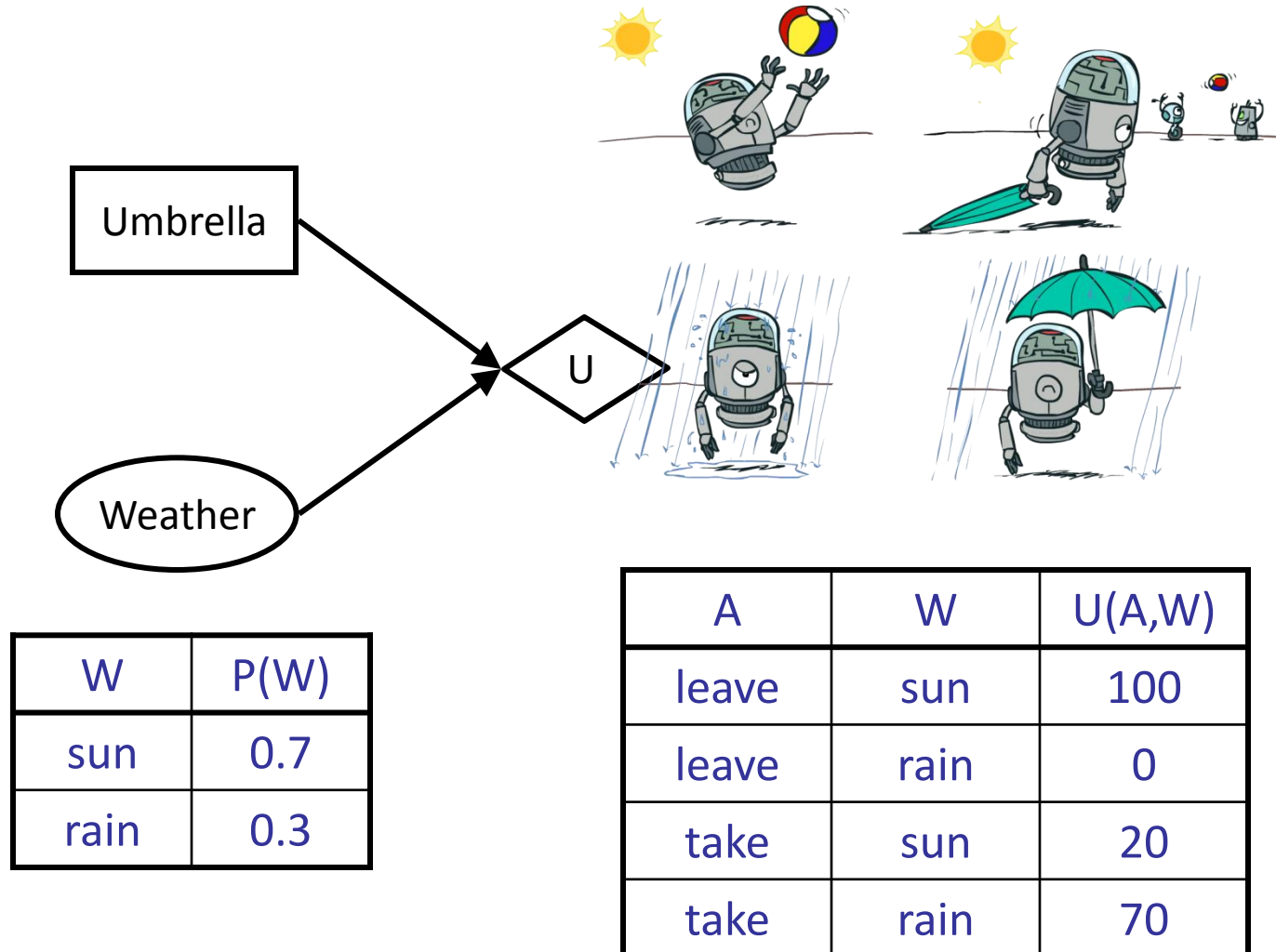
Umbrella = take

$$EU(\text{take}) = \sum_w P(w)U(\text{take}, w)$$

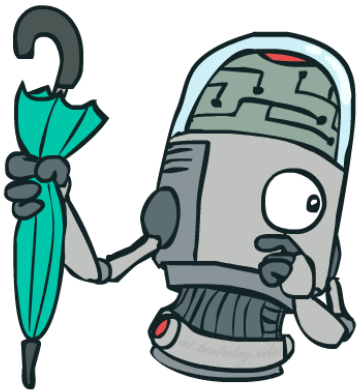
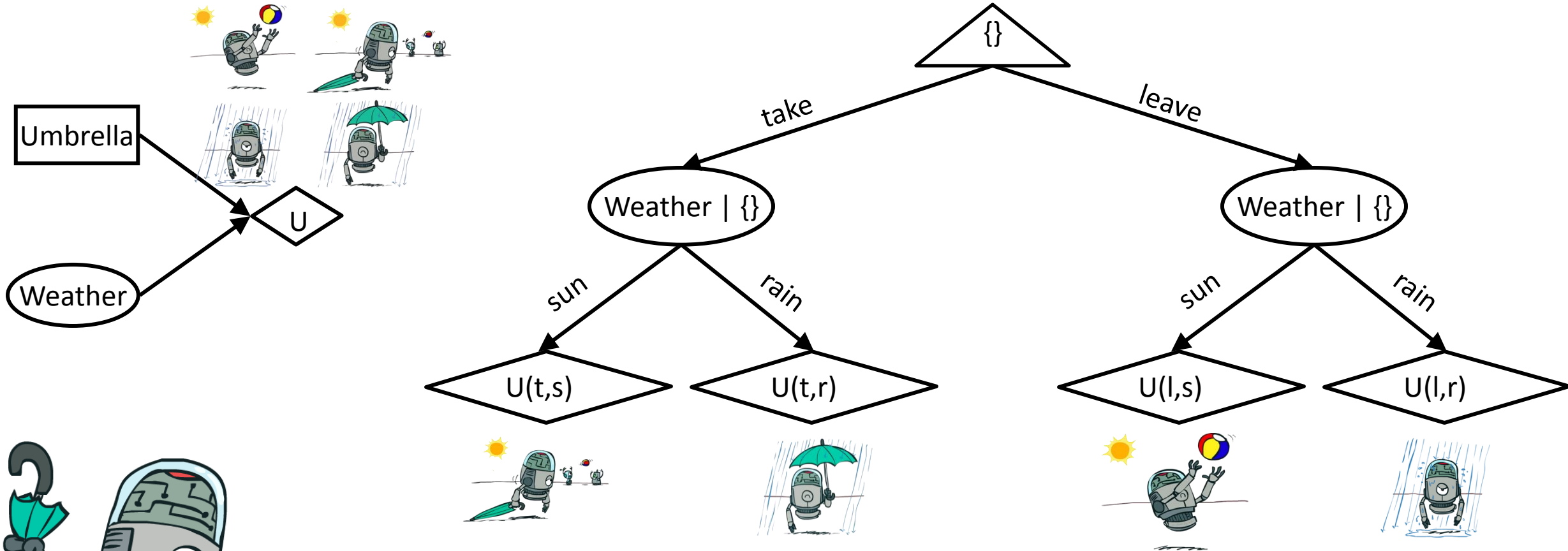
$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$MEU(\emptyset) = \max_a EU(a) = 70$$



# Decisions as Outcome Trees



- Almost exactly like expectimax / MDPs
- What's changed?



# Example: Decision Networks

Umbrella = leave

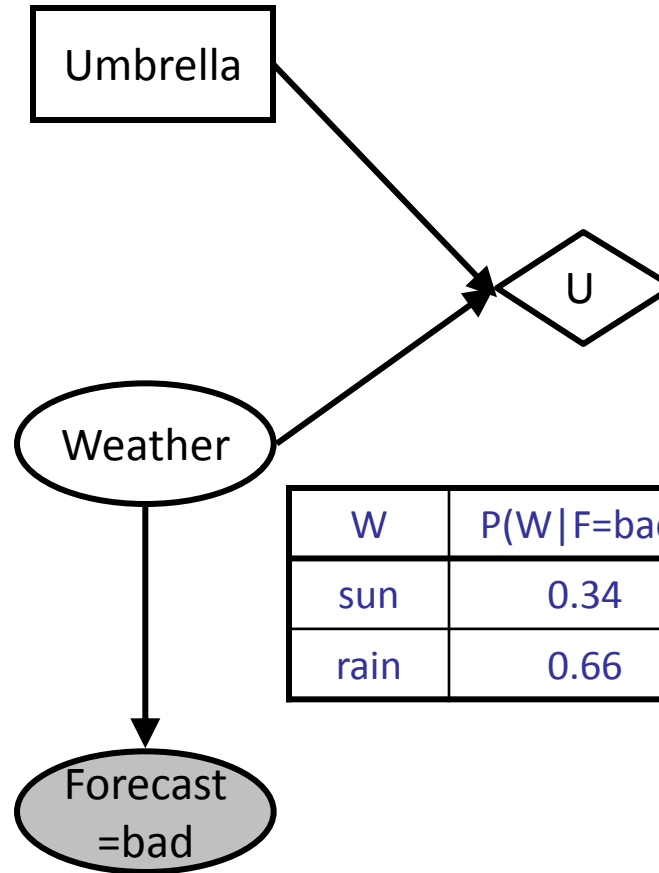
$$\begin{aligned} EU(\text{leave}|\text{bad}) &= \sum_w P(w|\text{bad})U(\text{leave}, w) \\ &= 0.34 \cdot 100 + 0.66 \cdot 0 = 34 \end{aligned}$$

Umbrella = take

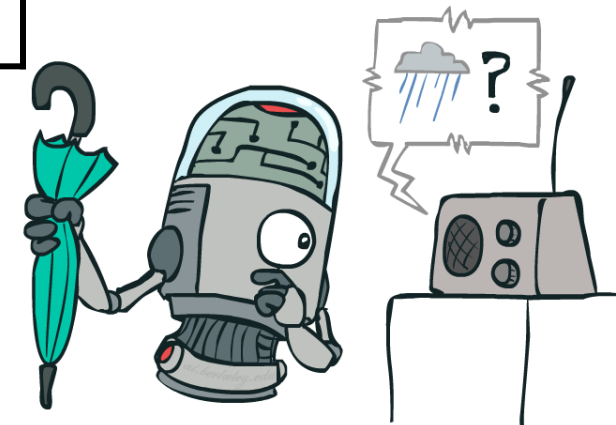
$$\begin{aligned} EU(\text{take}|\text{bad}) &= \sum_w P(w|\text{bad})U(\text{take}, w) \\ &= 0.34 \cdot 20 + 0.66 \cdot 70 = 53 \end{aligned}$$

Optimal decision = take

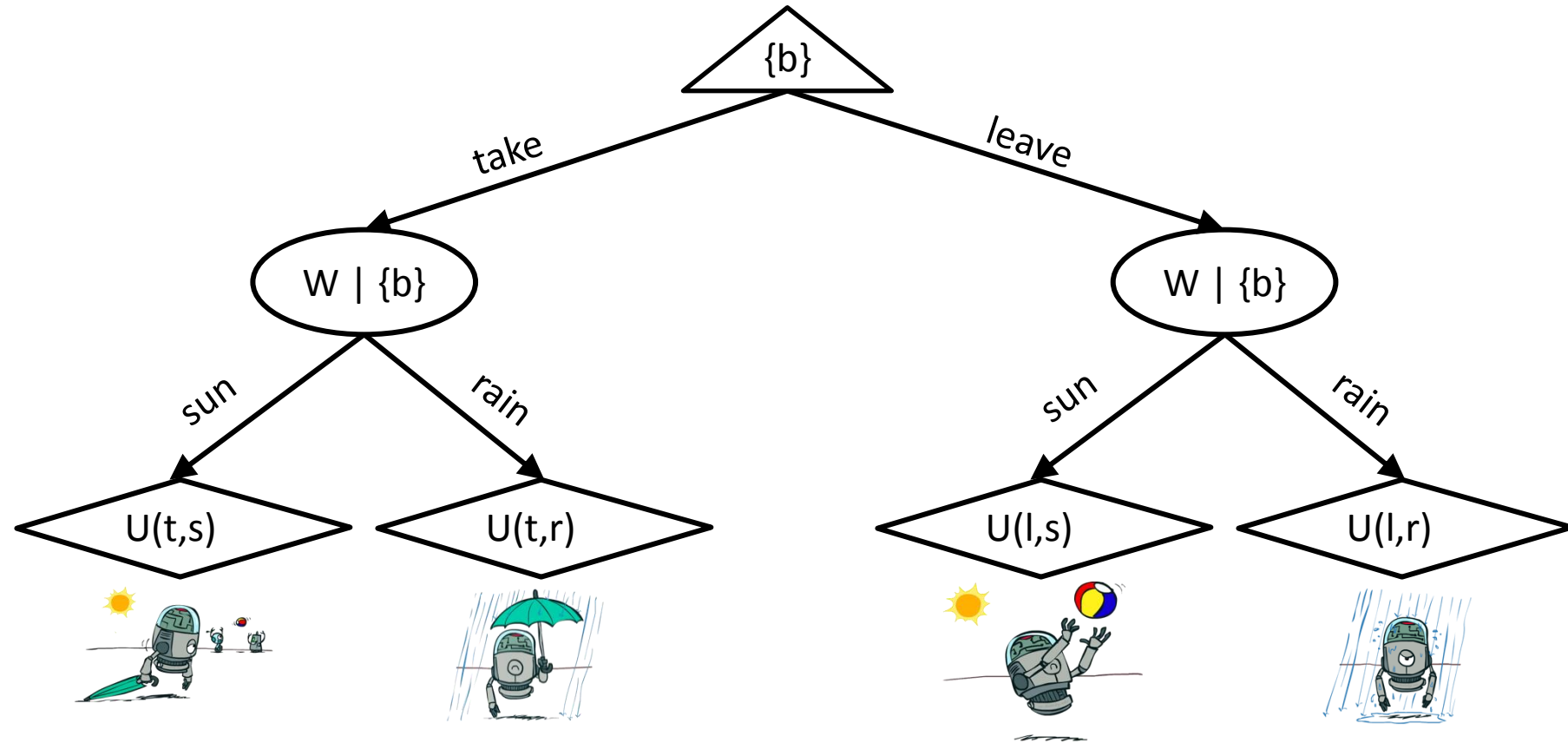
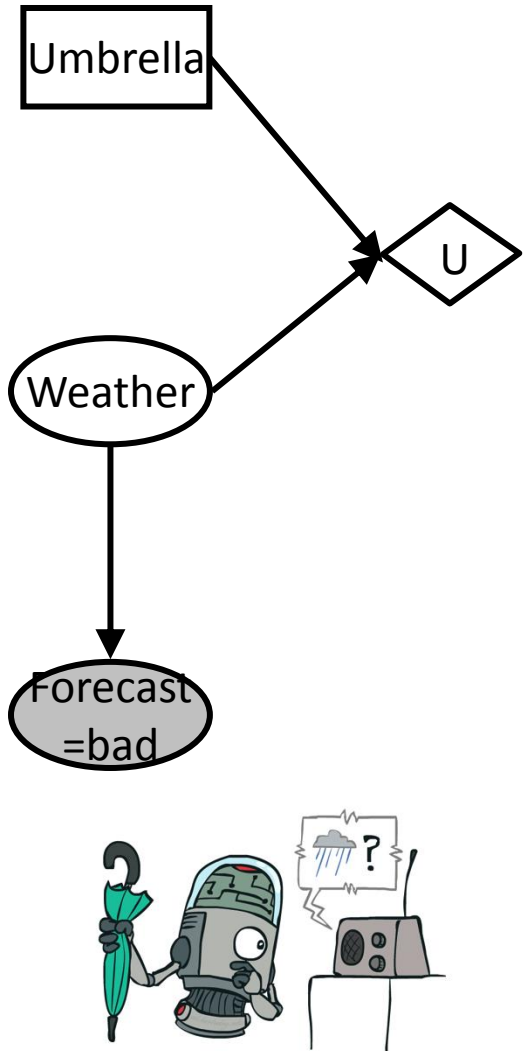
$$MEU(F = \text{bad}) = \max_a EU(a|\text{bad}) = 53$$



A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

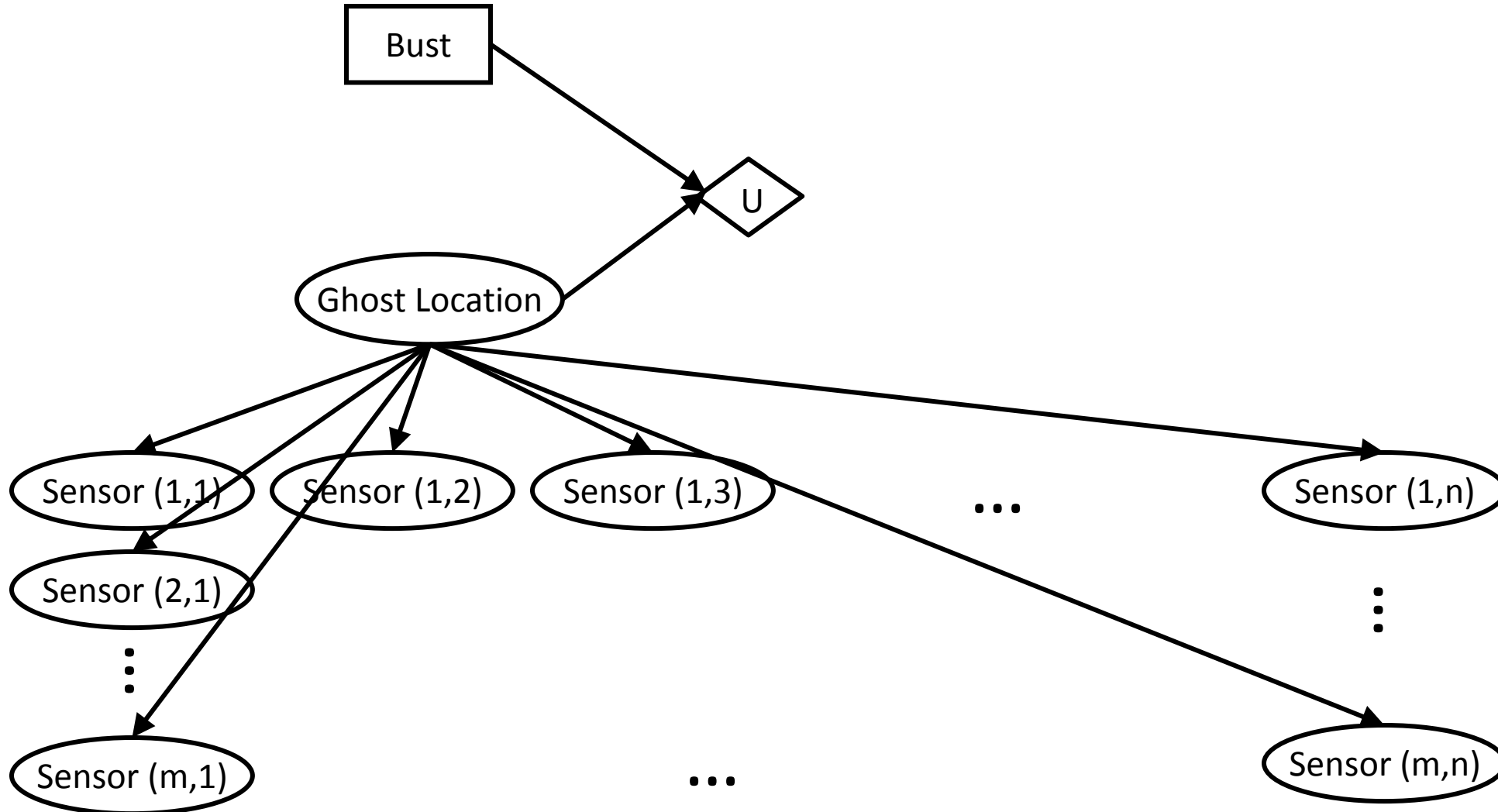


# Decisions as Outcome Trees

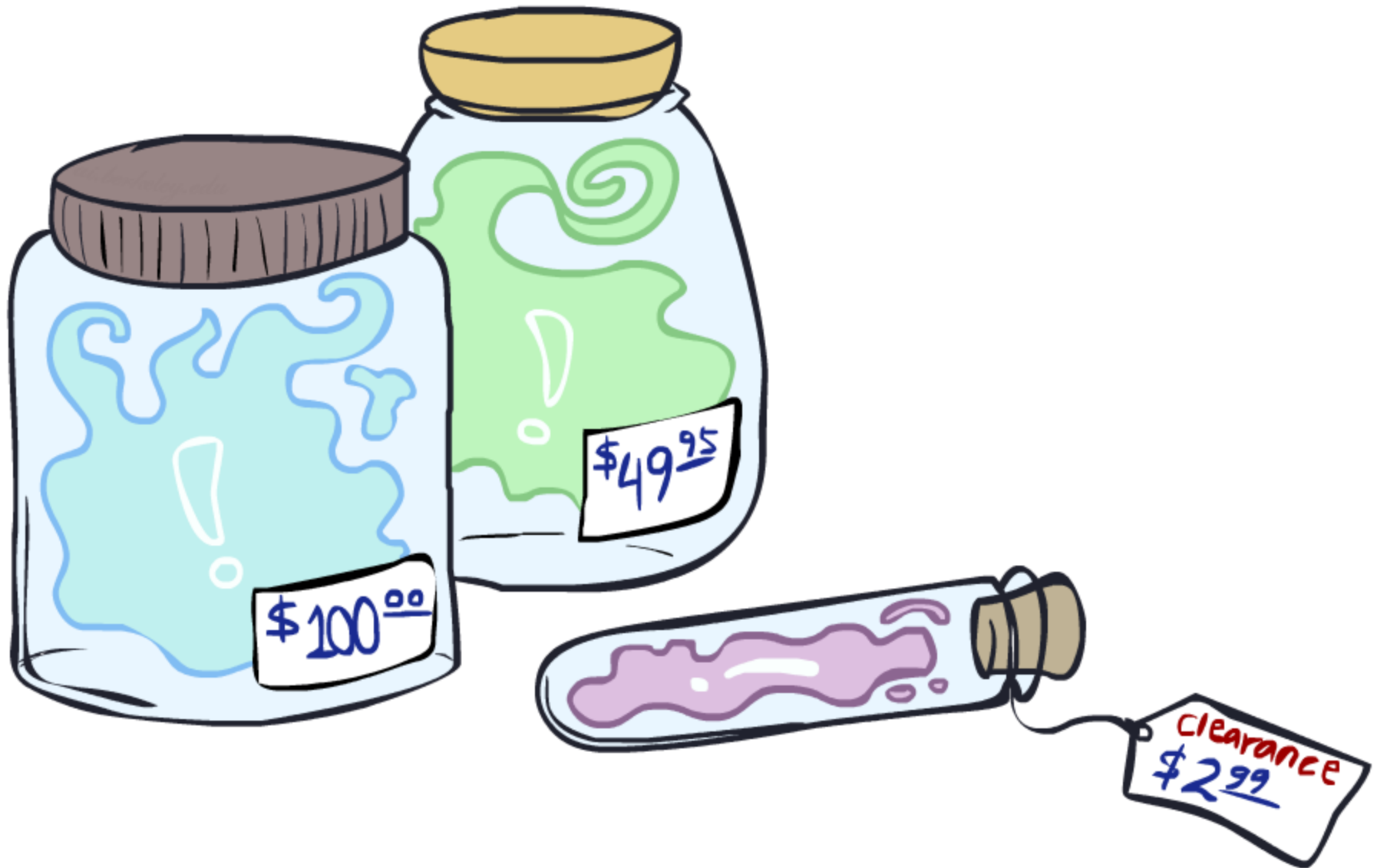


# Ghostbusters Decision Network

Demo: Ghostbusters with probability

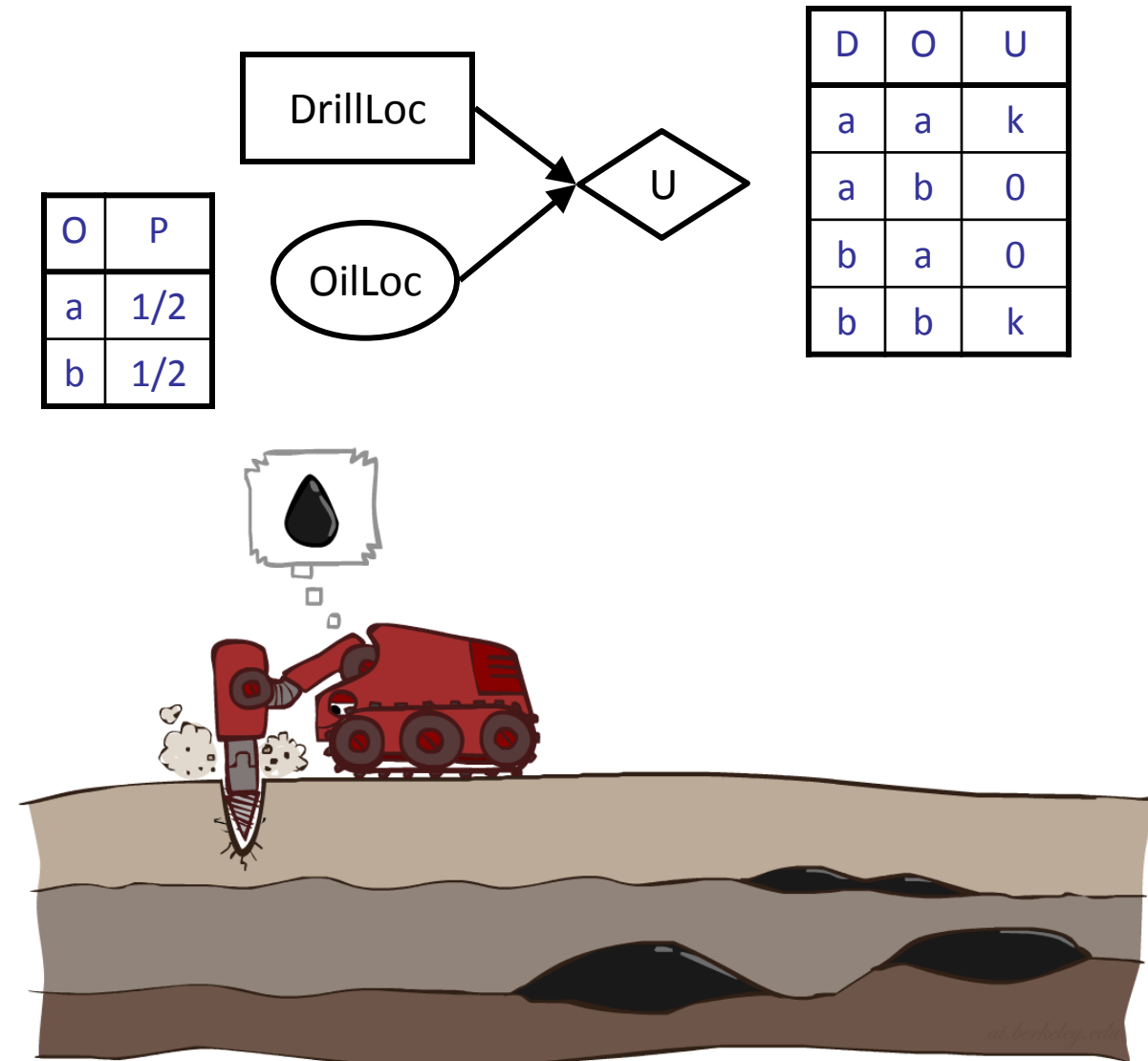


# Value of Information



# Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has  $EU = k/2$ ,  $MEU = k/2$
- Question: what's the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b," prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - $VPI(OilLoc) = k/2$
  - Fair price of information:  $k/2$



# VPI Example: Weather

MEU with no evidence

$$\text{MEU}(\emptyset) = \max_a \text{EU}(a) = 70$$

MEU if forecast is bad

$$\text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53$$

MEU if forecast is good

$$\text{MEU}(F = \text{good}) = \max_a \text{EU}(a|\text{good}) = 95$$

Forecast distribution

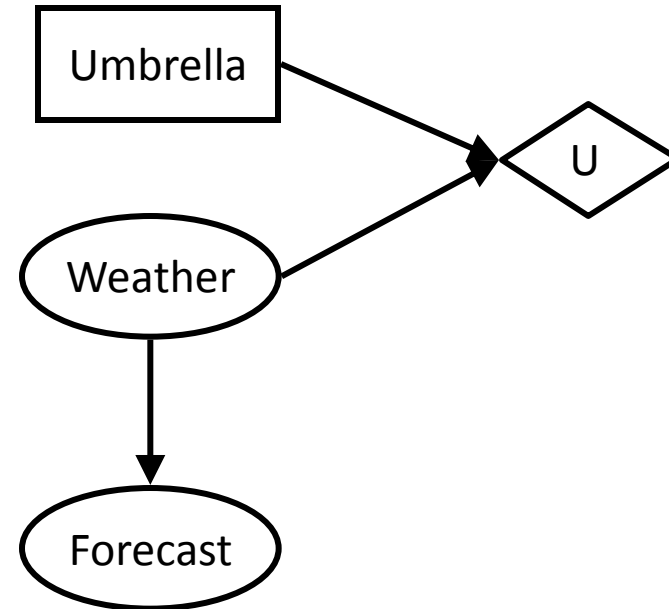
F	P(F)
good	0.59
bad	0.41



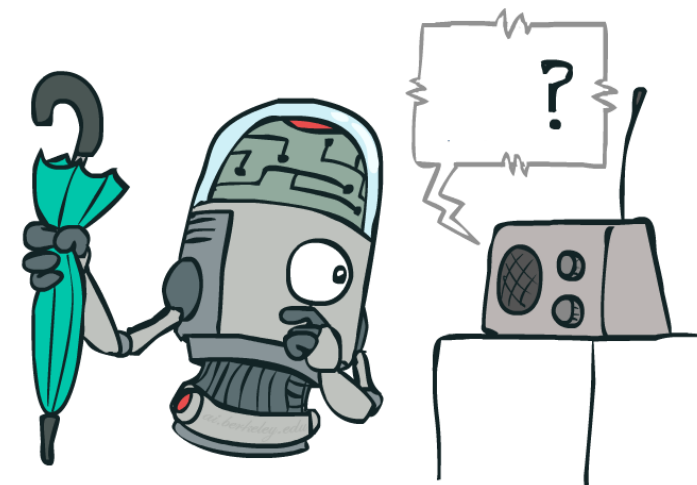
$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$77.8 - 70 = 7.8$$

$$\text{VPI}(E'|e) = \left( \sum_{e'} P(e'|e) \text{MEU}(e, e') \right) - \text{MEU}(e)$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70



# Value of Information

- Assume we have evidence  $E=e$ . Value if we act now:

$$MEU(e) = \max_a \sum_s P(s|e) U(s, a)$$

- Assume we see that  $E' = e'$ . Value if we act then:

$$MEU(e, e') = \max_a \sum_s P(s|e, e') U(s, a)$$

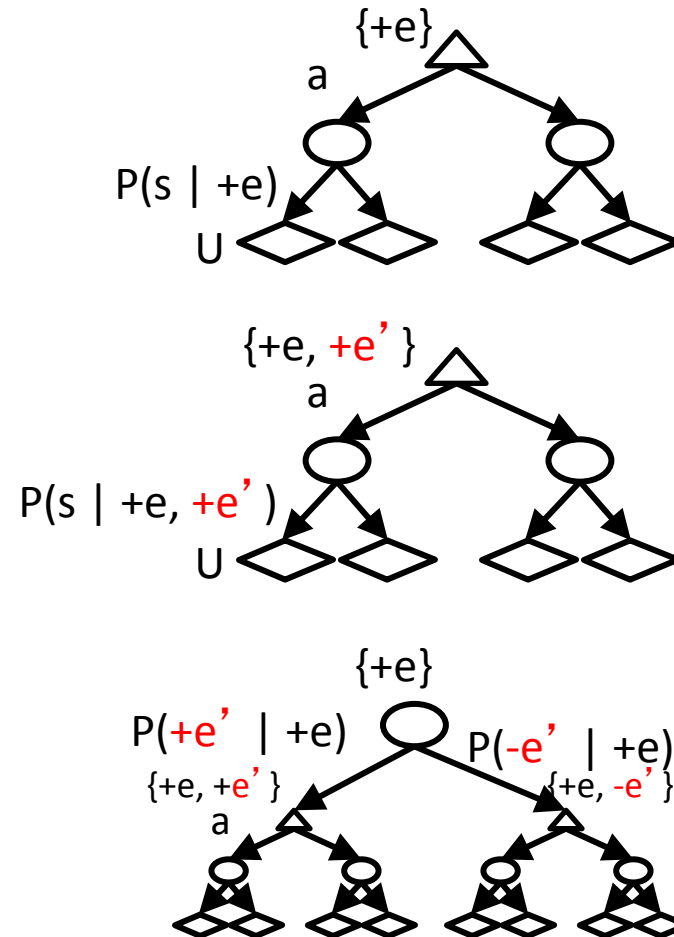
- BUT  $E'$  is a random variable whose value is unknown, so we don't know what  $e'$  will be

- Expected value if  $E'$  is revealed and then we act:

$$MEU(e, E') = \sum_{e'} P(e'|e) MEU(e, e')$$

- Value of information: how much MEU goes up by revealing  $E'$  first then acting, over acting now:

$$VPI(E'|e) = MEU(e, E') - MEU(e)$$



# VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$



- Nonadditive

(think of observing  $E_j$  twice)

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$



- Order-independent

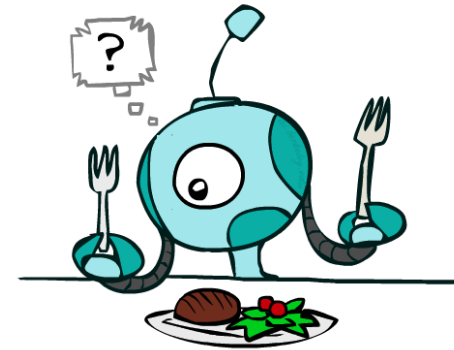
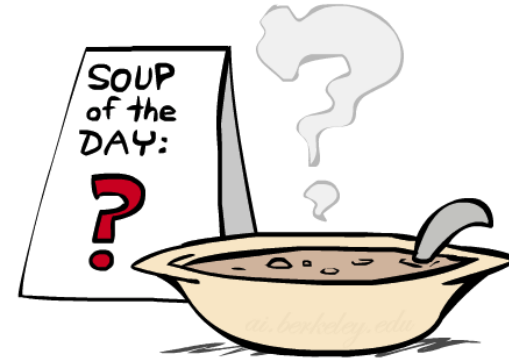
$$\begin{aligned} \text{VPI}(E_j, E_k|e) &= \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) \\ &= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k) \end{aligned}$$





# Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?
- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?
- You're playing the lottery. The prize will be \$0 or \$100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?



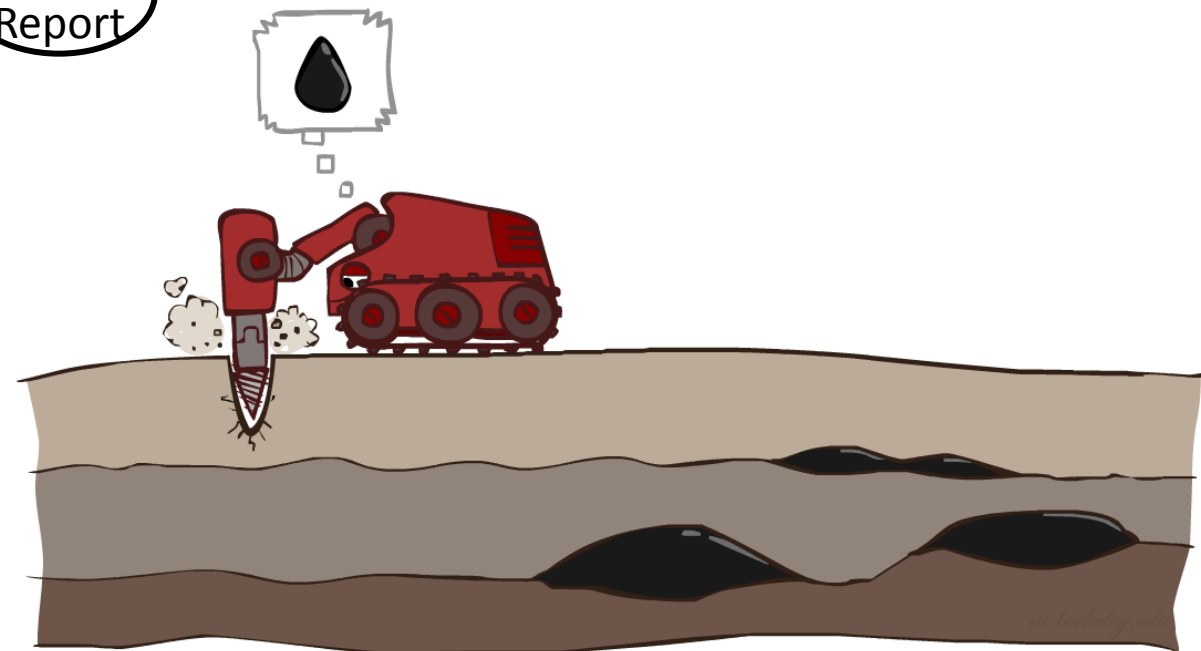
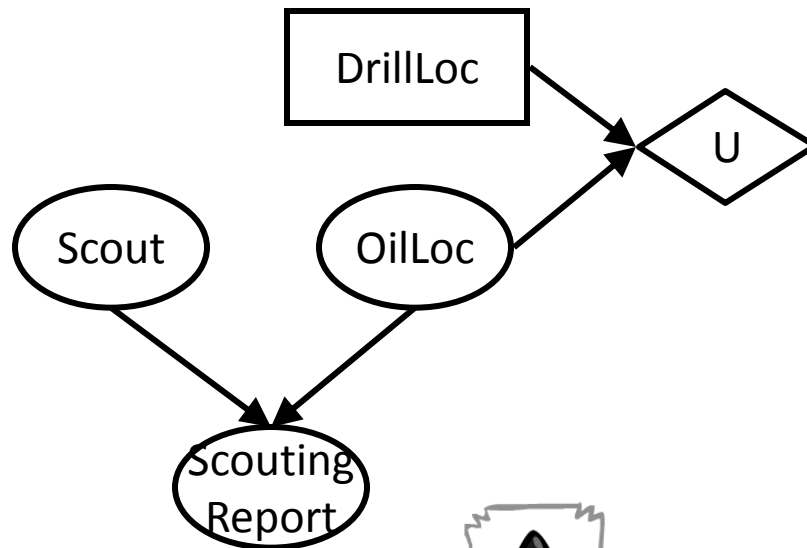
# Value of Imperfect Information?



- No such thing
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one

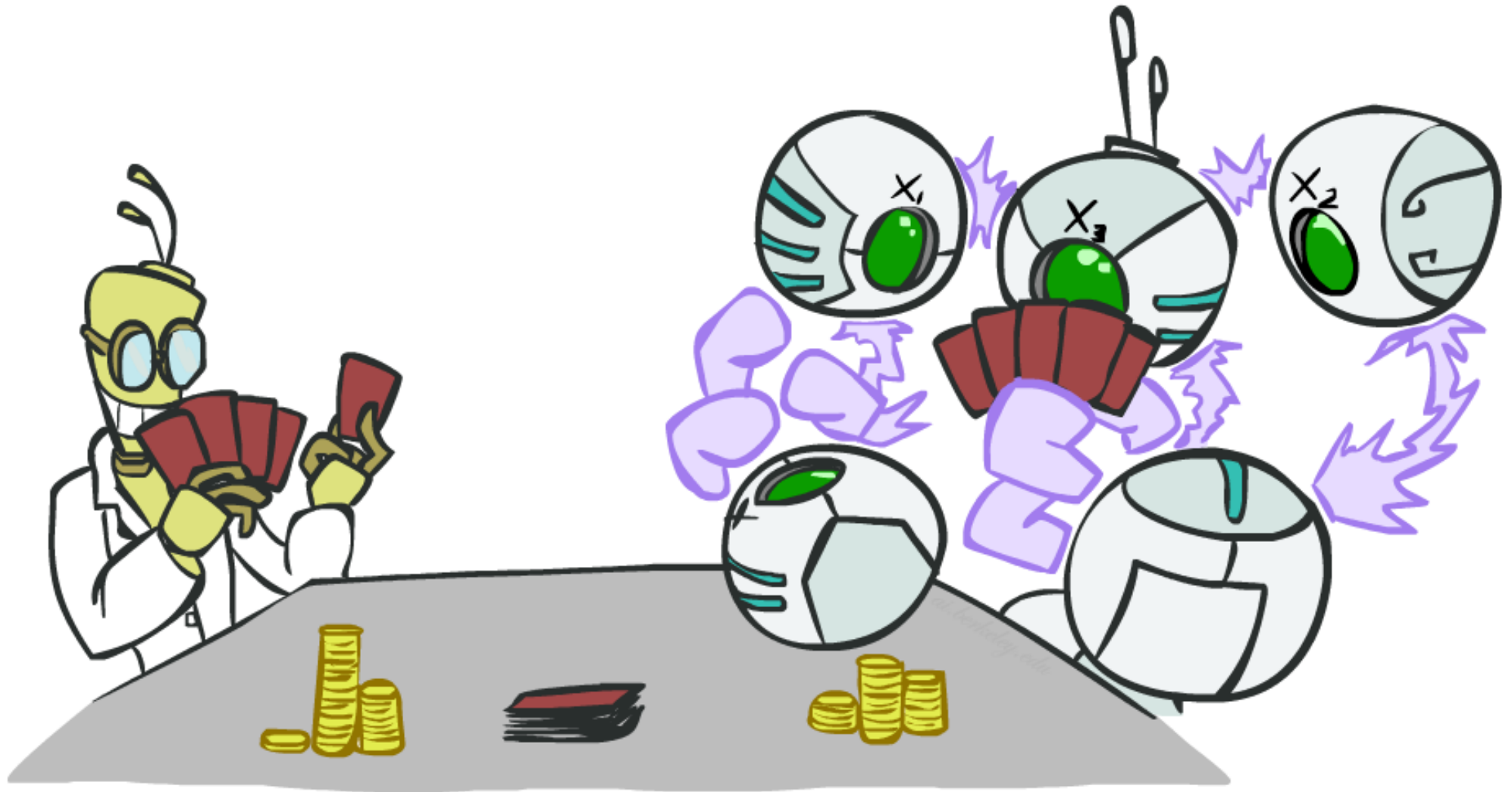
# VPI Question

- VPI(OilLoc) ?
- VPI(ScoutingReport) ?
- VPI(Scout) ?
- VPI(Scout | ScoutingReport) ?



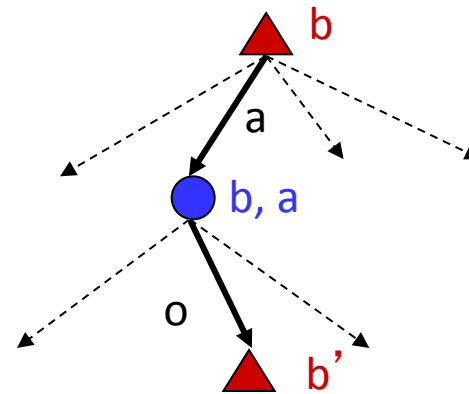
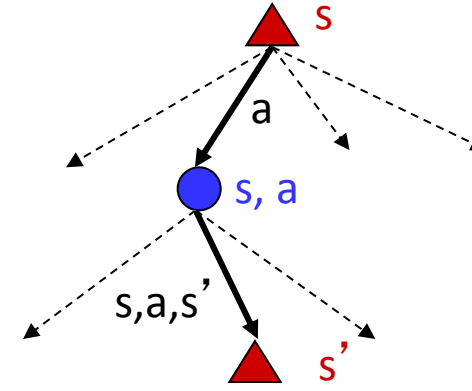
- Generally:  
If  $\text{Parents}(U) \perp\!\!\!\perp Z \mid \text{CurrentEvidence}$   
Then  $\text{VPI}(Z \mid \text{CurrentEvidence}) = 0$

# POMDPs



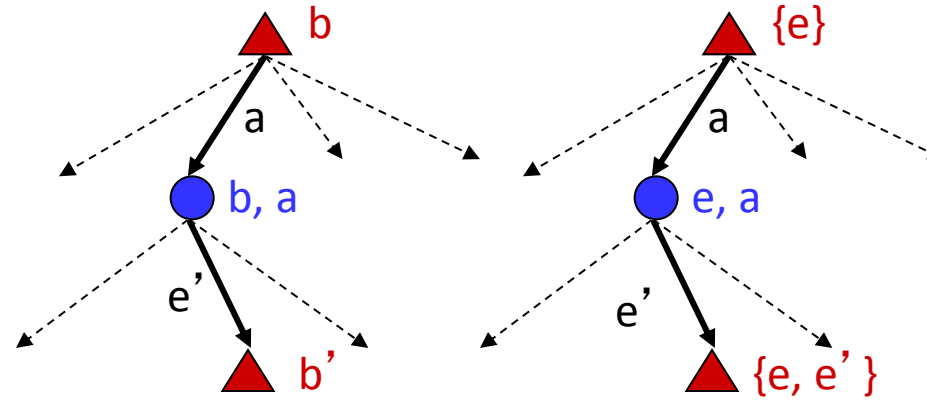
# POMDPs

- MDPs have:
  - States  $S$
  - Actions  $A$
  - Transition function  $P(s' | s, a)$  (or  $T(s, a, s')$ )
  - Rewards  $R(s, a, s')$
- POMDPs add:
  - Observations  $O$
  - Observation function  $P(o | s)$  (or  $O(s, o)$ )
- POMDPs are MDPs over belief states  $b$  (distributions over  $S$ )
- We'll be able to say more in a few lectures

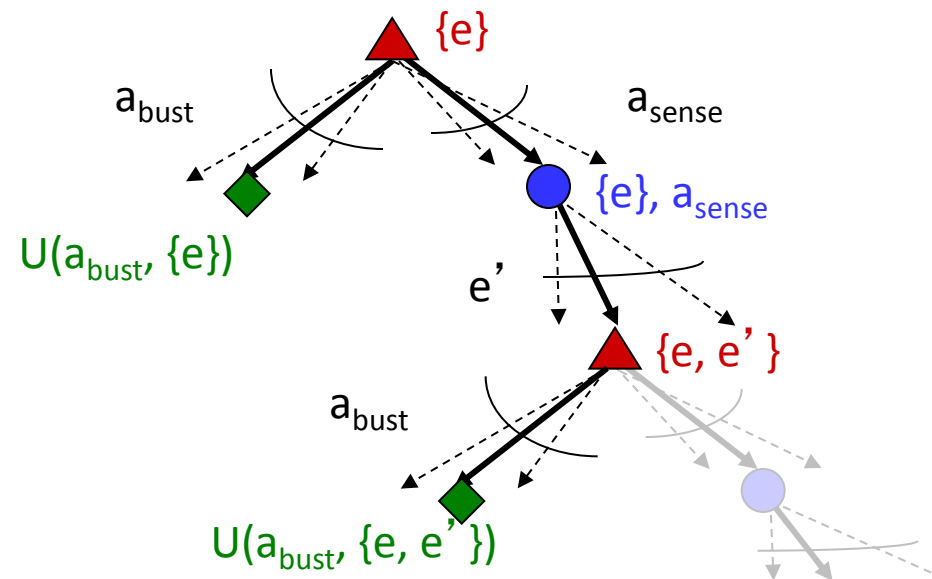


# Example: Ghostbusters

- In (static) Ghostbusters:
  - Belief state determined by evidence to date  $\{e\}$
  - Tree really over evidence sets
  - Probabilistic reasoning needed to predict new evidence given past evidence

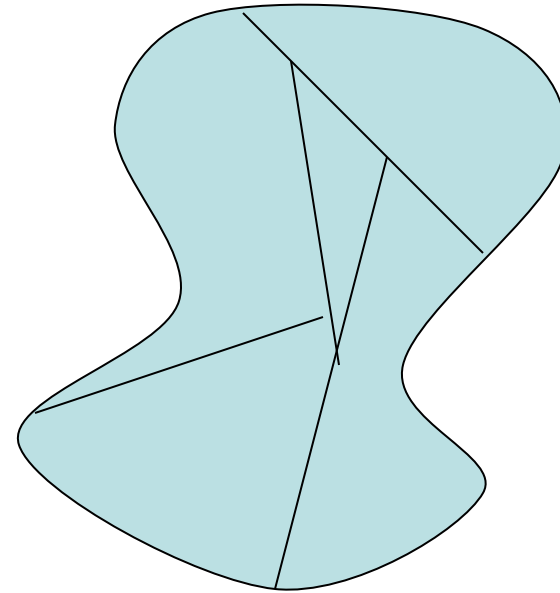


- Solving POMDPs
  - One way: use truncated expectimax to compute approximate value of actions
  - What if you only considered busting or one sense followed by a bust?
  - You get a VPI-based agent!



# More Generally\*

- General solutions map belief functions to actions
  - Can divide regions of belief space (set of belief functions) into policy regions (gets complex quickly)
  - Can build approximate policies using discretization methods
  - Can factor belief functions in various ways
- Overall, POMDPs are very (actually PSACE-) hard
- Most real problems are POMDPs, but we can rarely solve them in general!



# Summary

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- Decision Networks
- Value of Information
- POMDPs

