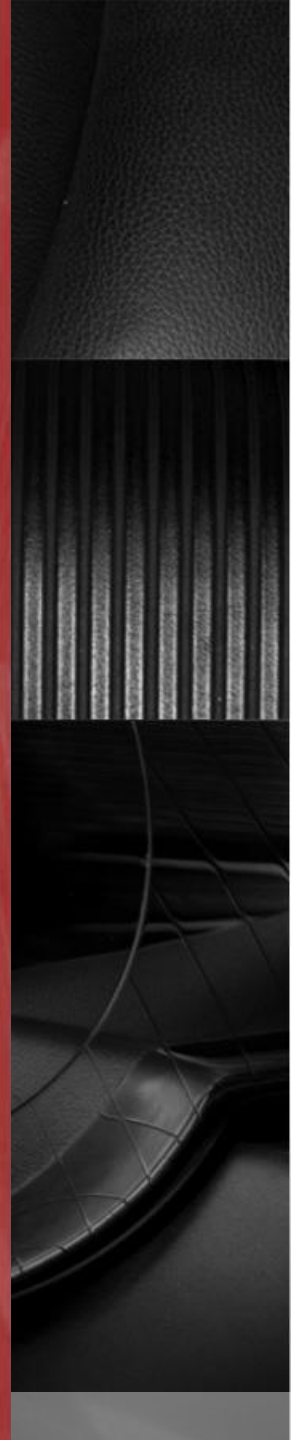
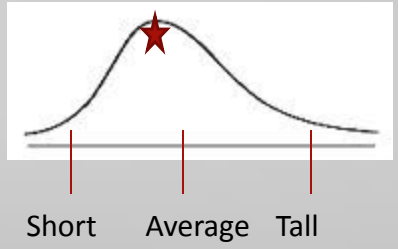
Fuzzy Logic



Fuzzy Logic

- Aristotle: A or (xor) not(A)
- Buddha: A and not(A)
- Example: My height
  - Ex-in-laws say I'm short
  - My family says I'm tall
  - Most people would say I'm on the short side of average



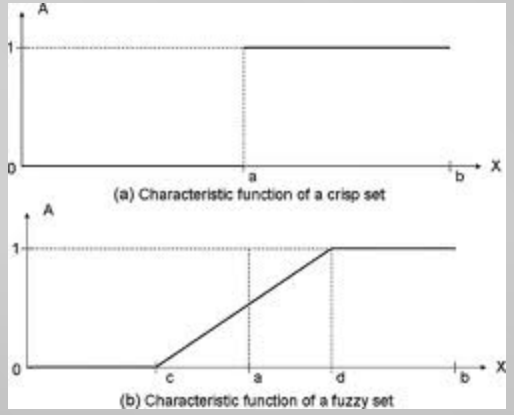
Fuzzy Logic

- Rather than a fact being either 1 or 0, true or false, fuzzy logic allows partial values, represented by real numbers, to indicate the possibility of truth or falsity
- Degrees of membership rather than crisp membership

Fuzzy Sets

- Membership Functions
  - Classical set theory is crisp

    x ϵ X OR x not ϵ X, but not both
  - Called the principle of dichotomy
- Membership functions (fuzzy) or Characteristic functions (crisp)



(a) Characteristic function of a crisp set

(b) Characteristic function of a fuzzy set

Fuzzy Sets

- Linguistic Variables and Hedges
  - A linguistic variable is a fuzzy variable

    If age is young
    And previous_accepts are several
    Then life_ins_accept is high

    There are 3 linguistic variables here:
    Age
    Previous_accepts
    Life_ins_accept

Fuzzy Sets

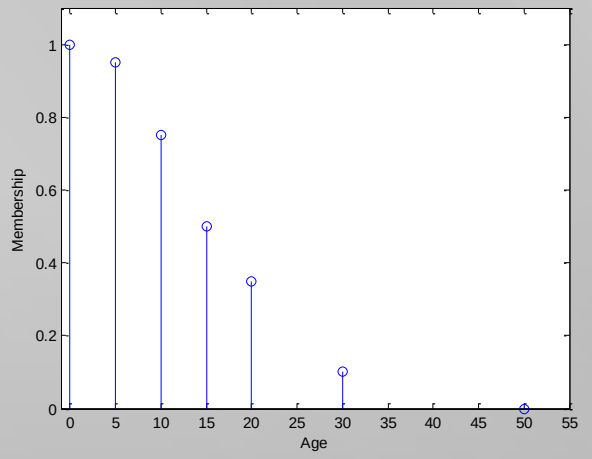
- Linguistic Variables and Hedges
  - We saw a continuous membership function a minute ago
  - Here is one way of defining a discrete membership function for age:

    Age is young:
    {(0/1.0), (5/0.95), (10/0.75), (15/0.50), (20/0.35), (30/0.10), (50/0.0)}

    x/y: x is the value for age, y is the degree of set membership

    Note: some books put this as $\mu_A(x)/x$, with the degree of membership first ($\mu_A(x)$) and the attribute value second (x)

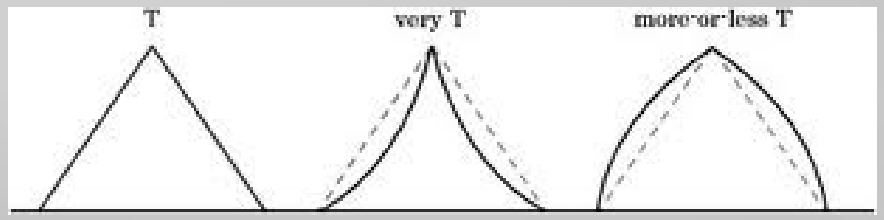
    To find out if a person is young or not, given an age not listed, interpolate between the values



Fuzzy Sets

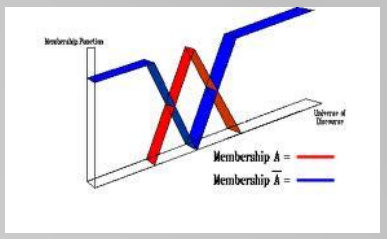
- Hedges:
  - All purpose modifiers: very, quite, extremely
  - Truth values: quite true, mostly false
  - Probabilities: likely, not very likely

    Roy's search and rescue rules – somewhat likely, etc.
  - Quantifiers: most, several, few
  - Possibilities: almost impossible, quite possible

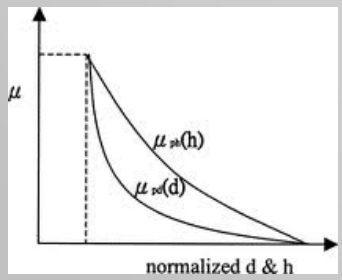


Fuzzy Sets

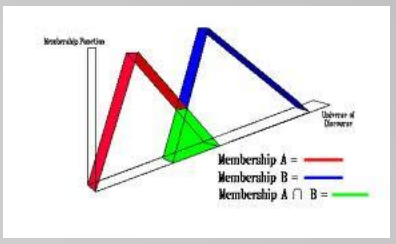
- Fuzzy Set Operations
  - Complement – $\mu_{\sim A}(x) = 1 - \mu_A(x)$



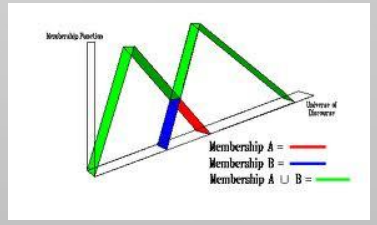
  - Containment – Elements of a subset vs. set will have lesser degrees of membership



  - Intersection - $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$



  - Union - $\mu_{AUB}(x) = \max[\mu_A(x), \mu_B(x)]$



Fuzzy Rules

- Crisp Rule:
  - If age < 30
    And previous _accepts > = 3
    Then life_ins_promo = yes
- Fuzzy Rules:
  - Rule 1: Accept is high
    If age is young
    And previous_accepts are several
    Then life_ins_accept is high
  - Rule 2: Accept is moderate
    If age is middle-aged
    And previous_accepts are some
    Then life_ins_promo is moderate
  - Rule 3: Accept is low
    If age is old
    Then life insurance accept is low
- May have multiple antecedent clauses, joined by ANDs and ORs
- May have multiple consequents – each one is affected equally by the antecedents

Fuzzy Inference

- The Process:
  - 1. Fuzzification
  - 2. Rule Inference
  - 3. Rule Composition
  - 4. Defuzzification

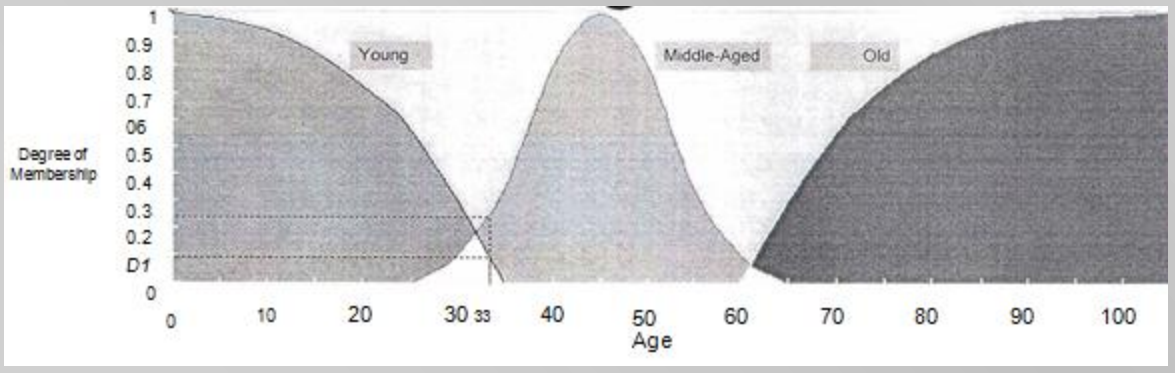
Fuzzy Inference

- Example:
  - Let's say we have a person who is 33 years old and has 5 previous accepts.

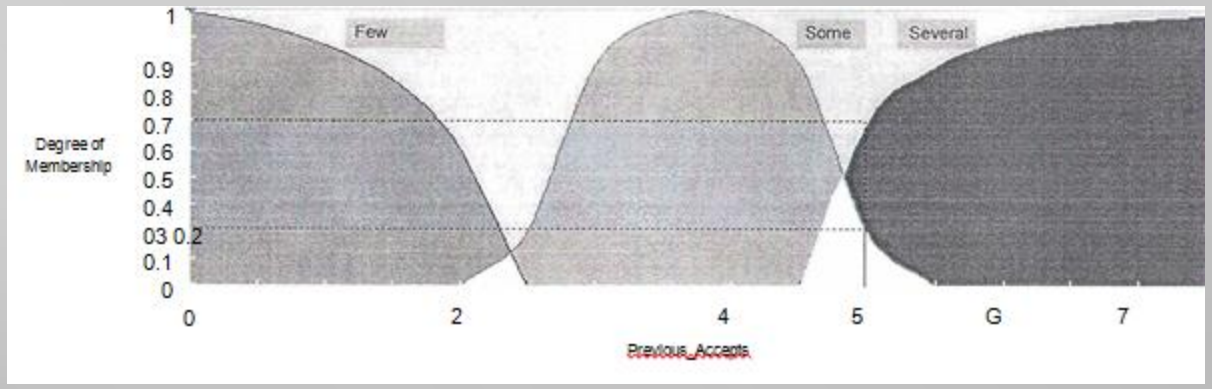
Table 13.1 . Life Insurance Promotion Data			
		Previous	Life Insurance
Instance #	Age	Accepts	Promotion
1	25	2	Yes
2	33	4	Yes
3	19	1	Yes
4	43	5	No
5	35	1	No
6	26	3	Yes
7	50	2	No
8	24	2	Yes
9	20	0	No
10	62	3	No
11	36	5	Yes
12	27	0	No
13	28	1	No
14	25	3	Yes

Fuzzification

- Define membership functions for all linguistic (fuzzy) variables:
  - Age

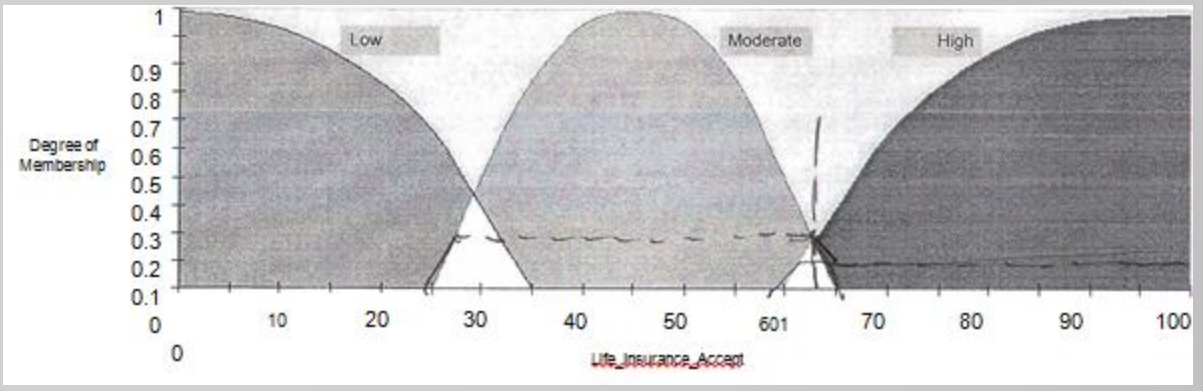


  - Previous_Accepts



Fuzzification

- Define membership functions for all linguistic (fuzzy) variables:
  - Life_Insurance_Accepts

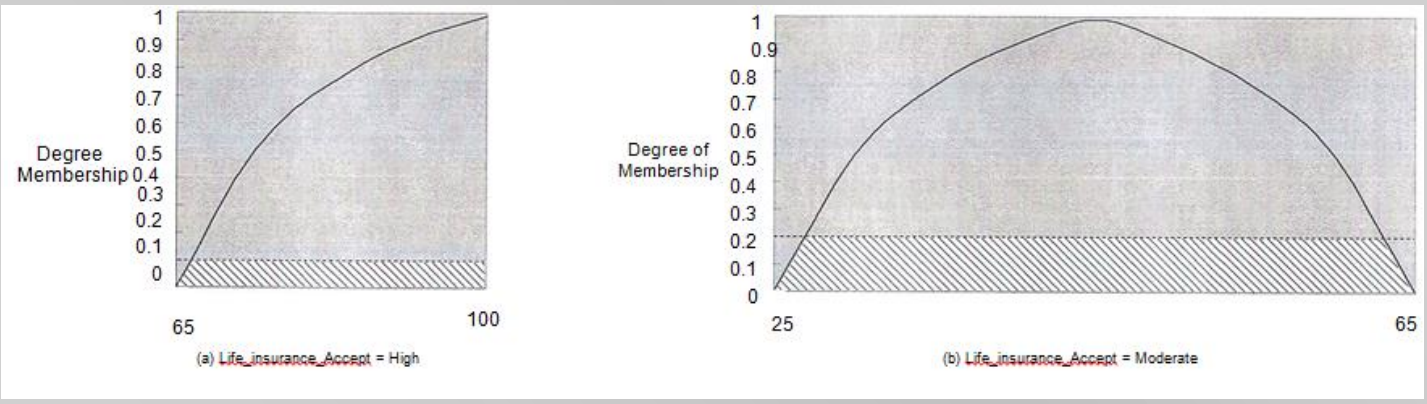


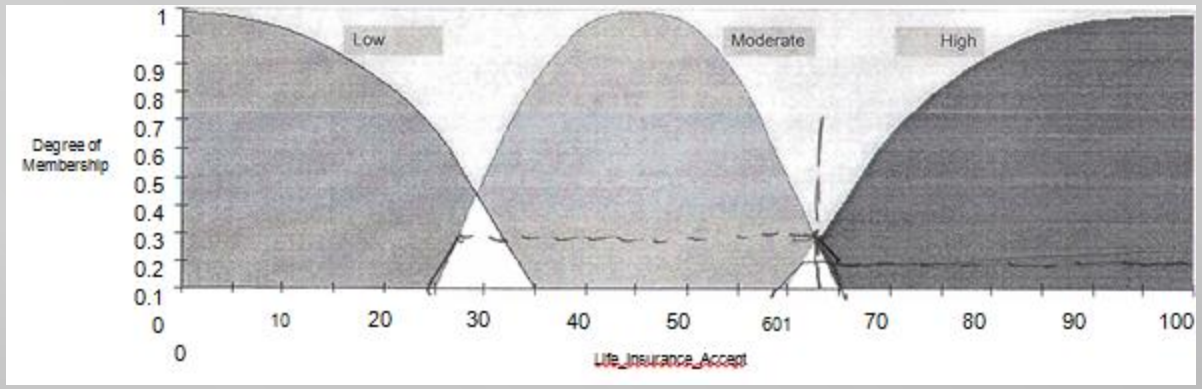
Rule Inference

- From our previous fuzzy rules… (Slide 9)
  - age = middle-aged (0.25)
young (0.10)
  - previous_accepts = some (0.20)
several (0.60)
  - Rule 1: age = young (0.10)
AND
prev_accepts = some (0.25)
  - These are ANDed, so use min:
0.10 degree of membership for life_ins = high
  - Rule 2: age = middle-aged (0.25)
AND
prev_accepts = some (0.20)
  - These are ANDed, so use min again:
0.20 degree of membership in life_ins = moderate
  - Rule 3: doesn't apply because there is no degree of membership for age = old

Rule Composition

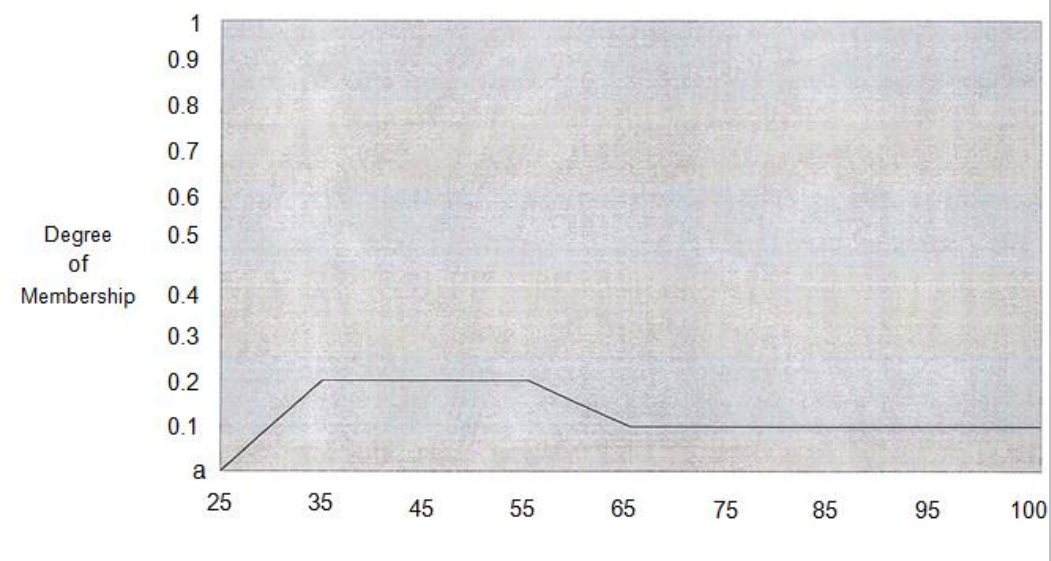
- Using the output of the fuzzy rules, and looking at the membership function for Life_Insurance_Accept we get the following graph:





Defuzzification

- Could use the largest value (max, or 0.20 in this case)
- OR
- Could compute the center of gravity (essentially the centroid, or mean)



Fuzzy Development Model

- Steps:
  - 1. Specify the problem and define linguistic variables
  - 2. Determine fuzzy sets and membership functions
  - 3. Elicit and construct fuzzy rules
  - 4. Encode fuzzy sets, rules, procedures
  - 5. Evaluate and tune the system

Fuzzy Logic Gone Wrong…

FIRST VILLAGER: We have found a witch. May we burn her?
ALL: A witch! Burn her!
BEDEVERE: Why do you think she is a witch?
SECOND VILLAGER: She turned me into a newt.
BEDEVERE: A newt?
SECOND VILLAGER (after looking at himself for some time): I got better.
ALL: Burn her anyway.
BEDEVERE: Quiet! Quiet! There are ways of telling whether she is a witch.
BEDEVERE: Tell me . . . what do you do with witches?
ALL: Burn them.
BEDEVERE: And what do you burn, apart from witches?
FOURTH VILLAGER: ... Wood?
BEDEVERE: So why do witches burn?
SECOND VILLAGER: (pianissimo) Because they're made of wood?
BEDEVERE: Good.
ALL: I see. Yes, of course.
BEDEVERE: So how can we tell if she is made of wood?
FIRST VILLAGER: Make a bridge out of her.
BEDEVERE: Ah . . . but can you not also make bridges out of stone?
ALL: Yes, of course. . . um . . . er . . .
BEDEVERE: Does wood sink in water?
ALL: No, no, it floats. Throw her in the pond.
BEDEVERE: Wait. Wait... tell me, what also floats on water?
ALL: Bread? No, no no. Apples... gravy. . . very small rocks. . .
BEDEVERE: No, no no,
KING ARTHUR: A duck!
(They all turn and look at ARTHUR. BEDEVERE looks up very impressed.) BEDEVERE: Exactly. So . . . logically. . .
FIRST VILLAGER (beginning to pick up the thread): If she. . . weighs the same as a duck. . . she's made of wood.
BEDEVERE: And therefore?
ALL: A witch!