

### Converting sentences from original problem to first order logic:

$\forall x, \text{kitten}(x) \wedge \text{lovesFish}(x) \Rightarrow \text{teachable}(x)$   
 $\forall x, \text{kitten}(x) \wedge \neg \text{tail}(x) \Rightarrow \neg \text{playsWithGorilla}(x)$   
 $\forall x, \text{kitten}(x) \wedge \text{whiskers}(x) \Rightarrow \text{lovesFish}(x)$   
 $\forall x, \text{kitten}(x) \wedge \text{teachable}(x) \Rightarrow \neg \text{greenEyes}(x)$   
 $\forall x, \text{kitten}(x) \wedge \text{whiskers}(x) \Rightarrow \text{tail}(x)$

Contrapositive from above:

$\forall x, \neg \text{teachable}(x) \Rightarrow \neg(\text{kitten}(x) \wedge \text{lovesFish}(x))$   
 $\forall x, \text{playsWithGorilla}(x) \Rightarrow \neg(\text{kitten}(x) \wedge \neg \text{tail}(x))$   
 $\forall x, \neg \text{lovesFish}(x) \Rightarrow \neg(\text{kitten}(x) \wedge \text{whiskers}(x))$   
 $\forall x, \text{greenEyes}(x) \Rightarrow \neg(\text{kitten}(x) \wedge \text{teachable}(x))$   
 $\forall x, \neg \text{tail}(x) \Rightarrow \neg(\text{kitten}(x) \wedge \text{whiskers}(x))$

Using DeMorgan's on the consequents of the above implications:

$\forall x, \neg \text{teachable}(x) \Rightarrow \neg \text{kitten}(x) \vee \neg \text{lovesFish}(x)$   
 $\forall x, \text{playsWithGorilla}(x) \Rightarrow \neg \text{kitten}(x) \vee \text{tail}(x)$   
 $\forall x, \neg \text{lovesFish}(x) \Rightarrow \neg \text{kitten}(x) \vee \neg \text{whiskers}(x)$   
 $\forall x, \text{greenEyes}(x) \Rightarrow \neg \text{kitten}(x) \vee \neg \text{teachable}(x)$   
 $\forall x, \neg \text{tail}(x) \Rightarrow \neg \text{kitten}(x) \vee \neg \text{whiskers}(x)$

All of these are already in Horn form, so we can do forward and backward chaining. Rewriting the reduced and valid rules and numbering them, so we can refer to them easily, the list becomes:

1.  $\forall x, \text{kitten}(x) \wedge \text{lovesFish}(x) \Rightarrow \text{teachable}(x)$
2.  $\forall x, \text{kitten}(x) \wedge \neg \text{tail}(x) \Rightarrow \neg \text{playsWithGorilla}(x)$
3.  $\forall x, \text{kitten}(x) \wedge \text{whiskers}(x) \Rightarrow \text{lovesFish}(x)$
4.  $\forall x, \text{kitten}(x) \wedge \text{teachable}(x) \Rightarrow \neg \text{greenEyes}(x)$
5.  $\forall x, \text{kitten}(x) \wedge \text{whiskers}(x) \Rightarrow \text{tail}(x)$
6.  $\forall x, \neg \text{teachable}(x) \Rightarrow \neg \text{kitten}(x) \vee \neg \text{lovesFish}(x)$
7.  $\forall x, \text{playsWithGorilla}(x) \Rightarrow \neg \text{kitten}(x) \vee \text{tail}(x)$
8.  $\forall x, \neg \text{lovesFish}(x) \Rightarrow \neg \text{kitten}(x) \vee \neg \text{whiskers}(x)$
9.  $\forall x, \text{greenEyes}(x) \Rightarrow \neg \text{kitten}(x) \vee \neg \text{teachable}(x)$
10.  $\forall x, \neg \text{tail}(x) \Rightarrow \neg \text{kitten}(x) \vee \neg \text{whiskers}(x)$

Given:

$\exists x, \text{kitten}(x) \wedge \text{greenEyes}(x) \wedge \neg \text{lovesFish}(x)$

Prove:

$\neg \text{teachable}(x)$

### **Forward Chaining:**

Facts:

11. kitten(x)
12. greenEyes(x)
13.  $\neg$ lovesFish(x)

Scanning the list of rules from top to bottom, the first rule antecedent (left hand side of the implication) that matches any facts is rule 8. From that we can conclude  $\neg$ whiskers(x) because the consequent says  $\neg$ kitten(x) or  $\neg$ whiskers(x), and we know that  $\neg$ kitten(x) is false. So we add  $\neg$ whiskers(x) to the fact base:

14.  $\neg$ whiskers(x)

Scanning again, Rule 9 matches. Can conclude  $\neg$ teachable(x):

15.  $\neg$ teachable(x)

And we've proved it.

### **Backward Chaining:**

Facts:

11. kitten(x)
12. greenEyes(x)
13.  $\neg$ lovesFish(x)

Since we are trying to prove  $\neg$ teachable(x), we look for rules that have that in the consequent (right hand side of the implication). Rule 9 does this. We look at the antecedent(s) to see what we need to prove next. The antecedent says greenEyes(x). This is given, so  $\neg$ teachable(x) is proved.

### **Resolution:**

To do resolution, we need to convert this to conjunctive normal form (CNF). Rules 1 through 5 become the statements:

1.  $\neg(\text{kitten}(x) \wedge \text{lovesFish}(x)) \vee \text{teachable}(x)$
2.  $\neg(\text{kitten}(x) \wedge \neg\text{tail}(x)) \vee \neg\text{playsWithGorilla}(x)$
3.  $\neg(\text{kitten}(x) \wedge \text{whiskers}(x)) \vee \text{lovesFish}(x)$
4.  $\neg(\text{kitten}(x) \wedge \text{teachable}(x)) \vee \neg\text{greenEyes}(x)$
5.  $\neg(\text{kitten}(x) \wedge \text{whiskers}(x)) \vee \text{tail}(x)$

Oddly (or perhaps not so oddly, since this is logic, after all), the translation of the rules from the problem and their contrapositives results in the same set of statements. The statements above still need to be flattened into conjunctive form, so by using DeMorgan's, they become:

1.  $\neg\text{kitten}(x) \vee \neg\text{lovesFish}(x) \vee \text{teachable}(x)$
2.  $\neg\text{kitten}(x) \vee \text{tail}(x) \vee \neg\text{playsWithGorilla}(x)$
3.  $\neg\text{kitten}(x) \vee \neg\text{whiskers}(x) \vee \text{lovesFish}(x)$
4.  $\neg\text{kitten}(x) \vee \neg\text{teachable}(x) \vee \neg\text{greenEyes}(x)$
5.  $\neg\text{kitten}(x) \vee \neg\text{whiskers}(x) \vee \text{tail}(x)$

We are still given:

6.  $\text{kitten}(x)$
7.  $\text{greenEyes}(x)$
8.  $\neg\text{lovesFish}(x)$

And want to prove:

$\neg\text{teachable}(x)$

So with proof by contradiction, we add the opposite,  $\text{teachable}(x)$ , to the knowledge base:

9.  $\text{teachable}(x)$

The resolution process:

We are told  $\text{kitten}(x)$ , so we can resolve that with all of the clauses, 1 through 5, and we get:

1.  $\neg\text{lovesFish}(x) \vee \text{teachable}(x)$
2.  $\text{tail}(x) \vee \neg\text{playsWithGorilla}(x)$
3.  $\neg\text{whiskers}(x) \vee \text{lovesFish}(x)$
4.  $\neg\text{teachable}(x) \vee \neg\text{greenEyes}(x)$
5.  $\neg\text{whiskers}(x) \vee \text{tail}(x)$
6.  $\text{kitten}(x)$
7.  $\text{greenEyes}(x)$
8.  $\neg\text{lovesFish}(x)$
9.  $\text{teachable}(x)$

Resolve clause 7,  $\text{greenEyes}(x)$ , with clause 4, and that results in:

10.  $\neg\text{teachable}(x)$

Resolve 10 with 9, and we get the empty set. Therefore, the proof by contradiction holds, and  $\neg\text{teachable}(x)$  has been proven by resolution.

NOTE:

To be really correct, I should have substituted a constant in for  $x$  in  $\text{kitten}(x)$ , say, *Scruffy*. Then all of the above would have used the (Skolem) constant as a substitution for  $x$ . Or I can just pretend my kitten is named " $x$ ". In this example it doesn't matter because all variables relate to the same object, the kitten. But in other problems, it could make a difference.