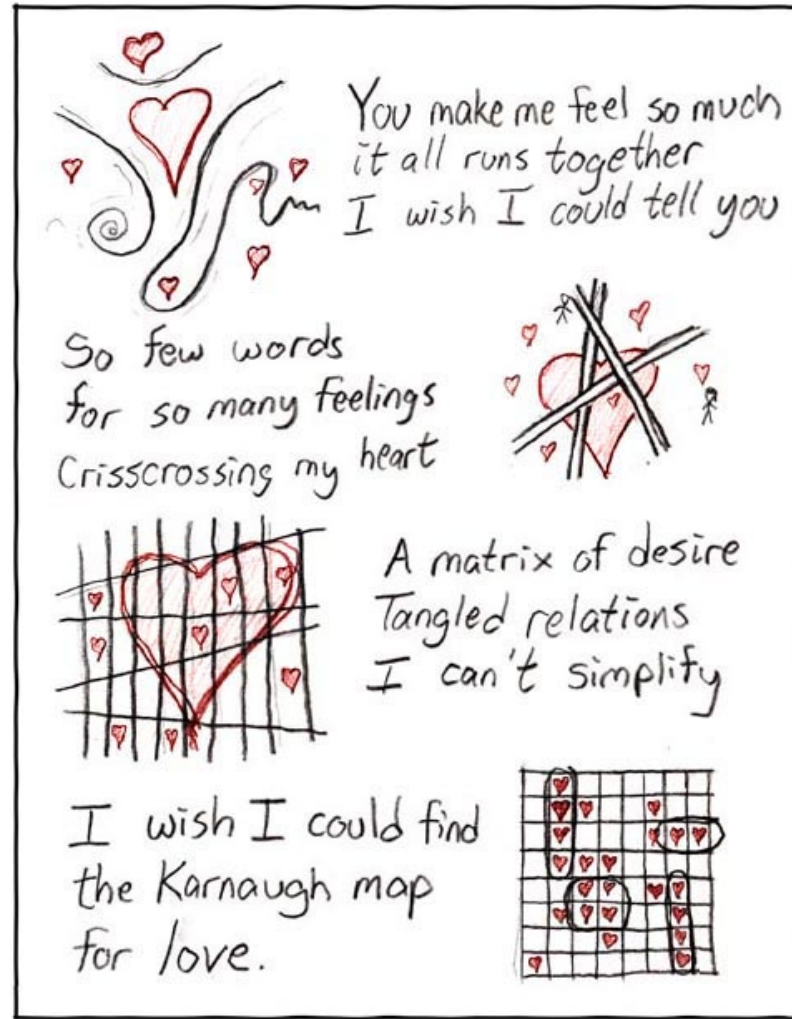


Karnaugh maps



<http://xkcd.com/62/>

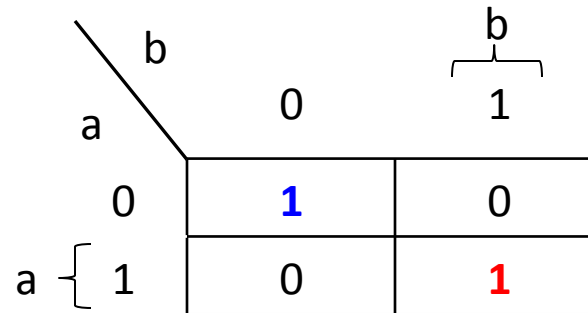
Overview

- Last time:
 - Simplifying Boolean expressions **using rules**
 - Finding reductions can be hard!
- This time:
 - **Karnaugh map**
 - Graphical method for Boolean simplification
 - **Truth table → map → Boolean expression**

Maps

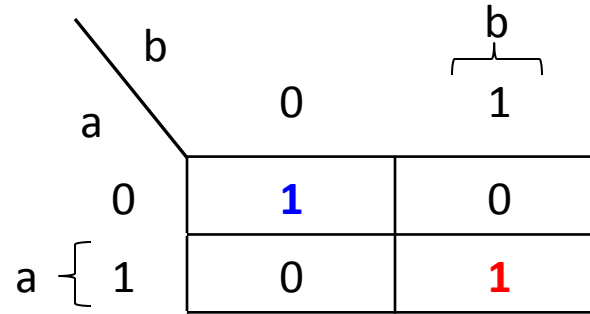
- Given n Boolean input variables
 - 2^n rows in the truth table
- Rearrange 2^n output cells into a grid

a	b	F(a,b)
0	0	1
0	1	0
1	0	0
1	1	1

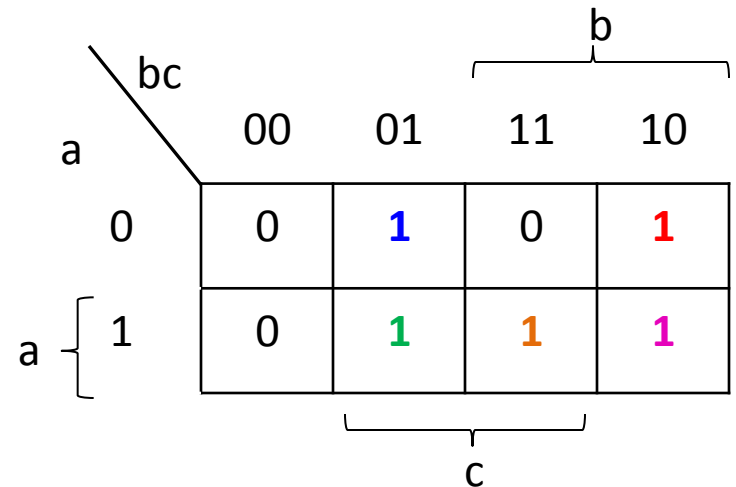


2 and 3-variable maps

a	b	F(a,b)
0	0	1
0	1	0
1	0	0
1	1	1

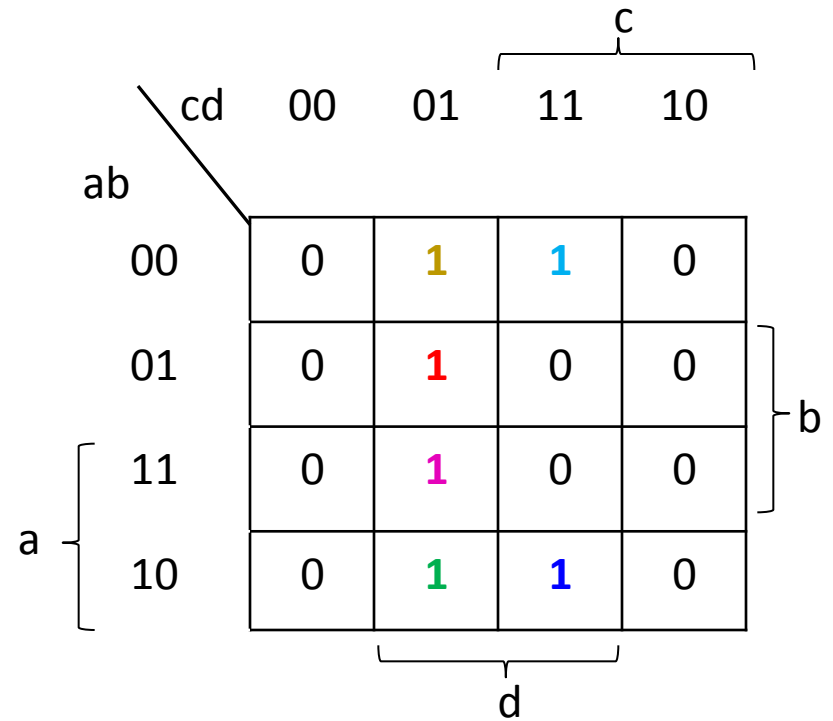


a	b	c	F(a,b,c)
0	0	0	0
0	0	1	1 (blue)
0	1	0	1 (red)
0	1	1	0
1	0	0	0
1	0	1	1 (green)
1	1	0	1 (orange)
1	1	1	1 (purple)



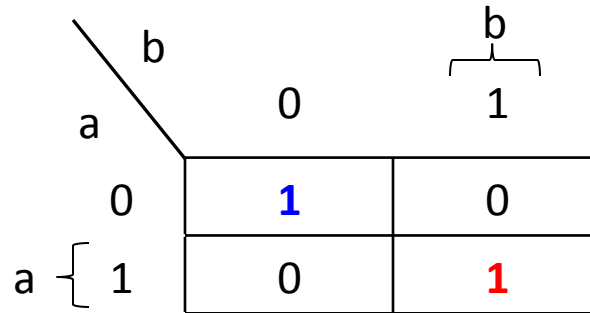
4-variable map

a	b	c	d	F(a,b,c,d)
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0



Find biggest groups

- Circle all ones
 - Use biggest groups of size 2^n
- Cyclic wrap around top and bottom



Find biggest groups

- Circle all ones
 - Use biggest groups of size 2^n
- Cyclic wrap around top and bottom

		b	
		0	1
a	0	1	0
	1	0	1

$$F(a,b) = a' b' + ab$$

Find biggest groups

- Circle all ones
 - Use biggest groups of size 2^n
- Cyclic wrap around top and bottom

	bc			
	00	01	11	10
a				
0	0	1	0	1
1	0	1	1	1

Find biggest groups

- Circle all ones
 - Use biggest groups of size 2^n
- Cyclic wrap around top and bottom

		b			
		00	01	11	10
a	0	0	1	0	1
	1	0	1	1	1

The diagram shows a 2x4 Karnaugh map for variables a, b, and c. The columns are labeled with bc values: 00, 01, 11, 10. The rows are labeled with a values: 0, 1. The cells containing '1' are at (a=0, bc=01), (a=0, bc=10), (a=1, bc=01), (a=1, bc=11), and (a=1, bc=10). These ones are grouped into three groups: a blue circle around the two ones in the bc=01 column; a green circle around the two ones in the a=1 row; and a red circle around the two ones in the bc=10 column. Brackets labeled 'b' and 'c' indicate the column and row groupings respectively.

$$F(a,b,c) = ab + b'c + bc'$$

Find biggest groups

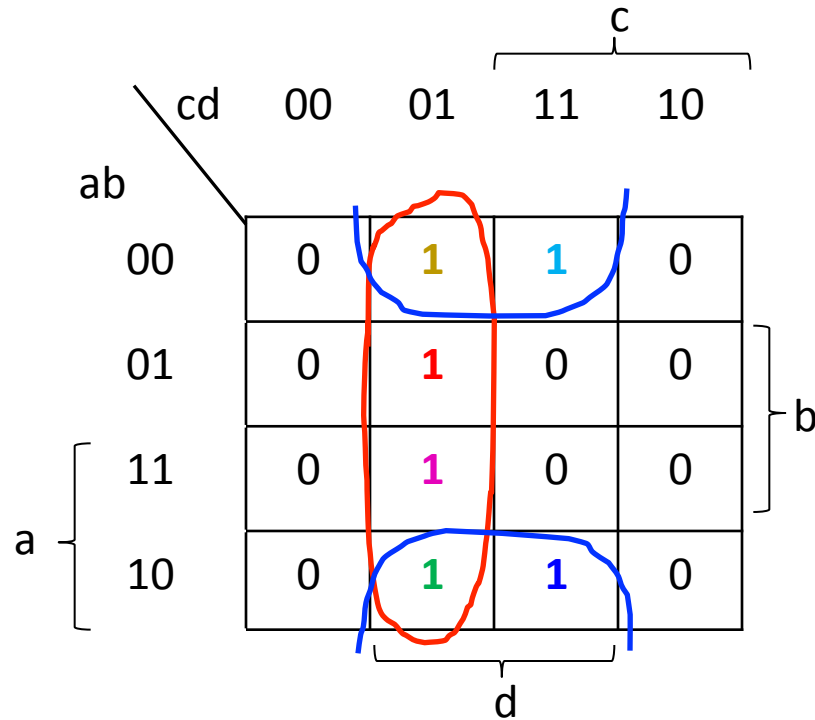
- Circle all ones
 - Use biggest groups of size 2^n
- Cyclic wrap around top and bottom

A Karnaugh map for the function F(a,b,c). The map is a 2x4 grid with rows labeled 'a' (0 and 1) and columns labeled 'bc' (00, 01, 11, 10). The cells contain values: (0,00)=0, (0,01)=1, (0,11)=0, (0,10)=1; (1,00)=0, (1,01)=1, (1,11)=1, (1,10)=1. Three groups are circled: a blue circle around (0,01) and (1,01); a green circle around (1,01) and (1,11); and a red circle around (0,10) and (1,10). Brackets indicate the dimensions of these groups: 'b' for the top row, 'c' for the bottom row, and '1' for the left column.

a \ bc	00	01	11	10
0	0	1	0	1
1	0	1	1	1

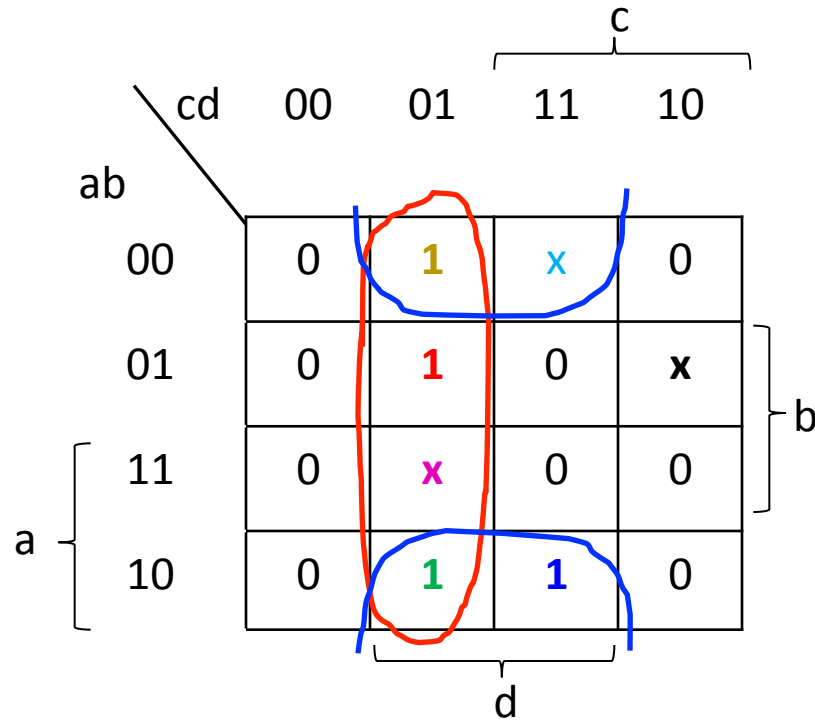
$$F(a,b,c) = ac + b'c + bc'$$

Find biggest groups



$$F(a,b,c,d) = b' d + c' d$$

Don't care conditions



$$F(a,b,c,d) = b' d + c' d$$

Don't care conditions

	c			
cd	00	01	11	10
ab				
00	x	0	0	0
01	x	0	x	x
11	1	0	1	1
10	1	0	0	0

$$F(a,b,c,d) = bc + c'd'$$

Summary

- Karnaugh maps
 - Graphical method for Boolean simplification
 - Relies on our natural **pattern matching** ability
 - **Truth table → map → Boolean expression**
 - Generally **much easier than using rules**
 - **Good for 2-4 variables**
 - Beyond that, hard to see patterns
 - Other algorithms / software