Indeterminant Form

$\lim\limits_{x \to c} f(x) = 0$ and $\lim\limits_{x \to c} g(x) = 0$

then

$\lim\limits_{x \to c} \frac{f(x)}{g(x)}$ is a $\frac{0}{0}$ indeterminate form

example $f(x) = x^2 - 4$ $g(x) = x - 2$

$$\lim_{x \to 2} \frac{f(x)}{g(x)} = \lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{\lim_{x \to 2} x^2-4}{\lim_{x \to 2} x-2} \to \frac{0}{0} \text{ indeterminate form}$$

If we have $\frac{0}{0}$ indeterminate form we can try to simplify

$$\lim_{x \to 2} \frac{x^2-4}{x-2} \overset{\text{factor}}{=} \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \to 2} (x+2) = 4$$

If we don't get $\frac{0}{0}$ the limit does not exist

$\lim\limits_{x \to c} g(x) = 0$

$\lim\limits_{x \to c} f(x) = L$ $L \neq 0$ then $\lim\limits_{x \to c} \frac{f(x)}{gx}$ does not exist

Is the limit a $\frac{0}{0}$ indeterminate?

What is the limit

$$\lim_{x \to 1} \frac{x-1}{x^2+1} \qquad \frac{\lim_{x \to 1} x-1}{\lim_{x \to 1} x^2+1} = \frac{0}{2} = \boxed{0}$$

Limit $= 0$

Not $\frac{0}{0}$ indeterminate

$$\lim_{x \to 1} \frac{x-1}{x^2-1} \qquad \frac{\lim_{x \to 1} x-1}{\lim_{x \to 1} x^2-1} = \frac{0}{0}$$

Yes, it is $\frac{0}{0}$ indeterminate form

Limit $= \frac{1}{2}$

so try to simplify

$$\frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)} = \frac{1}{x+1} \qquad \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

$$\lim_{x \to 1} \frac{x+1}{x^2-1} = \frac{\lim_{x \to 1} x+1}{\lim_{x \to 1} x^2-1} = \frac{2}{0}$$

No, not $\frac{0}{0}$ indeterminate

Limit Does not exist

limit of the difference quotient

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{if} \quad \lim_{h \to 0} [f(a+h) - f(a)] = 0$$

$f(x) = 4x - 5 \quad a = 3$

$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$\lim_{h \to 0} \frac{[4(3+h) - 5] - [4(3) - 5]}{h}$$

$$\lim_{h \to 0} \frac{12 + 4h - 5 - 12 + 5}{h}$$

$$\frac{\lim_{h \to 0} 4h}{\lim_{h \to 0} h} = \frac{0}{0} \checkmark$$

$\lim_{h \to 0} \frac{4h}{h}$ denominator 0 check if $\frac{0}{0}$ indeterminate

$$\lim_{h \to 0} = 4 = \boxed{4}$$