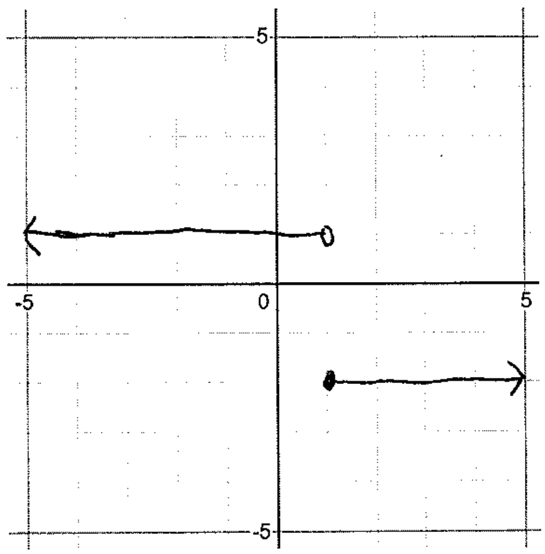
M142 Math for Bus. and Soc. Sci. II
Spring 2022 Practice Test 1 Name(First and Last)________________________

Academic Dishonesty: The possession of any unauthorized materials (e.g. calculators, cell phones, headphones, etc.) will result in you receiving a 0 on this exam. Show all of your work and justify all of your conclusions.

1. Use the graph of the function f shown to estimate the indicated limits and the function value.
Write "does not exist" if the limit exist.
Write "does not have a value" if the function does not have a value.



a. $\lim_{x\to 1^-} f(x) =$ 1

b. $\lim_{x\to 1^+} f(x) =$ −2

c. $\lim_{x\to 1} f(x) =$ Does not exist

d. $f(x) =$ −2

added justification

e. Is the function continuous at x = 1 justify

$\boxed{\text{No, } \lim_{x\to 1} f(x) \text{ DNE}}$

f. $\lim_{x\to -2^-} f(x) =$ 1

g. $\lim_{x\to -2^+} f(x) =$ 1

h. $\lim_{x\to -2} f(x) =$ 1

i. $f(-2) =$ 1

j. Is the function continuous at x = 2 justify

$\boxed{\text{YES, } \lim_{x\to -2} = f(-2)}$

2. Find the given limit or write does not exist.

$$\lim_{x\to -3} \frac{x^2+1}{x-2}$$

$$\frac{\lim\limits_{x\to -3} x^2+1}{\lim\limits_{x\to -3} x-2} = \frac{(-3)^2+1}{(-3)-2} = \frac{10}{-5} = -2$$

3. Given the limit $\lim\limits_{x\to -3} \sqrt[2]{x+2}$

a. Circle the correct expression after the limit properties have been applied.

i. $\sqrt[2]{\lim\limits_{x\to -3} x+2}$

ii. $\sqrt[2]{\lim\limits_{x\to -3} x+\lim\limits_{x\to -3} 2}$

iii. $2(\lim\limits_{x\to -3} x+\lim\limits_{x\to -3} 2)$

iv. $2\left(\lim\limits_{x\to -3} x+2\right)$

$\sqrt[2]{\lim\limits_{x\to -3}(x+2)}$

$\sqrt[2]{\lim\limits_{x\to -3} x+\lim\limits_{x\to -3} 2}$

4. Let $f(x) = \frac{x^2+2x+1}{(x+1)}$

Find the limit $\lim_{x\to -1} f(x)$ if it exists. If it does not exist write it does not exist.

Step 1

$$\lim_{x\to -1} \frac{x^2+2x+1}{(x+1)} \qquad \frac{\lim_{x\to -1} x^2+2x+1}{\lim_{x\to -1} x+1} = \frac{(-1)^2+2(-1)+1}{(-1)+1} = \frac{0}{0} \checkmark$$

Step 2

$$\frac{x^2+2x+1}{x+1} = \frac{(x+1)(x+1)}{(x+1)} = x+1$$

Step 3

$$\lim_{x\to -1} x+1 = (-1)+1 = \boxed{0}$$

5. Given $f(x) = \frac{2x^4+x^2+2x+1}{3x^3+x-1}$

a. Find the limit $\lim_{x\to\infty} f(x)$

$$\frac{2x^4}{3x^3} = \frac{2x}{3} \qquad \lim_{x\to\infty} \frac{2x}{3} = \infty$$

b. Find the limit $\lim_{x\to-\infty} f(x)$

$$\lim_{x\to-\infty} \frac{2x}{3} = -\infty$$

c. If there is a horizontal asymptote finish the equation below. If not leave it blank.

y = ______

limits at ∞ and $-\infty$ go to ∞ and $-\infty$, no horizontal asymptote

Once you have simplified your biggest powers
if you get a fraction with no x that is your horz asymptote eg $\frac{3}{2}$

if you get a fraction with x in the denominator, horiz asymptote is 0

6. Given the definition for the rate of change from the text book

DEFINITION Average Rate of Change

For $y = f(x)$, the average rate of change from $x = a$ to $x = a + h$ is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h} \qquad h \neq 0 \qquad (1)$$

Explain what the following limit tells us in terms of slope and rate of change

$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

That limit gives the derivative of f(x) evaluated at a.

The derivative gives the slope of the tangent line at a point which is the same as the instintanous rate of change at a point.

7. Find the following derivatives

$f(x) = x^2$

$f'(x) = 2x^{2-1} = 2x$

a. $f(x) = x^2$ $\qquad f'(x) = 2x$

$f(x) = \frac{1}{x^3} = x^{-3}$

$f'(x) = -3x^{-3-1} = -3x^{-4}$ or $\frac{-3}{x^4}$

b. $f(x) = \frac{1}{x^3}$ $\qquad f'(x) = -3x^{-4}$ or $-\frac{3}{x^4}$

either will be accepted

c. $f(x) = x^3 + x^2 - x^{1/2}$ $\qquad f'(x) = \frac{1}{2x^{\frac{1}{2}}}$

$3x^{3-1} = 3x^2 \qquad 2x^{2-1} = 2x \qquad \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{1}{2x^{\frac{1}{2}}}$

either

8. A child is making lemonade to sell at his lemonade stand. Their older sister is working on a math assignment to figure out how the average cost and marginal average cost if 20 cups of lemonade are produced, as well as estimate the average cost per cup if 21 were made using the previous information. The cost function in terms of cups made is C(x) = 5 + 2x

A. Find the average cost per cup if 20 cups are made.

$$\bar{C}(x) = \frac{C(x)}{x} = \frac{5+2x}{x}$$

$$\bar{C}(20) = \frac{5+2(20)}{20} = \frac{45}{20}$$

$$f(x) = \frac{5}{x} = 5x^{-1}$$

$$f'(x) = 5(-1)x^{-2} = -\frac{5}{x^2}$$

B. Find the marginal cost at a production level of 20 cups.

$$\bar{C}'(x) = \frac{d}{dx}\frac{5+2x}{x} = \frac{dy}{dx}\frac{5}{x}+2 = \frac{d}{dx}\frac{5}{x} + \frac{d}{dx}2 = -\frac{5}{x^2}+0$$

$$\bar{C}'(20) = -\frac{5}{20^2} = -\frac{5}{400}$$

$$f(x) = 2$$

$$f'(x) = 0$$

C. Use the results from parts (A) and (B) to estimate the average cost per cup if 21 cups were produced.

$$\bar{C}(20) + \bar{C}'(20$$

$$\boxed{\frac{45}{20} - \frac{5}{400}}$$

nice numbers or you dont have to simplify :)