

Academic Dishonesty: The possession of any unauthorized materials (e.g. calculators, cell phones, headphones, etc.) will result in you receiving a 0 on this exam. Show all of your work and justify all of your conclusions.

1. Find the Distance and Midpoint between (x_1, y_1) and (x_2, y_2) $(5, 1)$ and $(2, 3)$.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 5)^2 + (3 - 1)^2} = \sqrt{9 + 4} = \sqrt{13}$$

Distance

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{5 + 2}{2}, \frac{1 + 3}{2} \right) = \left(\frac{7}{2}, 2 \right)$$

2. Evaluate the quotient and write the result in the form $a + bi$.

$$\frac{3-i}{2-i}$$

$$i^2 = -1$$

complex conjugate

$$\frac{3-i}{2-i} \cdot \frac{(2+i)}{(2+i)} = \frac{6 + 3i - 2i - i^2}{4 - 2i + 2i - i^2} = \frac{6 + i - (-1)}{4 - (-1)} = \frac{7+i}{5} = \frac{7}{5} + \frac{1}{5}i$$

FOIL

3. The given equation involves a power of the variable. Find all real solutions of the equation.

a. $x^3 - 7 = 20$

$$x^3 = 27$$

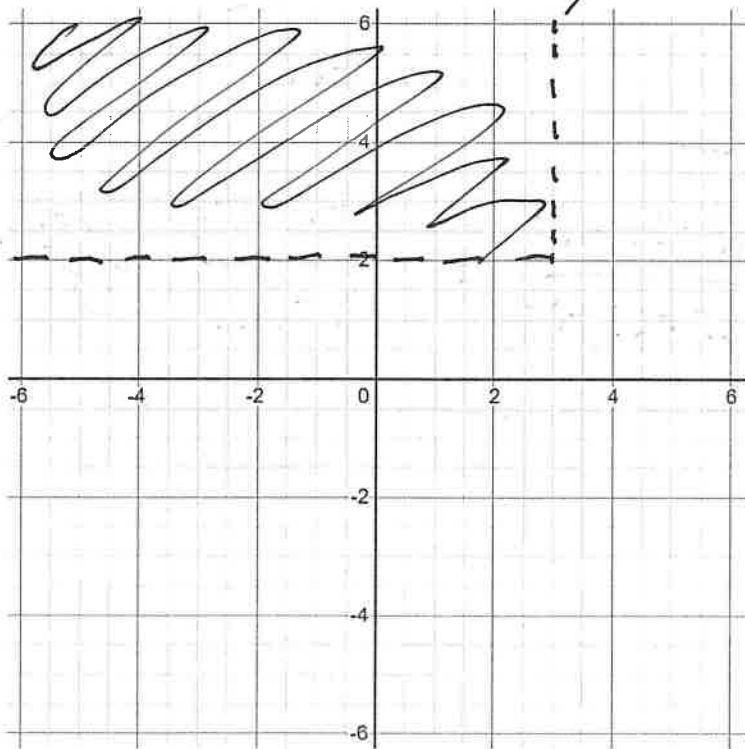
$$x = \sqrt[3]{27}$$

b. $x^2 = 9$

$$\sqrt{x^2} = \pm \sqrt{9}$$

$$x = \pm 3$$

4. Sketch the region given by the set.
 $\{(x, y) \mid x < 3 \text{ and } y > 2\}$



dotted because $<$ not \leq
 $x \text{ cant} = 3$

5. Given the line $y = \frac{1}{2}x + 2$

a. What is the slope? ^{slope}

$$y = m x + b$$

$$y = \left(\frac{1}{2}\right)x + 2$$

$$\boxed{\frac{1}{2}}$$

b. What is the y-intercept

$$y = m x + b \text{ --- y-intercept}$$

$$y = \frac{1}{2}x + \boxed{2}$$

$$\boxed{2}$$

6. Find the equation of the line through points P(2,-2) and Q(3,4)

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{3 - 2} = \frac{6}{1} = 6$$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = 6(x - 2)$$

$$\boxed{y + 2 = 6(x - 2)}$$

$$y + 2 = 6x - 12$$

$$\boxed{y = 6x - 14}$$

$$\text{or } \boxed{y - 6x + 14 = 0}$$

Ok, but if it asks for general or slope intercept keep going

7. Find all real solutions of the equation by completing the square. $x^2 + 8x + 7 = 0$

$$x^2 + 8x + 7 = 0$$

$$x^2 + 8x = -7$$

$$x^2 + 8x + \underline{16} = -7 + \underline{16}$$

$$(x+4)(x+4) = 9$$

$$\sqrt{(x+4)^2} = \sqrt{9}$$

$$x+4 = \pm\sqrt{9}$$

$$x = -4 \pm 3$$

$$x = -7 \text{ or } x = -1$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = 4^2 = 16$$

Check

$$(-7)^2 + 8(-7) + 7 = 0$$

$$49 - 56 + 7 = 0$$

$$0 = 0 \quad \checkmark$$

$$(-1)^2 + 8(-1) + 7 = 0$$

$$1 - 8 + 7 = 0$$

$$0 = 0 \quad \checkmark$$

8. Find all real solutions of the equation $x^4 - x^3 - 12x^2 = 0$

$$x^4 - x^3 - 12x^2 = 0$$

$$x^2(x^2 - x - 12) = 0$$

$$x^2(x-4)(x+3) = 0$$

$$x^2 = 0$$

$$(x-4) = 0$$

$$(x+3) = 0$$

$$x = 0$$

or

$$x = 4$$

or

$$x = -3$$

Check

$$\sqrt{x=0} \quad 0 - 0 - 0 = 0$$

$$\sqrt{x=4} \quad 256 - 64 - 192 = 0$$

$$\sqrt{x=-3} \quad 81 - (-27) - 12(9) = 108 - 108 = 0$$